Growth of Functions
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1. (a) $10^{n+1} = O(10^n)$, true, n plus a constant is negligible and equates to just n, thus it becomes exponential runtime, $O(k^n)...O(10^n)$
   (b) $5^{3n} = O(5^n)$, false, $(O(5^n) * O(5^n) * O(5^n))$ does not equal $O(5^n)$
2. (a) $g(n) = \lg n^3$ and $f(n) = 150 \lg n + 30$, $f(n) = \Omega(n)$: $g(n)$ out grows $f(n)$ so $f(n)$ is the lower bound. $f(n) = \Omega(g(n))$
   (b) $g(n) = \sqrt{n}$ and $f(n) = 23 \lg n^2$: $f(n)$ out grows $g(n)$ so $f(n)$ is the upper bound. $f(n) = O(g(n))$
   (c) $g(n) = \lg^2 n$ and $f(n) = 350 \lg n + O(1)$: $f(n)$ out grows $g(n)$ so $f(n)$ is the upper bound. $f(n) = O(g(n))$
   (d) $g(n) = n$ and $f(n) = \lg^2 n$: $g(n)$ out grows $f(n)$ so $f(n)$ is the lower bound. $f(n) = \Omega(g(n))$
   (e) $g(n) = n \lg n + n$ and $f(n) = 3 \lg n$: $g(n)$ out grows $f(n)$ so $f(n)$ is the lower bound. $f(n) = \Omega(g(n))$
   (f) $g(n) = 50$ and $f(n) = \lg 50$: neither of these functions grow, they are both constant time so $f(n)$ is the lower and upper bound. $f(n) = \Theta(g(n))$
   (g) $g(n) = 2^n$ and $f(n) = 30n^5$: $g(n)$ grows exponentially fast, and only factorial grows faster so $f(n)$ must be the lower bound. $f(n) = \Omega(g(n))$
   (h) $g(n) = 2^n$ and $f(n) = 4^n$: both functions grow exponentially fast, therefore $f(n)$ is the lower and upper bound. $f(n) = \Theta(g(n))$
3. (a) $O(n^2) = 10n^2 + 15$: true, constants are negligible and can be removed
   (b) $O(lgn) = 8\sqrt{n}$: false, $\sqrt{n}$ grows much faster than lgn, they are not the same complexity
   (c) $O(\sqrt{n}) = 14 \lg n + 1000$: false, 1000 and 14 can be removed because they are constants and once again $\sqrt{n}$ grows faster than lgn
   (d) $O(n^2 \lg n) = 2n^2(5 + \sqrt{n})$: false, for the same reasons as item (b) and (c), $\sqrt{n}$ and $\lg n$ are not equivalent in growth

(e) $O(n^2) = 5n^2 + 3\sqrt{n}$: true, n^2 is the upper bound for this expression so we take that as the big oh and ignore the constant multiplied by it and the $\sqrt{n}$ times 3 is also negligible in comparison to n^2

(f) $O(n) = 3\sqrt{n}\lg n + 15$: false, n grows much faster than $\sqrt{n}$ which is the upper bound for the expression

(g) $O(n^{-1/2}) = 5\lg n$: false, lgn and $n^{-1/2}$ do not grow at the same rate