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Laplace Transform

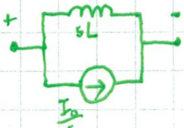
Laplace Transform $\mathcal{L}\{f(t)\} = \int_0^{\infty} f(t)e^{-st}\,dt$ Step Function $Ku(t-a)=0,\ t<a;\ Ku(t-a)=K,\ t>a$

Impulse $f'(0)\to\delta(t)$ as $\epsilon\to 0$ s.t. $\int_{-\infty}^{\infty} K\delta(t)\,dt = K$ and $\delta(t)=0,\ t\neq 0$ Shifting Property $\int_{-\infty}^{\infty} f(t)\delta(t-a)\,dt = f(a)$

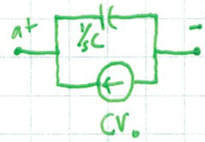
$\frac{du(t)}{dt} = \delta(t)$ Inverse transform complex $\mathcal{L}^{-1}\left\{\frac{K}{(s+\alpha-j\beta)^r} + \frac{K^*}{(s+\alpha+j\beta)^r}\right\} = \left[\frac{2|K|t^{r-1}}{(r-1)!}e^{-\alpha t}\cos(\beta t+\theta)\right]u(t)$ S-domain circuit

V: voltage-second. I: ampere-second. Resistor $V=IR$ Inductor $V=L[sI-i(0^-)]$ $\{v=L\frac{di}{dt}\}$

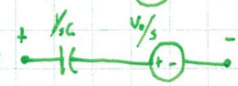
$I=\frac{V}{sL}+\frac{I_0}{s}$.



Capacitor $I=C[sV-v(0^-)]$ $\{i=C\frac{dv}{dt}\}$



$V=\frac{I}{sC}+\frac{V_0}{s}$



Inductor $V=j\omega LI = sLI$ Capacitor $V=\frac{1}{j\omega C}I=\frac{1}{sC}I$ Impedance $V=ZI$

Passive Filters

Lowpass (RL circuit w/ output @ R) $H(s)=\frac{R/L}{s+R/L}$ $|H(j\omega)|=\frac{R/L}{\sqrt{\omega^2+R^2/L^2}}$ $\theta(j\omega)=-\tan^{-1}\left(\frac{\omega L}{R}\right)$ $\omega_c=\frac{R}{L}$

RC circuit w/ output @ C $H(s)=\frac{\frac{1}{RC}}{s+\frac{1}{RC}}$ $|H(j\omega)|=\frac{\frac{1}{RC}}{\sqrt{\omega^2+(\frac{1}{RC})^2}}$ $\theta(j\omega)=-\tan^{-1}(\omega RC)$ $\omega_c=\frac{1}{RC}$

Highpass (RC circuit w/ output @ R) $H(s)=\frac{s}{s+\frac{1}{RC}}$ $|H(j\omega)|=\frac{\omega}{\sqrt{\omega^2+(\frac{1}{RC})^2}}$ $\theta(j\omega)=90^\circ-\tan^{-1}(\omega RC)$ $\omega_c=\frac{1}{RC}$

RL circuit w/ output @ L $H(s)=\frac{s}{s+R/L}$ $|H(j\omega)|=\frac{\omega}{\sqrt{\omega^2+(R/L)^2}}$ $\omega_c=\frac{R}{L}$

Loaded RL circuit $H(s)=\frac{Ks}{s+\omega_c}$, $K=\frac{R_L}{R+R_L}$, $\omega_c=KR/L$

Series RLC **Bandpass** output @ R $H(s)=\frac{(R/L)s}{s^2+(R/L)s+(1/LC)}$ $|H(j\omega)|=\frac{\omega(R/L)}{\sqrt{(1/LC-\omega^2)^2+(\omega R/L)^2}}$ $\theta(j\omega)=90-\tan^{-1}\left[\frac{\omega(R/L)}{(1/LC)-\omega^2}\right]$

$\omega_0=\sqrt{\frac{1}{LC}}$ $\omega_{c_{1,2}}=\mp\frac{R}{2L}+\sqrt{\left(\frac{R}{2L}\right)^2+\left(\frac{1}{LC}\right)}$ $\beta=\frac{R}{L}$

$\omega_0=\sqrt{\omega_{c_1}\cdot\omega_{c_2}}$ $Q=\frac{\omega_0}{\beta}=\frac{\sqrt{\omega_{c_1}\cdot\omega_{c_2}}}{\omega_{c_2}-\omega_{c_1}}=\sqrt{\frac{L}{CR^2}}$

