Pumping Lemma

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1. $\{0^n1^n \mid n \geq 1\}$

If we want to recognize the language 0^n1^n, let either x or $z = \varepsilon$. We will use either x and y or y and z to recognize the string of 0's then 1's.

Assume $y=0$. In this case the string $xyyy...z$ will have more 0's than 1's.

Now assume $y=1$, In this case we get a similar problem and end up with more 1's than 0's.

Now assume y consists of 1's and 0's. We will in this case have the same amount of 0's and 1's if we assume $x=0$ and $z=1$, however the 0's and 1's in y will be out of order.

2. $\{a^nb^mc^n \mid n$ and m are random integers$\}$

Assume $a^nb^mc^n$ is regular.

Since n and m are random, let $m=0$.

Then we have y^0 where $|y|=0$.

But, by the pumping lemma, for $a^nb^mc^n$ to be regular, $|y|$ must be greater than 0. This is a contradiction.

Therefore, $a^nb^nc^n$ is not regular.

3. $\{a^n b^m \mid n \le m\}$

Assume the language is regular.

Let $z = \varepsilon$. If this is the case $|x^n y^m| \le p$ or specifically in this case $|a^n b^m| \le p$. Now assume that n and m both equal p, this is possible because $n \le m$, not strictly greater. Assuming p for both $|a^p b^p| > p$, thus breaking the 3^{rd} rule of the pumping lemma. Contradicting our original assumption of the language being regular.

4. $\{a^n b^{3n} \mid n \ge 1\}$

Assume $a^n b^{3n}$ is regular.
Then, by the pumping lemma, $xy^iz \in a^n b^{3n}$
Let $x = a$, $y = b$, and $z = a$.
Then $xy^iz = a^n b^{3n} a \notin a^n b^{3n}$.
But, for this string to be regular, $a^n b^{3n} a$ must be a member of $a^n b^{3n}$, which is a contradiction.