Assignment 1

Name:Wang Dingrui

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EXERCISE 1: COMPLEX NUMBERS

1. Assume $x = a_1 + b_1 i$, $x = a_2 + b_2 i$, $x = a_3 + b_3 i$,

   we have $x + y + z = a_1 + a_2 + a_3 + (b_1 + b_2 + b_3)i$,

   To a complex number $n = a + bi$ we have $n^* = a - bi$,

   so

   $|x|^2 = x \cdot x^* = a_1^2 - b_1^2(i)^2 = a_1^2 - b_1^2 \cdot (-1) = a_1^2 + b_1^2$,

   $x^* y$
   $= (a_1 - b_1 i) \cdot (a_2 + b_2 i)$
   $= a_1 a_2 + a_1 b_2 i - a_2 b_1 i - b_1 b_2 i^2$
   $= a_1 a_2 + b_1 b_2 + (a_1 b_2 - a_2 b_1)i$,

   $Re(x^* y) = a_1 a_2 + b_1 b_2$,

   similarly, we have

   $|y|^2 = a_2^2 + b_2^2$,

   $|z|^2 = a_3^2 + b_3^2$,

   $Re(y^* z) = a_2 a_3 + b_2 b_3$

   $Re(x^* z) = a_1 a_3 + b_1 b_3$

   and

   $(x + y + z)^* = a_1 + a_2 + a_3 - (b_1 + b_2 + b_3)i$,

   $|x + y + z|^2$
   $= (x + y + z) \cdot (x + y + z)^*$
   $= ((a_1 + a_2 + a_3) + (b_1 + b_2 + b_3)i) \cdot ((a_1 + a_2 + a_3) - (b_1 + b_2 + b_3)i)$
   $= ((a_1 + a_2 + a_3)^2 - (b_1 + b_2 + b_3)^2 \cdot (-1))$
   $= ((a_1 + a_2 + a_3)^2 + (b_1 + b_2 + b_3)^2)$
   $= a_1^2 + a_2^2 + a_3^2 + 2a_1 a_2 + 2a_1 a_3 + 2a_2 a_3 + b_1^2 + b_2^2 + b_3^2 + 2b_1 b_2 + 2b_1 b_3 + 2b_2 b_3$
   $= a_1^2 + b_1^2 + a_2^2 + b_2^2 + a_3^2 + b_3^2 + 2(a_1 a_2 + b_1 b_2 + a_2 a_3 + b_2 b_3 + a_1 a_3 + b_1 b_3)$
   $= |x|^2 + |y|^2 + |z|^2 + 2[Re(x^* y) + Re(y^* z) + Re(x^* z)]$

   This shows that

   $|x + y + z|^2 = |x|^2 + |y|^2 + |z|^2 + 2[Re(x^* y) + Re(y^* z) + Re(x^* z)]$

2. $(i+2)(3-4i)/(2-i)$
$= (3i - 4i^2 + 2*3 - 2*4i)/(2-i)$
$= (3i - 4 \times (-1) + 2*3 - 2*4i)/(2-i)$
$= (3i + 4 + 6 - 8i)/(2-i)$
$= (10 - 5i)/(2-i)$
$= 5(2-i)/(2-i)$
$= 5$

3. $(i-4)/(2i-3)$
$= [(i-4)(2i+3)]/[(2i-3)(2i+3)]$
$= (2i^2 + 3i - 8i - 4*3)/((2i)^2 - 3*3)$
$= [2 \times (-1) + 3i - 8i - 4*3]/[4 \times (-1) - 3*3]$
$= (-2 - 5i - 12)/(-4 - 9)$
$= (-14 - 5i)/(-13)$
$= [(-1)(14 + 5i)]/(-1 \times 13)$
$= (14 + 5i)/13$
$= 14/13 + 5/13i$

   so, the real part is 14/13 and imaginary pary is 5/13.

4. i^{33}
$= i^{32}i$
$= i^{2\times 16}i$
$= (i^2)^{16}i$
$= (-1)^{16}i$
$= i$

   so, the absolute value of i^{33} is $|i|$

   $|i| = |0+i| = \sqrt{|0+i|^2} = \sqrt{(0+i)(0+i)^*} = \sqrt{(0+i)(0-i)} = \sqrt{-i^2} = \sqrt{-(-1)} = \sqrt{1} = 1$

5. i