Problem Set 4: Nondeterministic Finite Automaton

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1. **Conversion Process Detailed**

   (a) Q', the new set of states, will be the powerset of our original set of states Q.

   $$Q' = P(Q) = \{\emptyset, \{q_1\}, \{q_2\}, \{q_3\}, \{q_1, q_2\}, \{q_2, q_3\}, \{q_1, q_3\}, \{q_1, q_2, q_3\}\}$$

   (b) Σ', our new alphabet, will be the same as the prior alphabet with ϵ removed.

   $$\Sigma' = \{a, b\}$$

   (c) Now we first specify δ' without epsilon transfers in mind.

   $\delta' =$

	a	b
$\{q_1\}$	$\{q_3\}$	$\emptyset$
$\{q_2\}$	$\{q_1\}$	$\{q_3\}$
$\{q_3\}$	$\{q_2, q_3\}$	$\{q_2\}$
$\{q_1, q_2\}$	$\{q_1, q_3\}$	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_1, q_2, q_3\}$	$\{q_2, q_3\}$
$\{q_1, q_3\}$	$\{q_1, q_2, q_3\}$	$\{q_2\}$
$\{q_1, q_2, q_3\}$	$\{q_1, q_2, q_3\}$	$\{q_2, q_3\}$

   (d) F', our new set of accept states, is the set of states of R that contain an element of F, the original set of accept states.

   $$F' = \{\{q_3\}, \{q_2, q_3\}, \{q_1, q_3\}, \{q_1, q_2, q_3\}\}$$

   (e) Now we must account for epsilon transfers. We will denote the possible epsilon transfers as $E(\delta(q, a))$ for each element in R.

   $$E(R) = \{q | q \text{ can be reached from R by traveling along 0 or more } \epsilon \text{ values}\}$$

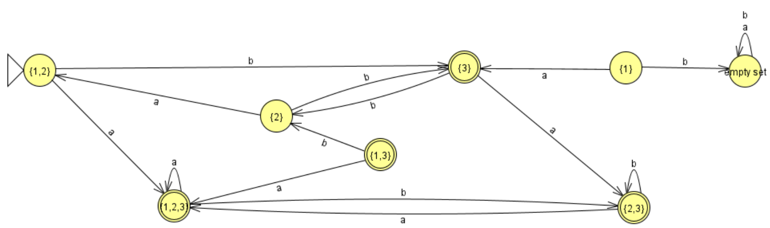
(f) Our new start state will be the result of $E(\delta(q_1, a))$, $\{q_1, q_2\}$.

(g) Finally we adjust the transition function using $E(R)$ for epsilon transfers and we are ready to build the DFA conversion.

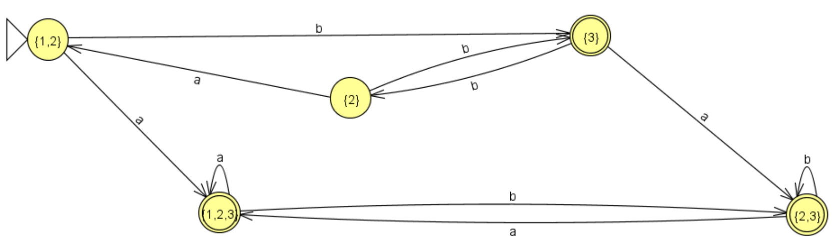
$\delta' =$

	a	b
$\{q_1\}$	$\{q_3\}$	$\emptyset$
$\{q_2\}$	$\{q_1, q_2\}$	$\{q_3\}$
$\{q_3\}$	$\{q_2, q_3\}$	$\{q_2\}$
$\{q_1, q_2\}$	$\{q_1, q_2, q_3\}$	$\{q_3\}$
$\{q_2, q_3\}$	$\{q_1, q_2, q_3\}$	$\{q_2, q_3\}$
$\{q_1, q_3\}$	$\{q_1, q_2, q_3\}$	$\{q_2\}$
$\{q_1, q_2, q_3\}$	$\{q_1, q_2, q_3\}$	$\{q_2, q_3\}$

Original DFA conversion



Unreachable states removed



2. **proof for closure under concatenation**

Let $N_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$ to recognize A, and
$N_2 = (Q_2, \Sigma, \delta_2, q_2, F_2)$ recognize B.

