MAT 4800 Homework # 6

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Problem #1

Suppose we are given the following function of,

$$g(x) = x^2 - 2$$

such that,

$$g'(x) = 2x$$

then finding the root for $g(x)$ is denoted by,

$$x^{(k+1)} = x^{(k)} - \frac{(x^{(k)})^2 - 2}{2x^{(k)}} = \frac{x^{(k)}}{2} + \frac{1}{x^{(k)}}$$

with $x^{(0)} = 1$, then

k	$x^{(k)}$
0	1
1	1.5
2	1.4166667
3	1.4142156862745099
4	1.4142135623746899
5	1.4142135623730951
6	1.4142135623730949
7	1.4142135623730951
8	1.4142135623730949
9	1.4142135623730951
10	1.4142135623730949

Table 1: Finding $\sqrt{2}$ using Newton's Method

Problem #2

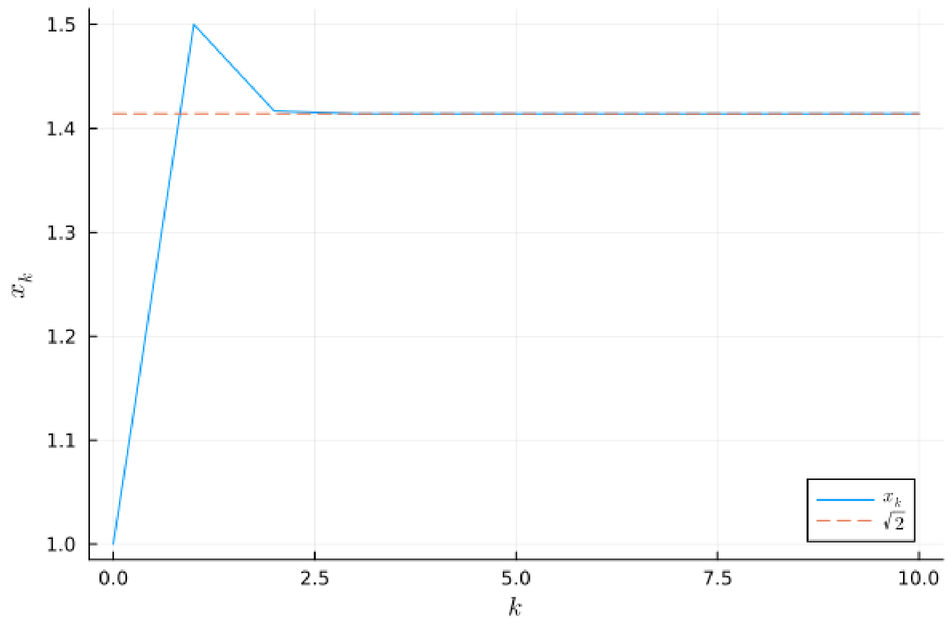

Figure 1: x_k vs. $\sqrt{2}$

Problem #3

Suppose we want to approximate the root of a function whose root is $2^{1/n}$, note that a simplified function that achieves such is,

$$g(x) = x^n - 2$$

and hence we compute,

$$g'(x) = nx^{n-1}$$

thus each $x^{(k+1)}$ for $k \in \mathbb{N}$ is computed as,

$$x^{(k+1)} = x^{(k)} - \frac{(x^{(k)})^n - 2}{n(x^{(k)})^{n-1}} = \frac{(n-1)x^{(k)}}{n} + \frac{2}{n(x^{(k)})^{n-1}}$$

Problem #4

Suppose we are given the following function of,

$$g(x) = x^5 - 2$$

such that,

$$g'(x) = 5x^4$$

then finding the root for $g(x)$, $2^{1/5}$ is denoted by,

$$x^{(k+1)} = x^{(k)} - \frac{g(x^{(k)})}{g'(x^{(k)})} = \frac{4x^{(k)}}{5} + \frac{2}{5(x^{(k)})^4}$$

with $x^{(0)} = 1$, then

k	$x^{(k)}$
0	1
1	1.2
2	1.1529012345679013
3	1.1487288865273251
4	1.1486983566199584
5	1.1486983549970351
6	1.1486983549970351
7	1.1486983549970351
8	1.1486983549970351
9	1.1486983549970351
10	1.1486983549970351

