# Guide de l'outil `Coq`
# Passage de la déduction naturelle à `Coq`

September 23, 2019

## Tactiques existantes en `Coq`

| *Déduction Naturelle* | *Nom* | *Équivalent* `Coq` | *Tactique* |
|---|---|---|---|
| $\dfrac{}{A \vdash A}$ | $Hyp$ | $\dfrac{}{\Gamma,\ H : A \vdash A}$ | `exact H.` |
| $\dfrac{\Gamma \vdash G}{\Gamma, A \vdash G}$ | $Aff$ | $\dfrac{\Gamma \vdash G}{\Gamma, H : A \vdash G}$ | `clear H.` |
| $\dfrac{\Gamma \vdash A \to G \quad \Gamma \vdash A}{\Gamma \vdash G}$ | $E_{\to}$ | $=$ | `cut A.` |
| $\dfrac{\Gamma,\ A \vdash G}{\Gamma \vdash A \to G}$ | $I_{\to}$ | $\dfrac{\Gamma,\ H : A \vdash G}{\Gamma \vdash A \to G}$ | `intro H.` |
| $\dfrac{\Gamma \vdash A \quad \Gamma \vdash B}{\Gamma \vdash A \wedge B}$ | $I_{\wedge}$ | $=$ | `split.` |
| $\dfrac{\Gamma \vdash A}{\Gamma \vdash A \vee B}$ | $I_{\vee}^{1}$ | $=$ | `left.` |
| $\dfrac{\Gamma \vdash B}{\Gamma \vdash A \vee B}$ | $I_{\vee}^{2}$ | $=$ | `right.` |
| $\dfrac{}{\Gamma \vdash A \vee \neg A}$ | $TiersExclu$ | $=$ | `apply (classic A).` |
| $\dfrac{\Gamma, x : A \vdash (G\ x)}{\Gamma \vdash \forall x : A.(G\ x)}$ | $I_{\forall}$ | $=$ | `intro x.` |
| $\dfrac{}{\Gamma \vdash x = x}$ | $I_{=}$ | $=$ | `reflexivity.` |
| $\dfrac{\Gamma \vdash [b \mid a]G}{\Gamma,\ a = b \vdash G}$ | $E_{=}$ | $\dfrac{\Gamma,\ H : a = b \vdash [b \mid a]G}{\Gamma,\ H : a = b \vdash G}$ | `rewrite -> H.` |

## Tactiques équivalentes en Coq

| *Déduction Naturelle* | *Nom* | *Équivalent* Coq | *Tactique* |
|---|---|---|---|
| $\dfrac{\Gamma \vdash \bot}{\Gamma \vdash G}$ | $E_\bot$ | $\dfrac{\dfrac{\dfrac{}{\Gamma,\ H : \bot \vdash G}}{\Gamma \vdash \bot \to G} \quad \Gamma \vdash \bot}{\Gamma \vdash G}$ | `cut False.`<br>`intro H.`<br>`contradiction.` |
| $\dfrac{\Gamma, \neg G \vdash \bot}{\Gamma \vdash G}$ | $E_\bot$ | $\dfrac{\dfrac{\dfrac{}{\Gamma,\ H : \bot \vdash G}}{\Gamma \vdash \bot \to G} \quad \Gamma \vdash \bot}{\Gamma \vdash G}$ | `cut (G \/ ~G).`<br>`intro Hgng.`<br>`elim Hgng.`<br>`intros Hg Hng.`<br>`exact Hg.`<br>`cut False.`<br>`intro H.`<br>`contradiction. ...`<br>`apply (classic G).` |
| $\dfrac{\Gamma \vdash A \wedge B}{\Gamma \vdash A}$ | $E^1_\wedge$ | $\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma,\ H : A \wedge B,\ HA : A,\ HB : B \vdash A}}{\Gamma,\ H : A \wedge B \vdash A \to B \to A}}{\Gamma,\ H : A \wedge B \vdash A}}{\Gamma \vdash A \wedge B \to A} \quad \Gamma \vdash A \wedge B}{\Gamma \vdash A}$ | `cut (A /\ B).`<br>`intro H.`<br>`elim H.`<br>`intros HA HB.`<br>`exact HA.` |
| $\dfrac{\Gamma \vdash A \wedge B}{\Gamma \vdash B}$ | $E^2_\wedge$ | idem | `cut (A /\ B).`<br>`intro H.`<br>`elim H.`<br>`intros HA HB.`<br>`exact HB.` |
| $\dfrac{\Gamma \vdash A \vee B \quad \Gamma, H1 : A \vdash G \quad \Gamma, H2 : B \vdash G}{\Gamma \vdash G}$ | $E_\vee$ | $\dfrac{\Gamma, H : A \vee B \vdash A \to G \quad \Gamma, H : A \vee B \vdash B \to G}{\Gamma, H : A \vee B \vdash G}$ | `elim H.` |
| $\dfrac{\Gamma, H : A \vdash \neg B \quad \Gamma, H : A \vdash B}{\Gamma \vdash \neg A}$ | $I_\neg$ | $\dfrac{\Gamma, H : A \vdash \bot}{\Gamma \vdash \neg A}$ | `unfold not.`<br>`intro H.` |
| $\dfrac{\Gamma, H : A \to B \vdash A}{\Gamma, H : A \to B \vdash B}$ | *Apply* | $=$ | `apply H.` |

