

SENG 474 - Assignment 1

1. Age

$$\text{Young} \rightarrow \text{Entropy}(4, 2, 2) = -4/8 \log(4/8) - 2/8 \log(2/8) - 2/8 \log(2/8) = 0.4515$$

$$\text{Pre-presbyopic} \rightarrow \text{Entropy}(5, 2, 1) = -5/8 \log(5/8) - 2/8 \log(2/8) - 1/8 \log(1/8) = 0.3909$$

$$\text{Presbyopic} \rightarrow \text{Entropy}(6, 1, 1) = -6/8 \log(6/8) - 1/8 \log(1/8) - 1/8 \log(1/8) = 0.3194$$

$$\text{Average Entropy} = 0.4515(8/24) + 0.3909(8/24) + 0.3194(8/24) = 0.3870$$

Spectacle-Prescrip

$$\text{Myope} \rightarrow \text{Entropy}(7, 2, 3) = -7/12 \log(7/12) - 2/12 \log(2/12) - 3/12 \log(3/12) = 0.4168$$

$$\text{Hypermetrope} \rightarrow \text{Entropy}(8, 3, 1) =$$

$$-8/12 \log(8/12) - 3/12 \log(3/12) - 1/12 \log(1/12) = 0.3578$$

$$\text{Average Entropy} = 0.4168(12/24) + 0.3578(12/24) = 0.3873$$

Astigmatism

$$\text{No} \rightarrow \text{Entropy}(7, 5, 0) = -7/12 \log(7/12) - 5/12 \log(5/12) - 0/12 \log(0/12) = 0.2950$$

$$\text{Yes} \rightarrow \text{Entropy}(8, 0, 4) = -8/12 \log(8/12) - 0/12 \log(0/12) - 4/12 \log(4/12) = 0.2764$$

$$\text{Average Entropy} = 0.2950(12/24) + 0.2764(12/24) = 0.2857$$

Tear-prod-rate

$$\text{Normal} \rightarrow \text{Entropy}(3, 5, 4) = -3/12 \log(3/12) - 5/12 \log(5/12) - 4/12 \log(4/12) = 0.4680$$

$$\text{Reduced} \rightarrow \text{Entropy}(12, 0, 0) = -12/12 \log(12/12) - 0/12 \log(0/12) - 0/12 \log(0/12) = 0$$

$$\text{Average Entropy} = 0.4680(12/24) + 0(12/24) = 0.2340$$

Root: We select Tear-prod-rate for the root as the average entropy is the lowest.

Age

$$\text{Young} \rightarrow \text{Entropy}(0, 2, 2) = -0/4 \log(0/4) - 2/4 \log(2/4) - 2/4 \log(2/4) = 0.3010$$

$$\text{Pre-presbyopic} \rightarrow \text{Entropy}(1, 2, 1) = -1/4 \log(1/4) - 2/4 \log(2/4) - 1/4 \log(1/4) = 0.4515$$

$$\text{Presbyopic} \rightarrow \text{Entropy}(2, 1, 1) = -2/4 \log(2/4) - 1/4 \log(1/4) - 1/4 \log(1/4) = 0.4515$$

$$\text{Average Entropy} = 0.3010(4/12) + 0.4515(4/12) + 0.4515(4/12) = 0.4013$$

Spectacle-Prescrip

$$\text{Myope} \rightarrow \text{Entropy}(2, 3, 1) = -2/6 \log(2/6) - 3/6 \log(3/6) - 1/6 \log(1/6) = 0.4392$$

$$\text{Hypermetrope} \rightarrow \text{Entropy}(1, 2, 3) = -1/6 \log(1/6) - 2/6 \log(2/6) - 3/6 \log(3/6) = 0.4392$$

$$\text{Average Entropy} = 0.4392(6/12) + 0.4392(6/12) = 0.4392$$

Astigmatism

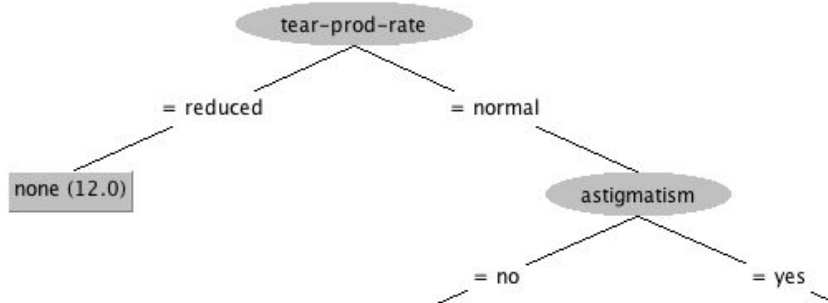
$$\text{No} \rightarrow \text{Entropy}(1, 5, 0) = -1/6 \log(1/6) - 5/6 \log(5/6) - 0/6 \log(0/6) = 0.1957$$

$$\text{Yes} \rightarrow \text{Entropy}(2, 0, 4) = -2/6 \log(2/6) - 0/6 \log(0/6) - 4/6 \log(4/6) = 0.2764$$

$$\text{Average Entropy} = 0.1957(6/12) + 0.2764(6/12) = 0.2361$$

Second Level: We select Astigmatism for the second level as the average entropy is the lowest.

