Law of Sines Proof

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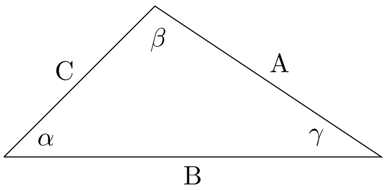
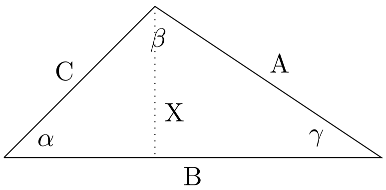
<u>Theorem</u>:

Given any triangle with side lengths A, B, and C, with angles α between sides B and C, β between sides A and C, and γ between sides A and B, the relationship $\frac{sin(\alpha)}{A} = \frac{sin(\beta)}{B} = \frac{sin(\gamma)}{C}$ always holds.



<u>Proof</u>:

Imagine we draw a line from the β corner down to the B side length to make a right triangle and label the new line X.



Since we have created two right triangles that share a length X we can sove for X with two different equations:

$$sin(\alpha) = \frac{X}{C} \text{ and } sin(\gamma) = \frac{X}{A}$$
$$C * sin(\alpha) = X \text{ and } A * sin(\gamma) = X$$
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If we set the equations equal to each other we can gain most of the relationship:

$$C * sin(\alpha) = A * sin(\gamma)$$
$$C * \frac{sin(\alpha)}{A} = sin(\gamma)$$
$$\frac{sin(\alpha)}{A} = \frac{sin(\gamma)}{C}$$

In these steps we have draw a vertical line stemming from one corner of the triangle to make two right triangles incorporating the other two corners. Doing this we have shown that the *sine* of the angle each corner produces over the side length opposite to each corner is equal.

With this proven we can assert that not only does $\frac{sin(\alpha)}{A} = \frac{sin(\gamma)}{C}$, but $\frac{sin(\alpha)}{A} = \frac{sin(\beta)}{B}$ which we could prove with a line stemming from the γ corner to side length C.

Therefore, by transitive property, if $\frac{sin(\alpha)}{A} = \frac{sin(\gamma)}{C}$ and $\frac{sin(\alpha)}{A} = \frac{sin(\beta)}{B}$, then $\frac{sin(\alpha)}{A} = \frac{sin(\beta)}{B} = \frac{sin(\gamma)}{C}$.

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