Parallel RLC Bandpass output @ C||L $H(s)=\frac{\frac{s}{RC}}{s^2+\frac{s}{RC}+\frac{1}{LC}}$ $|H(j\omega)|=\frac{\frac{\omega}{RC}}{\sqrt{(\frac{1}{LC}-\omega^2)^2+(\frac{\omega}{RC})^2}}$ $\omega_0=\sqrt{\frac{1}{LC}}$, $\beta=\frac{1}{RC}$

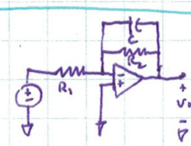
$\omega_{c_{1,2}}=\mp\frac{1}{2RC}+\sqrt{\left(\frac{1}{2RC}\right)^2+\frac{1}{LC}}$ $Q=\frac{\omega_0}{\beta}=\sqrt{\frac{R^2C}{L}}$

Bandreject RLC series (output @ C & L) $H(s)=\frac{s^2+\frac{1}{LC}}{s^2+\frac{R}{L}s+\frac{1}{LC}}$ $|H(j\omega)|=\frac{\left|\frac{1}{LC}-\omega^2\right|}{\sqrt{(\frac{1}{LC}-\omega^2)^2+(\frac{\omega R}{L})^2}}$ $\theta(j\omega)=-\tan^{-1}\left[\frac{\frac{\omega R}{L}}{\frac{1}{LC}-\omega^2}\right]$

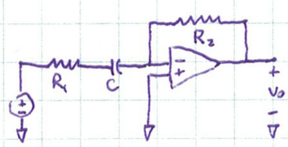
$\omega_0=\sqrt{\frac{1}{LC}}$ $\omega_{c_{1,2}}=\mp\frac{R}{2L}+\sqrt{\left(\frac{R}{2L}\right)^2+\frac{1}{LC}}$ $\beta=\frac{R}{L}$ $Q=\sqrt{\frac{L}{R^2C}}$

Active Filters

1st order lowpass filter $H(s)=\frac{-Z_f}{Z_i}=-K\frac{\omega_c}{s+\omega_c}$, $K=\frac{R_2}{R_1}$ $\omega_c=\frac{1}{R_2C}$



1st order highpass filter $H(s)=\frac{-Z_f}{Z_i}=-K\frac{s}{s+\omega_c}$, $K=\frac{R_2}{R_1}$, $\omega_c=\frac{1}{R_1C}$



Scaling $R'=k_mR$, $L'=\frac{k_m}{k_f}L$, $C'=\frac{1}{k_mk_f}C$

Broadband Bandpass $v_i\to$ [Low pass filter] $\to$ [High pass filter] $\to$ [Inverting Amplifier] $\to v_o$ $H(s)=\frac{-K\omega_{c_2}s}{s^2+(\omega_{c_1}+\omega_{c_2})s+\omega_{c_1}\omega_{c_2}}$ $|H(j\omega)|=\frac{R_f}{R_i}$

$\omega_{c_2}=\frac{1}{R_LC_L}$ $\omega_{c_1}=\frac{1}{R_HC_H}$ In series, multiply transfer functions

Bandreject $v_i\to$ [Low-pass filter] / [High pass filter] $\to$ [Summing amplifier] $\to v_o$ $H(s)=\frac{R_f}{R_i}\left[\frac{s^2+2\omega_{c_1}s+\omega_{c_1}\omega_{c_2}}{(s+\omega_{c_1})(s+\omega_{c_2})}\right]$

In parallel, add transfer functions

You should be able to derive all of them!!!

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Active Filters (Cont.) Butterworth low-pass filter $|H(j\omega)| = \frac{1}{\sqrt{1+(\omega/\omega_c)^{2n}}}$ Transfer Function for 2^{nd}-order Low-Passed Filter $H(s) = \frac{\frac{1}{R^2C_1C_2}}{s^2+\frac{2}{RC_1}s+\frac{1}{R^2C_1C_2}}$

1^{st} Order Response RL Natural Response: $L\frac{di}{dt}+Ri=0$ $\quad i(0^-)=i(0^+)=I_0$ $\quad C\frac{dv}{dt}+\frac{v}{R}=0$

$i(t)=I_0e^{-(R/L)t}$, $t\geq 0$ $\quad \tau = R/L$ RC Natural Response: $v(0^-)=v(0^+)=V_0$ $\quad \tau = RC$

