Noah Boulton

SENG 474 - Assignment 1

1. Age

   Young → Entropy(4, 2, 2) = $-4/8\ log(4/8 - 2/8\ log(2/8) - 2/8\ log(2/8) = 0.4515$

   Pre-presbyopic → Entropy(5, 2, 1) = $-5/8\ log(5/8) - 2/8\ log(2/8) - 1/8\ log(1/8) = 0.3909$

   Presbyopic → Entropy(6, 1, 1) = $-6/8\ log(6/8) - 1/8\ log(1//8) - 1/8\ log(1//8) = 0.3194$

   Average Entropy = $0.4515(8/24) + 0.3909(8/24) + 0.3194(8/24) = 0.3870$

   Spectacle-Prescrip

   Myope → Entropy(7, 2, 3) = $-7/12\ log(7/12) - 2/12\ log(2/12) - 3/12\ log(3/12) = 0.4168$

   Hypermetrope → Entropy(8, 3, 1) =
   $-8/12\ log(8/12) - 3/12\ log(3/12) - 1/12\ log(1/12) = 0.3578$

   Average Entropy = $0.4168(12/24) + 0.3578(12/24) = 0.3873$

   Astigmatism

   No → Entropy(7, 5, 0) = $-7/12\ log(7/12) - 5/12\ log(5/12) - 0/12\ log(0/12) = 0.2950$

   Yes → Entropy(8, 0, 4) = $-8/12\ log(8/12) - 0/12\ log(0/12) - 4/12\ log(4/12) = 0.2764$

   Average Entropy = $0.2950(12/24) + 0.2764(12/24) = 0.2857$

   Tear-prod-rate

   Normal → Entropy(3, 5, 4) = $-3/12\ log(3/12) - 5/12\ log(5/12) - 4/12\ log(4/12) = 0.4680$

   Reduced → Entropy(12, 0, 0) = $-12/12\ log(12/12) - 0/12\ log(0/12) - 0/12\ log(0/12) = 0$

   Average Entropy = $0.4680(12/24) + 0(12/24) = 0.2340$

   Root: We select Tear-prod-rate for the root as the average entropy is the lowest.

   Age

   Young → Entropy(0, 2, 2) = $-0/4\ log(0/4) - 2/4\ log(2/4) - 2/4\ log(2/4) = 0.3010$

   Pre-presbyopic → Entropy(1, 2, 1) = $-1/4\ log(1/4) - 2/4\ log(2/4) - 1/4\ log(1/4) = 0.4515$

   Presbyopic → Entropy(2, 1, 1) = $-2/4\ log(2/4) - 1/4\ log(1/4) - 1/4\ log(1/4) = 0.4515$

   Average Entropy = $0.3010(4/12) + 0.4515(4/12) + 0.4515(4/12) = 0.4013$

   Spectacle-Prescrip

   Myope → Entropy(2, 3, 1) = $-2/6\ log(2/6) - 3/6\ log(3/6) - 1/6\ log(1/6) = 0.4392$

   Hypermetrope → Entropy(1, 2, 3) = $-1/6\ log(1/6) - 2/6\ log(2/6) - 3/6\ log(3/6) = 0.4392$

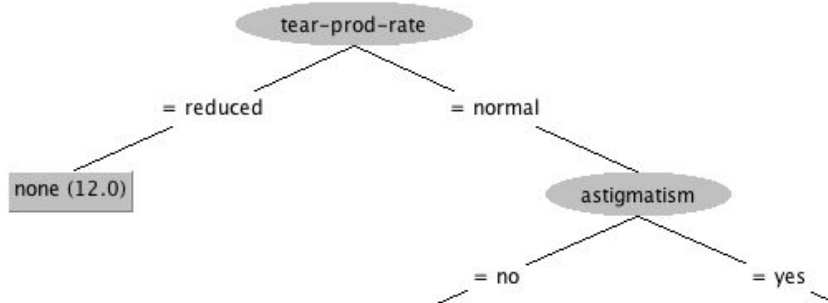   Average Entropy = $0.4392(6/12) + 0.4392(6/12) = 0.4392$

   Astigmatism

   No → Entropy(1, 5, 0) = $-1/6\ log(1/6) - 5/6\ log(5/6) - 0/6\ log(0/6) = 0.1957$

   Yes → Entropy(2, 0, 4) = $-2/6\ log(2/6) - 0/6\ log(0/6) - 4/6\ log(4/6) = 0.2764$

Average Entropy = $0.1957(6/12) + 0.2764(6/12) = 0.2361$

Second Level: We select Astigmatism for the second level as the average entropy is the lowest.

