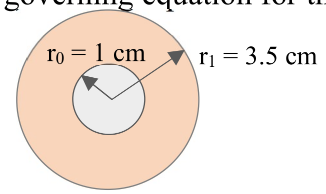
3.

A cylindrical p **Problem** ipe with wall thickness 2.5 cm and inner radius 1 cm and carries a fluid at a temperature of 550 °C and the outer wall of the pipe is at 25°C. The governing equation for the temperature distribution in the pipe wall is:



$$r\frac{d^2T}{dr^2} + \frac{dT}{dr} - H/\mathrm{k} = 0$$

subject to boundary conditions of T(1) = 600 °C and T(3.5) = 25 °C.
H is a heat generation term. (The material of the pipe generates heat, it may be made from a radioactive material.) k is thermal conductivity.

a) Break the above ODE into coupled 1st-order ODEs. (5 pts)

Insert Your answer here:

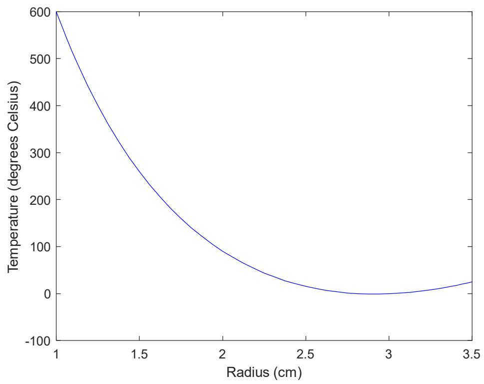
$$\frac{dT}{dr} = S \quad (1)$$

$$r\left(\frac{dS}{dT}\right) + S - \frac{H}{K} = 0 \quad (2)$$

b) When we assume steady state, set H/k=500. Write a MATLAB function that solves the problem with the shooting method and plot the temperature as a function of r. Label the axis with units and paste the figure into the box. (25 pts)

```
function [] = E2p3NoahGa()
r0 = 1;
r1 = 3.5;
if nargin == 0, guess = 0; end

S0 = fzero(@Rooter,guess);
[r, y] = ode45(@coupledfirsts,[r0 r1],[600 S0]);
plot(r,y(:,1),'-b');
xlabel('Radius (cm)');
ylabel('Temperature (degrees Celsius)');

if nargout == 0, r = []; y = []; end
end

function dy = coupledfirsts(r,y)
%y = [T S]
HoverK = 500;
dTdr = y(2);
dSdr = (-y(2)./r) + (HoverK)./r;
dy = [dTdr; dSdr];
end

function err = Rooter(guess)
[r, y] = ode45(@coupledfirsts,[1 3.5],[600 guess]);
err = 25 - y(end,1);
end
```

Insert your figure here:



Initial condition dT/dt is -956.7802

c) Set H/k=200. How will the peak temperature and its gradient at 1 cm change with this parameter? Discuss the thermodynamic principle (embedded in the coupled ODEs in a) that you used to make the prediction and check with the code. (5 pts)

Insert Your answer here:

By halving H/k we could have raised the thermal conductivity of the material. I would expect the Gradient to be less steep in this case as the rate of change of temp throughout the material would be less. When we plot the H/K= 200, we can see that the temperature throughout the material is higher than 25 degrees Celsius at all times. Compared to the H/k = 500, where there is steep dropoff of temperature when increasing distance from the Hottest inside wall.