Construct $N = (Q', \Sigma, \delta', q_1, F_2)$ to recognize $A \circ B$.

(a) $Q' = Q_1 \cup Q_2$. The states of N are the states of N_1 and N_2.

(b) The start state for N will be the start state of N_1.

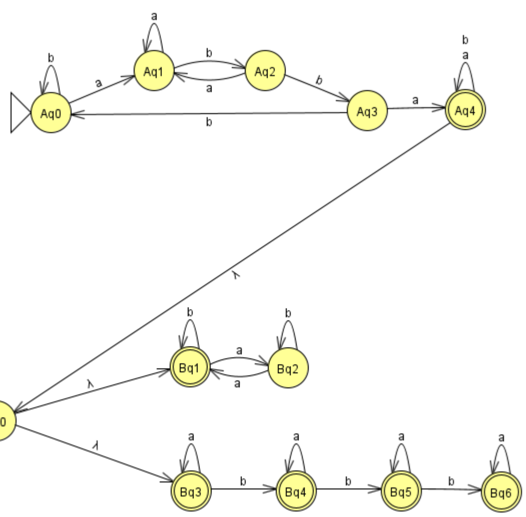
(c) The accept states for N will be the accept states of N_2.

(d) We will define δ' so that for any $q \in Q'$ and any $a \in \Sigma_\epsilon$,

$$\delta(q,a) = \begin{cases} \delta_1(q,a) & q \in Q_1 \text{ and } q \notin F_1 \\ \delta_1(q,a) & q \in F_1 \text{ and } a \neq \epsilon \\ \delta_1(q,a) \cup \{q_2\} & q \in F_1 \text{ and } a = \epsilon \\ \delta_2(q,a) & q \in Q_2 \end{cases}$$

Therefore we have constructed N that recognizes $A \circ B$ proving that regular languages are closed under concatenation.

Resulting NFA



3. proof for closure under union

Let $N_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$ to recognize D, and
$N_2 = (Q_2, \Sigma, \delta_2, q_2, F_2)$ recognize E.

Construct $N = (Q', \Sigma, \delta', q_0', F')$ to recognize $D \cup E$.

(a) $Q' = q_0' \cup Q_1 \cup Q_2$. The states of N are the states of N_1 and N_2, and additionally some new start state q_0'.

(b) q_0' will be the start state for N.

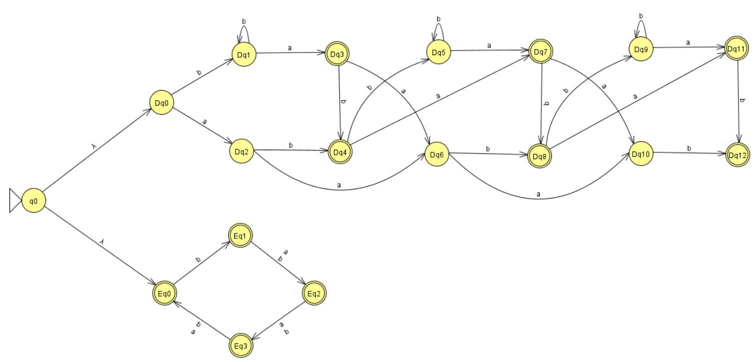
(c) $F' = F_1 \cup F_2$. The accept states of N are the accept states of N_1 and N_2.

(d) We will define δ' so that for any $q \in Q'$ and any $a \in \Sigma_\epsilon$,

$$\delta(q,a) = \begin{cases} \delta_1(q,a) & q \in Q_1 \\ \delta_2(q,a) & q \in Q_2 \\ q_1, q_2 & q = q_0' \text{ and } a = \epsilon \\ \emptyset & q = q_0' \text{ and } a \neq \epsilon \end{cases}$$

Therefore we have constructed N that recognizes $D \cup E$ proving that regular languages are closed under union.