Tree:

tear-prod-rate = reduced: none
tear-prod-rate = normal
| astigmatism = no
| | ...
| astigmatism = yes

2. Rule 1:   if ___ then "yes" → Accuracy = 9/14 = 0.6429
                If outlook = overcast then "yes" → Accuracy = 4/4 = 1.0
   Rule 2:   if ___ then "yes" → Accuracy = 9/14 = 0.6429
                If outlook = rainy then "yes" → Accuracy = ⅗ = 0.6
                If outlook = rainy and windy = false then "yes" → Accuracy = 3/3 = 1.0

3. P(None | Age = Pre-presbyopic, Spectacle-prescrip = Hypermetrope, Astigmatism = Yes, Tear-prod-rate = Reduced) = α⬚P(Pre-presbyopic | None)⬚P(Hypermetrope | None)⬚P(Yes | None)⬚P(Reduced | None)⬚P(None) =
$\alpha x(5 + 1/15 + 3)x(8 + 1/15 + 2)x(8 + 1/15 + 2)x(12 + 1/15 + 2)x(15 + 1/24 + 3) = \alpha x(0.042336657)$
P(Soft | Age = Pre-presbyopic, Spectacle-prescrip = Hypermetrope, Astigmatism = Yes, Tear-prod-rate = Reduced) = α⬚P(Pre-presbyopic | Soft)⬚P(Hypermetrope | Soft)⬚P(Yes | Soft)⬚P(Reduced | Soft)⬚P(Soft) =
$\alpha \square(2 + 1/5 + 3)x(3 + 1/5 + 2)x(0 + 1/5 + 2)x(0 + 1/5 + 2)x(5 + 1/24 + 3) = \alpha x(0.000971817)$

P(Hard | Age = Pre-presbyopic, Spectacle-prescrip = Hypermetrope, Astigmatism = Yes, Tear-prod-rate = Reduced) = α⬚P(Pre-presbyopic | Hard)⬚P(Hypermetrope | Hard)⬚P(Yes | Hard)⬚P(Reduced | Hard)⬚P(Hard) =
$\alpha x(1 + 1/4 + 3)x(1 + 1/4 + 2)x(4 + 1/4 + 2)x(0 + 1/4 + 2)x(4 + 1/24 + 3) = \alpha x(0.002449539)$

$1 = \alpha(0.042336657 + 0.000971817 + 0.002449539)$
$\alpha = 1/(0.042336657 + 0.000971817 + 0.002449539) = 21.85409581$

P(None | Age = Pre-presbyopic, Spectacle-prescrip = Hypermetrope, Astigmatism = Yes, Tear-prod-rate = Reduced) = α☐P(Pre-presbyopic | None)☐P(Hypermetrope | None)☐P(Yes | None)☐P(Reduced | None)☐P(None) = 0.9252= 92.52%

P(Soft | Age = Pre-presbyopic, Spectacle-prescrip = Hypermetrope, Astigmatism = Yes, Tear-prod-rate = Reduced) = α☐P(Pre-presbyopic | Soft)☐P(Hypermetrope | Soft)☐P(Yes | Soft)☐P(Reduced | Soft)☐P(Soft) = 0.0212 = 2.12%

P(Hard | Age = Pre-presbyopic, Spectacle-prescrip = Hypermetrope, Astigmatism = Yes, Tear-prod-rate = Reduced) = α☐P(Pre-presbyopic | Hard)☐P(Hypermetrope | Hard)☐P(Yes | Hard)☐P(Reduced | Hard)☐P(Hard) = 0.0535 = 5.35%

The instance is classified as **None**.