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Problem Set 1
Formal Languages

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1. (a) $\{A|A > 30\}$
   (b) $\{B|B \in \mathbb{N} \land B > 10 \land B < 50 \land B = 2n, n \in \mathbb{N}\}$
   (c) $\{C|C = \{\emptyset\}\}$
   (d) $\{D|D = \emptyset\}$
   (e) $\{E = a * nb * n|n \in \mathbb{N}\}$

   I wasn't sure if I could use the concatenation format x and y, xy that you showed in class here but that's what I went with

2. (a) g : X x Y $\implies$ $\mathbb{N}$
   (b)

	A	B	C
1	11	12	13
2	12	13	14
3	13	14	15
4	14	15	16

   (c) g(B,3) = 14

3. (a) X x Y contains all possible combinations of ordered tuples in the format of (x,y) therefore any set containing values in the form of ordered tuples in the format of (x,y) will be a subset of X x Y if the same alphabet is used. I could prove this by listing all possible values and show that each value given in R is contained but it would be 40 values total and I think this explanation is concise enough.
   (b) R is not a function, there are multiple inputs for each output and multiple outputs for each input

4. $\forall a \forall b \Big[(a,b) \in R\Big] \land \Big[\neg(a,b) \in R\Big]$

5. (a) R = {(1,1),(4,2),(9,3),(16,4)}

i. not reflexive, the only case where it could ever be considered reflexive is at 0 or 1, every other case to infinity and negative infinity do not work, as 0 and 1 are the only numbers that exist where the squared value of itself is equal to itself.

ii. not symmetric, the first number in the pair will always be larger, for it to be reversed would break the rules set forth, once again it only technically works at 0 and 1 if you consider the two values being the same to be symmetric.

iii. not transitive, best represented with an example, suppose we have 3 numbers, A, B, and C, and suppose the values for (A,B) and (B,C) are in the set meaning A = B^2 and B = C^2, for it to be transitive, (A,C) must be included as well, however this would mean A = B^2, B = C^2 and A = C^2, this is not mathematically possible for any 3 numbers and would break the rules of the set that were specified, this applies for any numbers used.

(b) R = {(-3,-5),(-2,-4),(-1,-3),(0,-2),(1,-1),(2,0),(3,1),(4,2),(5,3)}

i. not reflexive, the values a and b cannot be the same and satisfy the rules at the same time since the numbers must not equal each other for the difference of a and b to be 2

ii. not symmetric, a must strictly always be larger, for b to be larger and satisfy the symmetric requirements it would no longer follow the rule a-b=2

iii. not transitive, for the set to be transitive the difference of the numbers will always exceed 2, as each tuple was built on the pretense of a-b=2, (6,4), (4,2), (6,2), 6-2=4 and so on, the gap between the number will grow past 2 for the set to be transitive, and therefore not possible.

6. (1,1),(2,2),(3,3),(4,4),(1,2),(2,1),(2,3),(3,2)

7. yes

8. $x^0 + x^1 + x^2 + ... + x^n = \sum_{i=0}^{n} x^i$

base case: n=1, if n =1 $\sum_{i=0}^{n} x^i$, which simplifies to x + 1,

which is less than $\frac{1}{1-x}$ for all x values 0 < x < 1,
n = 0.01 = 1.01 < 1.01010101....,
n = 0.99 = 1.99 < 100

inductive step: $\sum_{i=0}^{n+1} x^i = \sum_{i=0}^{n} x^i + x^{n+1}$,

$\sum_{i=0}^{n} x^i + x^{n+1} < \frac{1}{1-x}$,

suppose n = 4,

$x^0 + x^1 + x^2 + x^3 + x^4 + x^5 < \dfrac{1}{1-x}$ for all 0<x<1,
n = 0.01 = 1.0101 < 1.01010101....,
n = 0.99 = 5.851985 < 100

The two values are closest at the smallest possible value but it never gets closer than that and this has been proven at the n+1 step therefore by induction proving it as a whole.