Tree:



tear-prod-rate = reduced: none

tear-prod-rate = normal

| astigmatism = no

| | ...

| astigmatism = yes

2. Rule 1: if ____ then "yes" → Accuracy = 9/14 = 0.6429
If outlook = overcast then "yes" → Accuracy = 4/4 = 1.0
- Rule 2: if ____ then "yes" → Accuracy = 9/14 = 0.6429
If outlook = rainy then "yes" → Accuracy = 3/5 = 0.6
If outlook = rainy and windy = false then "yes" → Accuracy = 3/3 = 1.0

3. $P(\text{None} \mid \text{Age} = \text{Pre-presbyopic}, \text{Spectacle-prescrip} = \text{Hypermetrope}, \text{Astigmatism} = \text{Yes}, \text{Tear-prod-rate} = \text{Reduced}) = \alpha \square P(\text{Pre-presbyopic} \mid \text{None}) \square P(\text{Hypermetrope} \mid \text{None}) \square P(\text{Yes} \mid \text{None}) \square P(\text{Reduced} \mid \text{None}) \square P(\text{None}) =$
 $\alpha \square (5 + 1/15 + 3) \square (8 + 1/15 + 2) \square (8 + 1/15 + 2) \square (11 + 1/15 + 2) \square (15 + 1/24 + 3) = \alpha \square (0.039079991)$
 $P(\text{Soft} \mid \text{Age} = \text{Pre-presbyopic}, \text{Spectacle-prescrip} = \text{Hypermetrope}, \text{Astigmatism} = \text{Yes}, \text{Tear-prod-rate} = \text{Reduced}) = \alpha \square P(\text{Pre-presbyopic} \mid \text{Soft}) \square P(\text{Hypermetrope} \mid \text{Soft}) \square P(\text{Yes} \mid \text{Soft}) \square P(\text{Reduced} \mid \text{Soft}) \square P(\text{Soft}) =$
 $\alpha \square (2 + 1/5 + 3) \square (3 + 1/5 + 2) \square (0 + 1/5 + 2) \square (0 + 1/5 + 2) \square (5 + 1/24 + 3) = \alpha \square (0.000971817)$

$P(\text{Hard} \mid \text{Age} = \text{Pre-presbyopic}, \text{Spectacle-prescrip} = \text{Hypermetrope}, \text{Astigmatism} = \text{Yes}, \text{Tear-prod-rate} = \text{Reduced}) = \alpha \square P(\text{Pre-presbyopic} \mid \text{Hard}) \square P(\text{Hypermetrope} \mid \text{Hard}) \square P(\text{Yes} \mid \text{Hard}) \square P(\text{Reduced} \mid \text{Hard}) \square P(\text{Hard}) =$
 $\alpha \square (1 + 1/4 + 3) \square (1 + 1/4 + 2) \square (4 + 1/4 + 2) \square (0 + 1/4 + 2) \square (4 + 1/24 + 3) = \alpha \square (0.002449539)$

$$1 = \alpha(0.039079991 + 0.000971817 + 0.002449539)$$

$$\alpha = 1/(0.039079991 + 0.000971817 + 0.002449539) = 23.52866604$$

$P(\text{None} \mid \text{Age} = \text{Pre-presbyopic}, \text{Spectacle-prescrip} = \text{Hypermetrope}, \text{Astigmatism} = \text{Yes}, \text{Tear-prod-rate} = \text{Reduced}) = \alpha P(\text{Pre-presbyopic} \mid \text{None}) P(\text{Hypermetrope} \mid \text{None}) P(\text{Yes} \mid \text{None}) P(\text{Reduced} \mid \text{None}) P(\text{None}) = 0.9195 = 91.95\%$

$P(\text{Soft} \mid \text{Age} = \text{Pre-presbyopic}, \text{Spectacle-prescrip} = \text{Hypermetrope}, \text{Astigmatism} = \text{Yes}, \text{Tear-prod-rate} = \text{Reduced}) = \alpha P(\text{Pre-presbyopic} \mid \text{Soft}) P(\text{Hypermetrope} \mid \text{Soft}) P(\text{Yes} \mid \text{Soft}) P(\text{Reduced} \mid \text{Soft}) P(\text{Soft}) = 0.0229 = 2.29\%$

$P(\text{Hard} \mid \text{Age} = \text{Pre-presbyopic}, \text{Spectacle-prescrip} = \text{Hypermetrope}, \text{Astigmatism} = \text{Yes}, \text{Tear-prod-rate} = \text{Reduced}) = \alpha P(\text{Pre-presbyopic} \mid \text{Hard}) P(\text{Hypermetrope} \mid \text{Hard}) P(\text{Yes} \mid \text{Hard}) P(\text{Reduced} \mid \text{Hard}) P(\text{Hard}) = 0.0576 = 5.76\%$

The instance is classified as **None**.