Table 2: Finding $2^{1/5}$ using Newton's Method

Problem #5

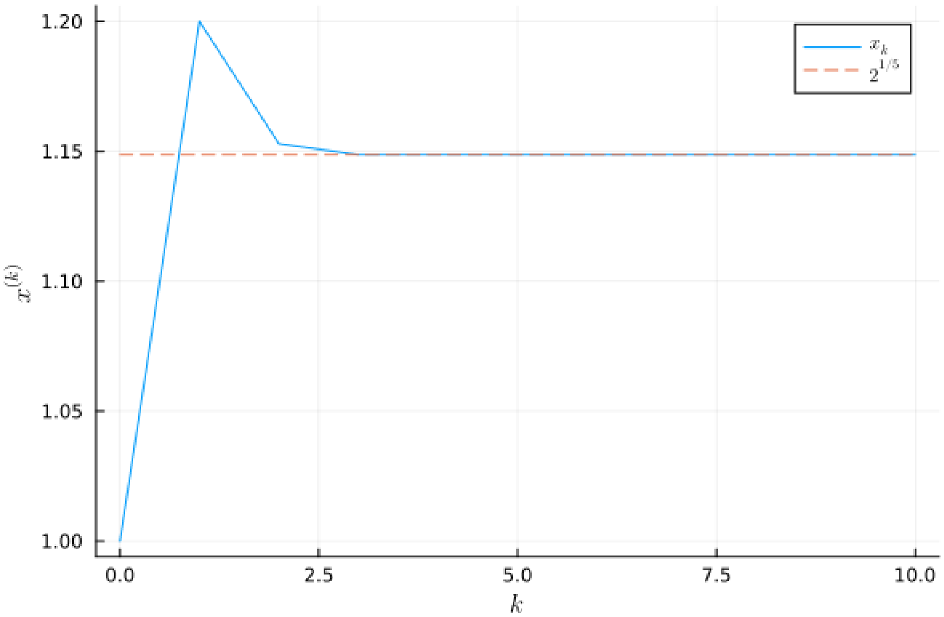

Figure 2: $x^{(k)}$ vs. $2^{1/5}$

Problem #6

Suppose we are given the following function of,

$$g(x) = x^n - a$$

such that,

$$g'(x) = nx^{n-1}$$

then finding the root for $g(x)$, $a^{1/n}$ is denoted by,

$$x^{(k+1)} = x^{(k)} - \frac{g(x^{(k)})}{g'(x^{(k)})} = \frac{(n-1)x^{(k)}}{n} + \frac{a}{n(x^{(k)})^{n-1}}$$

Problem #7

Here we let $a = 8$ and $n = 3$, such that we want to approximate the root $8^{1/3}$, using the Newton Scheme above with $x_0 = 1$ we get,

k	$x^{(k)}$
0	1
1	3.3333333333333335
2	2.4622222222222221
3	2.0813412476715789
4	2.0031374991412871
5	2.0000049116755041
6	2.0000000000120624
7	2.0000000000000000
8	2.0000000000000000
9	2.0000000000000000
10	2.0000000000000000

Table 3: Finding $8^{1/3}$ using Newton's Method

Problem #8

We compute the error for each $x^{(k)}$ as,

k	$e^{(k)}$
0	1
1	1.3333333333333335
2	0.4622222222222221
3	0.08134124767157891
4	0.0031374991412871367
5	4.911675504093438e-6
6	1.20623511179474e-11
7	0
8	0
9	0
10	0

Table 4: Error for Approximating $8^{1/3}$ using Newton's Method

Problem #9

Here we approximate π by using the function,

$$g(x) = \tan(x)$$

and,

$$g'(x) = \sec^2(x)$$

then finding the root for $g(x)$, π is denoted by,

$$x^{(k+1)} = x^{(k)} - \frac{g(x^{(k)})}{g'(x^{(k)})} = x^{(k)} - \frac{\tan(x)}{\sec^2(x)} = x^{(k)} - \sin(x^{(k)})\cos(x^{(k)})$$

using $x^{(0)} = 3$ to get,

k	$e^{(k)}$
0	3.0000000000000000
1	3.1397077490994629
2	3.1415926491252555
3	3.1415926535897931
4	3.1415926535897931
5	3.1415926535897931
6	3.1415926535897931
7	3.1415926535897931
8	3.1415926535897931
9	3.1415926535897931
10	3.1415926535897931

Table 5: Approximating π using Newton's Method

Problem #10

We can do something similar to above for approximating π, by instead using

$$g(x) = \sin(x)$$

and,

$$g'(x) = \cos(x)$$

then finding the root for $g(x)$, π is denoted by,

$$x^{(k+1)} = x^{(k)} - \frac{g(x^{(k)})}{g'(x^{(k)})} = x^{(k)} - \frac{\sin(x)}{\cos(x)} = x^{(k)} - \tan(x^{(k)})$$

using $x^{(0)} = 3$ to get,

k	$e^{(k)}$
0	3.0000000000000000
1	3.1425465430742778
2	3.1415926533004770
3	3.1415926535897931
4	3.1415926535897931
5	3.1415926535897931
6	3.1415926535897931
7	3.1415926535897931
8	3.1415926535897931
9	3.1415926535897931
10	3.1415926535897931

Table 6: Approximating π using Newton's Method