Resulting NFA



4. **proof for closure under concatenation, union, and star operations**

concatenation

Let $N_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$ to recognize M, and
$N_2 = (Q_2, \Sigma, \delta_2, q_2, F_2)$ recognize N.

Construct $N = (Q', \Sigma, \delta', q_1, F_2)$ to recognize $M \circ N$.

(a) $Q' = Q_1 \cup Q_2$. The states of N are the states of N_1 and N_2.

(b) The start state for N will be the start state of N_1.

(c) The accept states for N will be the accept states of N_2.

(d) We will define δ' so that for any $q \in Q'$ and any $a \in \Sigma_\epsilon$,

$$\delta(q,a) = \begin{cases} \delta_1(q,a) & q \in Q_1 \text{ and } q \notin F_1 \\ \delta_1(q,a) & q \in F_1 \text{ and } a \neq \epsilon \\ \delta_1(q,a) \cup \{q_2\} & q \in F_1 \text{ and } a = \epsilon \\ \delta_2(q,a) & q \in Q_2 \end{cases}$$

Now that we have constructed N to recognize $M \circ N$ and proved closure for concatenation let us rename $M \circ N$ to MN to signify it is the machine that is the result of $M \circ N$ while maintaining that it is one machine.

union

Now let $N_3 = (Q_3, \Sigma, \delta_3, q_3, F_3)$ to recognize O. Let us recall at this point that $MN = M \circ N$ and $MN = (Q, \Sigma, \delta, q_0, F)$.

Construct a new $N = (Q', \Sigma, \delta', q_0', F')$ to recognize $MN \cup O$.

(a) $Q' = \{q_0'\} \cup Q \cup Q_3$. The states of N are the states of MN and N_3, and additionally some new start state q_0'.

(b) q_0' will be the start state for N.

(c) $F' = F \cup F_3$. The accept states of N are the accept states of MN and N_3.

(d) We will define δ' so that for any $q \in Q'$ and any $a \in \Sigma_\epsilon$,

$$\delta(q,a) = \begin{cases} \delta(q,a) & q \in Q \\ \delta_3(q,a) & q \in Q_3 \\ \{q, q_3\} & q = q_0' \text{ and } a = \epsilon \\ \emptyset & q = q_0' \text{ and } a \neq \epsilon \end{cases}$$

Now that we have constructed N to recognize $MN \cup O$ and proved closure for union let us rename $MN \cup O$ to MNO to signify it is the machine that is the result of $MN \cup O$ while maintaining once again that it is now one machine.

star

Let us recall at this point that $MNO = (M \circ N) \cup O$ and $MNO = (Q, \Sigma, \delta, q_0, F)$.

Construct a new $N = (Q', \Sigma, \delta', q_0', F')$ to recognize $[(M \circ N) \cup O]*$.

(a) $Q' = \{q_0'\} \cup Q$. The states of N are the states of MNO and the addition of some new start state q_0'.

(b) The state q_0' is the start state.

(c) $F = \{q_0'\} \cup F$. The accept states for N will be the accept states of MNO with the addition of q_0' also being an accept state as well as a start state.

(d) We will define δ' so that for any $q \in Q'$ and any $a \in \Sigma_\epsilon$,

$$\delta(q,a) = \begin{cases} \delta(q,a) & q \in Q \text{ and } q \notin F \\ \delta(q,a) & q \in F \text{ and } q \neq \epsilon \\ \delta(q,a) \cup \{q_0\} & q \in F \text{ and } q = \epsilon \\ \{q_0\} & q = q_0' \text{ and } q = \epsilon \\ \emptyset & q = q_0' \text{ and } q \neq \epsilon \end{cases}$$

Let us now call this final machine we have constructed $[(MN)O*$. We have now constructed $MNO*$ that recognizes the language $[(M \circ N) \cup O]*$, thereby proving the language is closed under all 3 of these operations.

Resulting NFA