

To find the reduced state on the first k qubits of A and B, tracing out the last $n - k$ qubits on each side, we can start with the $2n$ -qubit state $\rho = |\Psi_n\rangle\langle\Psi_n|$ and perform the partial trace operation.

The state $|\Psi_n\rangle$ can be written as:

$$|\Psi_n\rangle = \frac{1}{\sqrt{2^n}} \sum_i |i\rangle_A \otimes |i\rangle_B,$$

where $|i\rangle_A$ and $|i\rangle_B$ are basis states for Alice's and Bob's qubits, respectively.

To obtain the reduced state on the first k qubits of A and B, we need to trace out the last $n - k$ qubits on each side. Let's denote Alice's first k qubits as $A_1 A_2 \dots A_k$ and Bob's first k qubits as $B_1 B_2 \dots B_k$. We will trace out the remaining $n - k$ qubits, which are denoted as $A_{k+1} A_{k+2} \dots A_n$ for Alice and $B_{k+1} B_{k+2} \dots B_n$ for Bob.

To perform the partial trace, we need to sum over all possible states of the traced-out qubits and compute the inner product with the corresponding basis states on Alice's and Bob's sides.

The reduced state on the first k qubits of A and B can be calculated as follows:

$$\begin{aligned} \rho_{A_1 A_2 \dots A_k B_1 B_2 \dots B_k} &= \text{Tr}_{A_{k+1} A_{k+2} \dots A_n B_{k+1} B_{k+2} \dots B_n}(\rho) \\ &= \text{Tr}_{A_{k+1} A_{k+2} \dots A_n B_{k+1} B_{k+2} \dots B_n}(|\Psi_n\rangle\langle\Psi_n|) \\ &= \frac{1}{2^n} \sum_i \sum_j \text{Tr}_{A_{k+1} A_{k+2} \dots A_n B_{k+1} B_{k+2} \dots B_n}(|i\rangle_A \langle j|_A \otimes |i\rangle_B \langle j|_B) \\ &= \frac{1}{2^n} \sum_i \sum_j \text{Tr}_{A_{k+1} A_{k+2} \dots A_n}(|i\rangle_A \langle j|_A) \otimes \text{Tr}_{B_{k+1} B_{k+2} \dots B_n}(|i\rangle_B \langle j|_B) \\ &= \frac{1}{2^n} \sum_i \sum_j (\text{Tr}_{A_{k+1} A_{k+2} \dots A_n}(|i\rangle_A \langle j|_A)) \otimes (\text{Tr}_{B_{k+1} B_{k+2} \dots B_n}(|i\rangle_B \langle j|_B)) \\ &= \frac{1}{2^n} \sum_i \sum_j (\delta_{ij} \mathbb{I}_{A_1 A_2 \dots A_k}) \otimes (\delta_{ij} \mathbb{I}_{B_1 B_2 \dots B_k}) \\ &= \frac{1}{2^n} \sum_i \mathbb{I}_{A_1 A_2 \dots A_k} \otimes \mathbb{I}_{B_1 B_2 \dots B_k} \\ &= \frac{1}{2^n} \mathbb{I}_{A_1 A_2 \dots A_k} \otimes \mathbb{I}_{B_1 B_2 \dots B_k}, \end{aligned}$$

where $\mathbb{I}_{A_1 A_2 \dots A_k}$ and $\mathbb{I}_{B_1 B_2 \dots B_k}$ are identity operators acting on Alice's and Bob's k qubits, respectively.

Therefore, the reduced state on the first k qubits of A and B is given by $\rho_{A_1 A_2 \dots A_k B_1 B_2 \dots B_k}$. The reduced state on the first k qubits of A and B, tracing out the last $n - k$ qubits on each side, is given by:

$$\rho_{A_1 A_2 \dots A_k B_1 B_2 \dots B_k} = \frac{1}{2^n} \mathbb{I}_{A_1 A_2 \dots A_k} \otimes \mathbb{I}_{B_1 B_2 \dots B_k},$$

where $\mathbb{I}_{A_1 A_2 \dots A_k}$ and $\mathbb{I}_{B_1 B_2 \dots B_k}$ are identity operators acting on Alice's and Bob's k qubits, respectively.