Noah Buchanan
Problem Set 2
AI at 5:25 PM

February 2, 2021

1. Two events are considered independent in probability if the occurrence of one does not affect the other events chance of occurring and vice versa. An example of this would be drawing a card from a deck, and flipping a coin, neither of these events has any impact on the other therefore are considered independent of each other.

2. Joint probability is the probability of two events occurring simultaneously, it is also important to note both events must be independent of eachother, otherwise it is just conditional probability. Marginal probability is the probability of an event regardless of the outcome of another factor or variable. Conditional probability is the probability of an event to occur following the occurrence of another event, or in other words, the probability of A given B has already happened.

3. (a) $\frac{4}{9}$

   (b) $\frac{6}{9}$

   (c) $\frac{8}{9}$

4. (a) $\frac{5}{14}$

(b) $\frac{1}{14}$

(c) $\frac{9}{14}$

(d) $\frac{6}{14}$

(e) $\frac{7}{14}$

5. (a) $\frac{1}{16}$

(b) $\frac{3}{8}$

(c) I assumed the wording "at least 1 AI and at least 1 NLP": $\frac{7}{8}$

(d) $\frac{1}{15}$

(e) $\frac{2}{5}$

(f) $\frac{14}{15}$

(g) Yes

(h) No

6. (a) 2598960

(b) 4

(c) $\frac{4}{2598960}$

(d) 624

(e) $\frac{624}{2598960}$

7. (a) 0.23

   (b) P(student$\in X_1$) $= \frac{0.14}{0.23}$, P(student$\in X_2$) $= \frac{0.09}{0.23}$

8. (a) $\frac{0.04}{0.18}$

   (b) $\frac{0.05}{0.18}$

   (c) $\frac{0.09}{0.18}$

9. $$\begin{bmatrix} (2\cdot 1)+(2\cdot 3)+(3\cdot 4) & (2\cdot 2)+(2\cdot 2)+(3\cdot 4) \\ (1\cdot 1)+(1\cdot 3)+(3\cdot 4) & (1\cdot 2)+(1\cdot 2)+(3\cdot 4) \\ (4\cdot 1)+(2\cdot 3)+(5\cdot 4) & (4\cdot 2)+(2\cdot 2)+(5\cdot 4) \end{bmatrix} = \begin{bmatrix} 20 & 20 \\ 16 & 16 \\ 30 & 32 \end{bmatrix}$$

10. cannot perform 3x2 $\cdot$ 3x3 $\cdot$ 3x3, the dimensions are not compatible

11. Formula: $P(C = C_j) = \frac{P(F|C_j)P(C_j)}{\sum_{i=1}^{N} P(F|C_i)P(C_i)}$

    $c_1 = \{the = \frac{1}{4}, red = \frac{1}{4}, cat = \frac{1}{4}, dog = \frac{1}{4}\}$

    $c_2 = \{the = \frac{1}{5}, red = \frac{2}{5}, cat = \frac{1}{5}, dog = \frac{1}{5}\}$

    $C = \{c_1, c_2\}$

F = the cat the cat the red dog

Solution: First we must start by determining the initial probability of c_1 and c_2.

$$P(c_1) = \frac{1}{2}$$

$$P(c_2) = \frac{1}{2}$$

Now that we have these numbers we are ready to use the formula.

$$P(C = c_1|F) = \frac{\left(\frac{1}{2}\right) * \left(\frac{1}{4}\right)^7}{\left[\left(\frac{1}{2}\right) * \left(\frac{1}{4}\right)^7\right] + \left[\left(\frac{1}{2}\right) * \left(\left(\frac{1}{5}\right)^6 \left(\frac{2}{5}\right)\right)\right]}$$

$$= 0.7045$$

$$P(C = c_2|F) = \frac{\left(\frac{1}{2}\right) * \left(\left(\frac{1}{5}\right)^6 \left(\frac{2}{5}\right)\right)}{\left[\left(\frac{1}{2}\right) * \left(\frac{1}{4}\right)^7\right] + \left[\left(\frac{1}{2}\right) * \left(\left(\frac{1}{5}\right)^6 \left(\frac{2}{5}\right)\right)\right]}$$

$$= 0.2955$$

Now we have the probabilities of c_1 and c_2 given the sentence "the cat the cat the red dog". c_1 has a higher probability so we will classify F to c_1