$v(t)=V_0e^{-t/\tau}$, $t\geq 0$ RL Step Response $V_s = Ri + L\frac{di}{dt}$ $\quad i(t)=\frac{V_s}{R}+\left(I_0-\frac{V_s}{R}\right)e^{-(R/L)t}$ RC Step Response: $C\frac{dv_c}{dt}+\frac{v_c}{R}=I_s$

$v_c(t)=I_sR+(V_0-I_sR)e^{-t/RC}$, $t\geq 0$

2^{nd} Order Response Parallel RLC Circuit $\frac{v}{R}+\frac{1}{L}\int_0^t v\,d\tau + I_0 + C\frac{dv}{dt}=0$

$\frac{d^2v}{dt}+\frac{1}{RC}\frac{dv}{dt}+\frac{v}{LC}=0$ solution is of the form $v=Ae^{st}$ Characteristic Equation: $s^2+\frac{s}{RC}+\frac{1}{LC}=0$ $\quad s=-\alpha\pm\sqrt{\alpha^2-\omega_0^2}$

$\alpha=\frac{1}{2RC}$ $\quad \omega_0^2=\frac{1}{LC}$ Overdamped $(\alpha>\omega_0)$ $v(t)=A_1e^{s_1t}+A_2e^{s_2t}$ Critically Damped $(\alpha=\omega_0)$ $v(t)=(A_1+A_2t)e^{-\alpha t}$

Underdamped $(\alpha<\omega_0)$ $v(t)=e^{-\alpha t}(A_1\cos(\omega_dt)+A_2\sin(\omega_d(t)))$ use $v(0)$ & $\frac{dv(0)}{dt}=-\frac{(V_0+RI_0)}{RC}$ (Remember $i=C\frac{dv}{dt}$)

Step Response of Parallel RLC Circuit $\frac{d^2i_L}{dt}+\frac{1}{RC}\frac{di_L}{dt}+\frac{i_L}{LC}=\frac{I}{LC}$ Overdamped $(\alpha>\omega_0)$ $i_L(t)=I(\infty)+A_1e^{s_1t}+A_2e^{s_2t}$

Critically Damped $i_L(t)=I(\infty)+(A_1t+A_2)e^{-\alpha t}$ Underdamped $i_L(t)=e^{-\alpha t}(B_1\cos(\omega_dt)+B_2\sin(\omega_dt))+I(\infty)$

$\omega_0^2=\omega_d^2+\alpha^2$ Series RLC Circuit $Ri+L\frac{di}{dt}+\frac{1}{C}\int_0^t i\,d\tau+V_0=0 \Rightarrow \frac{d^2i}{dt^2}+\frac{R}{L}\frac{di}{dt}+\frac{i}{LC}=0$ Characteristic

Equation: $s^2+\frac{R}{L}s+\frac{1}{LC}=0$ $\quad \alpha=\frac{R}{2L}$ $\quad \omega_0^2=\frac{1}{LC}$ Natural Response: Overdamped $(\alpha>\omega_0)$ $i(t)=A_1e^{s_1t}+A_2e^{s_2t}$

Critically Damped $i(t)=D_1te^{-\alpha t}+D_2e^{-\alpha t}$ Under Damped $B_1e^{-\alpha t}\cos(\omega_dt)+B_2e^{-\alpha t}\sin(\omega_dt)$ Step Response: Overdamped:

$v_c=V_c(\infty)+A_1'e^{s_1t}+A_2'e^{s_2t}$ Critically Damped $v_c=v_c(\infty)+D_1'te^{-\alpha t}+D_2'e^{-\alpha t}$ Underdamped $v_c=v_c(\infty)+B_1'e^{-\alpha t}\cos(\omega_dt)$

$B_2'e^{-\alpha t}\sin(\omega_dt)$

Convolution $x(t)\rightarrow\boxed{h(t)}\rightarrow y(t)$ $\quad y(t)=\int_{-\infty}^{\infty}h(\lambda)x(t-\lambda)\,d\lambda=\int_{-\infty}^{\infty}h(t-\lambda)x(\lambda)\,d\lambda=(h*x)(t)$ $\quad f*\delta=f$