# Tactiques propres à Coq

| *Déduction Naturelle* | *Nom* | *Équivalent* `Coq` | *Tactique* |
|---|---|---|---|
| $\dfrac{\Gamma \vdash \neg A \quad \Gamma \vdash A}{\Gamma \vdash G}$ | $E_\neg$ | $=$ | `absurd A.` |
| $\dfrac{}{\Gamma \vdash A \vee \neg A}$ | $TiersExclu$ | $=$ | `apply (classic A).` |
| $\dfrac{\Gamma \vdash \neg\neg A}{\Gamma \vdash A}$ | $Pierce$ | $=$ | `apply (NNPP A).` |
| $\dfrac{\Gamma, H1 : A, H2 : A \to B, H3 : B \vdash G}{\Gamma, H1 : A, H2 : A \to B \vdash G}$ | $ModusPonens$ | $\dfrac{\Gamma, H1 : A, H2 : A \to B \vdash B \to G}{\Gamma, H1 : A, H2 : A \to B \vdash G}$ | `generalize (H2 H1).` |
| $\dfrac{\Gamma, x : A \vdash (G\ x)}{\Gamma \vdash \forall x : A.(G\ x)}$ | $I_\forall$ | $=$ | `intro x.` |
| $\dfrac{\Gamma \vdash \forall x : A.(G\ x) \quad \Gamma \vdash y : A}{\Gamma \vdash (G\ y)}$ | $E_\forall$ | $\dfrac{\Gamma, y : A \vdash \forall x : A.(G\ x)}{\Gamma, y : A \vdash (G\ y)}$ | `generalize y.` |
| $\dfrac{\Gamma \vdash (G\ y) \quad \Gamma \vdash y : A}{\Gamma \vdash \exists x : A.(G\ x)}$ | $I_\exists$ | $=$ | `exists y.` |
| $\dfrac{\Gamma \vdash \exists x : A.(P\ x) \quad \Gamma, y : A, H : (P\ y) \vdash G}{\Gamma \vdash G}$ | $E_\exists$ | $\dfrac{\Gamma, H : \exists x : A.(P\ x) \vdash \forall y : A.(P\ y) \to G}{\Gamma, H : \exists x : A.(P\ x) \vdash G}$ | `elim H.` |
| $\dfrac{\Gamma \vdash (G\ 0) \quad \Gamma \vdash \forall m : Nat.(G\ m) \to (G\ (S\ m))}{\Gamma \vdash \forall n : Nat.(G\ n)}$ | $E_{Nat}$ | $\approx$ | `intro n ;elim n.` |
| $\dfrac{\Gamma \vdash (G\ 0) \quad \Gamma \vdash \forall m : Nat.(G\ (S\ m))}{\Gamma \vdash \forall n : Nat.(G\ n)}$ | $Cas\ sur\ Nat$ | $\approx$ | `intro n ;case n.` |
| $\dfrac{\forall k \in [1, N] : \quad \Gamma, H : T = (C_k\ u_1 \ldots u_{n_k}) \vdash G}{\Gamma, T : (I\ v_1 \ldots v_n) \vdash G}$ | $I\ inductif$ | $\approx$ | `inversion T.` |
| $\dfrac{}{\Gamma, H : t[\![(C\ u_1 \ldots u_n)]\!] = t[\![(C'\ v_1 \ldots v_p)]\!] \vdash G}$ | $C \neq C'$ | $=$ | `discriminate H.` |
| $\dfrac{\Gamma \vdash u_1 = v_1 \to \ldots u_n = v_n \to G}{\Gamma, H : (C\ u_1 \ldots u_n) = (C\ v_1 \ldots v_n) \vdash G}$ | $C\ injectif$ | $\approx$ | `injection H.` |
| $\dfrac{\Gamma \vdash G'}{\Gamma \vdash G}$ | $G \rhd G'$ | $=$ | `simpl.` |
| $\dfrac{\Gamma, H : a = b \vdash [a \mid b]G}{\Gamma, H : a = b \vdash G}$ | $b = a$ | $=$ | `rewrite <- H.` |
| $\dfrac{\Gamma, HA : A, HB : B \vdash G}{\Gamma, H : A \wedge B \vdash G}$ | $E'_\wedge$ | $=$ | `destruct H as (HA,HB).` |
| $\dfrac{\Gamma, HA : A \vdash G \quad \Gamma, HB : B \vdash G}{\Gamma, H : A \vee B \vdash G}$ | $E'_\vee$ | $=$ | `destruct H as [HA \| HB].` |