$\mathcal{F}\{f*g\}=k\cdot\mathcal{F}\{f\}\cdot\mathcal{F}\{g\}$

Active Filters (Cont.) Cascading Identical Filters $H(s)=\frac{(-1)^n}{(s+1)^n}$, $\omega_{cn}=\sqrt{\sqrt[n]{2}-1}$ Order of Butterworth Filters $\left(\frac{\omega_s}{\omega_p}\right)^n=\frac{\sqrt{10^{0.1A_s}-1}}{\sqrt{10^{0.1A_p}-1}}=\frac{\sigma_s}{\sigma_p}$ $\quad n=\frac{\log_{10}(\sigma_s/\sigma_p)}{\log_{10}(\omega_s/\omega_p)}$ Low-Q (Broadband) High-Q (Narrowband)

Fourier Series Fourier Series $f(t)=a_v+\sum_{n=1}^{\infty}a_n\cos(n\omega_0t)+b_n\sin(n\omega_0t)$ $\quad a_v=\frac{1}{T}\int_{t_0}^{t_0+T}f(t)dt$ $\quad a_k=\frac{2}{T}\int_{t_0}^{t_0+T}f(t)\cos(k\omega_0t)\,dt$

$b_k=\frac{2}{T}\int_{t_0}^{t_0+T}f(t)\sin(k\omega_0t)\,dt$ Even function $f(x)=f(-x)$ odd function $f(x)=-f(-x)$ $f_{e_1}(x)\cdot f_{e_2}(x)$ is even $f_{o_1}(x)\cdot f_{o_2}(x)$ is even

$f_e(x)\cdot f_o(x)=$ odd Even function $a_v=\frac{2}{T}\int_0^{T/2}f(t)dt$ $\quad a_k=\frac{4}{T}\int_0^{T/2}f(t)\cos(k\omega_0t)\,dt$ $\quad b_k=0$ odd function $a_v=0$ $a_k=0$

$b_k=\frac{4}{T}\int_0^{T/2}f(t)\sin(k\omega_0t)\,dt$ Half-Wave Symmetry (if $f(t)=-f(t-T/2)$) $a_v=0$ $a_k=0$ (k even) $a_k=\frac{4}{T}\int_0^{T/2}f(t)\cos(k\omega_0t)\,dt$

$b_k=0$ (k even) $b_k=\frac{4}{T}\int_0^{T/2}f(t)\sin(k\omega_0t)\,dt$ (k odd) QuarterWave... Fourier Series $f(t)=a_v+\sum_{n=1}^{\infty}A_n\cos(n\omega_0t-\theta_n)$ where

$a_n-jb_n=\sqrt{a_n^2+b_n^2}\angle\theta_n$ $\quad \mathcal{L}^{-1}\{A_n\angle-\theta_n\}=A_n\cos(n\omega_0t-\theta_n)$ Exponential $f(t)=\sum_{n=-\infty}^{\infty}C_ne^{jn\omega_0t}$, $C_n=\frac{1}{T}\int_{t_0}^{t_0+T}f(t)e^{-jn\omega_0t}dt$

Using Fourier Series in Circuit Analysis:

1. Find Fourier series representation of input (if it's periodic)
2. Find complex-domain expression for output & input (use transfer function)
3. Pick 1 harmonic at a time, find in phasor-domain
4. Convert back using inverse Fourier

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Fourier Transform $f(t) = \sum_{n=-\infty}^{\infty} C_n e^{jnw_0 t}$, as $nw_0 \to w$ as $T \to \infty$ $F(w) = \int_{-\infty}^{\infty} f(t) e^{-jwt} dt$ Inverse $f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(w) e^{jwt} dw$

Parseval-s Theorem
Energy spectral density $\int_{-\infty}^{\infty} f^2(t) dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |F(w)|^2 dw$

Two-Port Circuits $Z_{voltage, current}$ $Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} \Omega$, $Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} \Omega$, $Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} \Omega$, $Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} \Omega$

Cramer's Rule For $Ax=b$ $x_i = \frac{\det(A_i)}{\det(A)}$

Balanced Three-Phase Circuits Positive seq $V_a = V_m \angle 0°$, $V_b = V_m \angle -120°$, $V_c = V_m \angle 120°$

Finding Output

1. using convolution $y(t) = x(t) * h(t)$
2. Using Fourier Transforms $y(t) = \mathcal{F}^{-1}\{X(w) H(w)\}$
3. Using Laplace Transforms $y(t) = \mathcal{L}^{-1}\{X(s) H(s)\}$
4. Using Fourier Eigenfunctions if $x(t) = e^{jw_0 t}$, then $y(t) = H(w_0) e^{jw_0 t}$
5. Using Laplace Eigenfunctions if $x(t) = e^{st}$, then $y(t) = H(s) e^{st}$