Optimization II

Project 2 - Dynamic Programming

Group 22 Members

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Introduction

Airlines must fly with all their seats booked in order to maximize their revenue. This requires a pricing strategy that encourages demand for tickets and may involve overbooking to fill empty seats left by passengers who do not show up. The goal is to find the optimal pricing policy and number of tickets to offer for sale on a particular flight to maximize expected discounted profit, which comes from the revenue of ticket sales minus overbooking costs.

We analyzed and determined the best overbooking policy for the two ticket classes in airplanes - Coach and First-Class.

To analyze pricing strategies, we will compare two approaches. The first involves varying the overbooking level to determine the one with the highest expected profit (Fixed Cap Strategy). The second approach involves overbooking the coach to fill up the first class but also includes the flexibility to not put the flight up for booking on a given day (Variable Cap Strategy). Dynamic programming will be used to arrive at a pricing policy that maximizes profit each day by setting prices, determining whether to sell or not, and transferring passengers between classes or kicking them off the flight due to overbooking.

Methodology

To estimate real world data, we approximate the problem in a way that is simple for a dynamic program to solve. We assume there are 365 days left before the flight, which has 20 first class and 100 coach tickets available. We allow for coach to be overbooked up to some n number of seats, but not first class. On any given day, we can sell up to one first class ticket and one coach ticket. One can sell the first class ticket at $425 or $500, which will result in a 8% or 4% chance of being sold, respectively. Similarly, a coach ticket at $300 or $350, which will result in a 65% or 35% chance of being sold, respectively. The chances of each ticket being sold given the price is independent. Sales prices are discounted at 15% APY. Lastly, if a first class seat is not available for sale, then the probability of selling a coach seat increases by 3%.

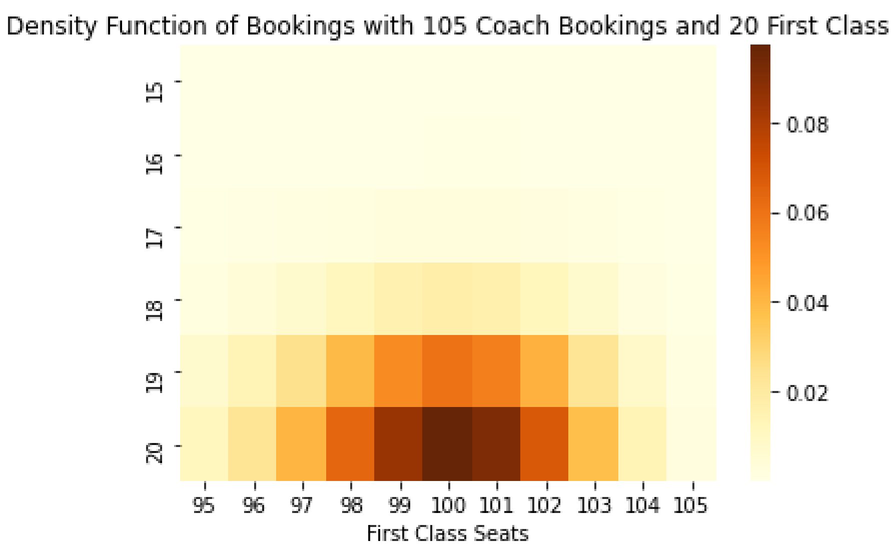
This can be easily converted into a dynamic program. Our goal will be to maximize net present value. Our state space is defined as the number of first class seats left *f*=1,2,3,...,20*,* the number of coach seats left, *c*=1,2,3,..,n, and the time period *t*=1,2,...,365. Let F denote the event that a first class ticket is sold, and C denote the event that a coach ticket is sold. Lastly, let p_c be the price of a coach ticket, and p_f be the price of a first class ticket. Then we have bellman equation

$$
\begin{aligned}
V(f, c, t) \quad &= \max_{p_f, p_c}\{P(F|p_f) * p_f + P(C|p_c) * p_c + \delta * \quad (P(F|p_f) * P(F|p_c) * V(f-1, c-1, t+1) \\
&\qquad +(1 - P(F|p_f)) * P(C|p_c) * V(f, c-1, t) \\
&\qquad +P(F|p_f) * (1 - P(C|p_c)) * V(f-1, c, t+1) \\
&\qquad +(1 - P(F|p_f)) * (1 - P(C|p_c)) * V(f, c, t+1))\},
\end{aligned}
$$

Where

$$
\delta = \frac{1}{(1 + 0.15/365)}
$$

To define the terminal condition, we must consider the probability of no-shows. Each booking has a 5% chance of cancellation if it is coach, or a 3% if it is first class. Each cancellation is again independent, ie. We can model the number of shows as two independent Bernoulli distributions, with joint density function displayed below.



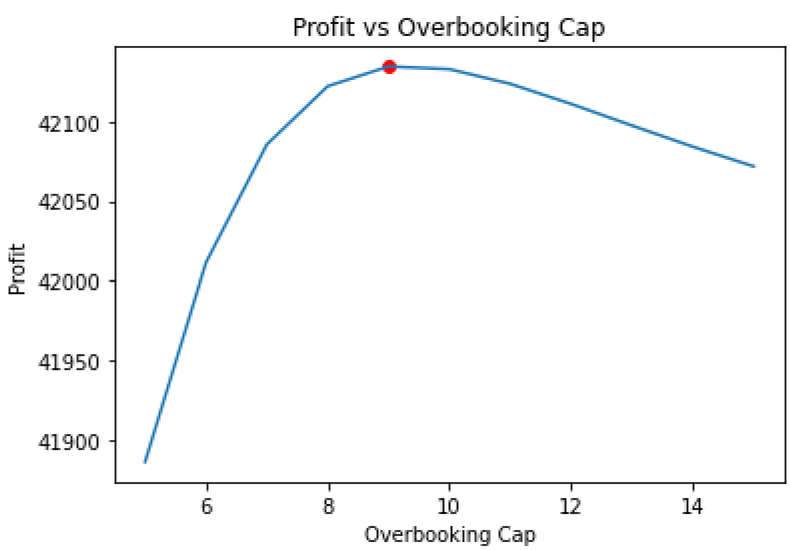
Either customers can be reseated in an available first class seat, which will cost the airline $50, or if no seat is available, they can be kicked off the flight, which incurs a cost of $425. We can therefore set the terminal condition to be the expected costs of overbooking, as according to this distribution.

To analyze pricing strategies, we will compare two approaches. The first involves varying the overbooking level to determine the one with the highest expected profit (Fixed Cap Strategy). The second approach involves overbooking the coach to fill up the first class but also includes the flexibility to not put the flight up for booking on a given day (Variable Cap Strategy). Dynamic programming will be used to arrive at a pricing policy that maximizes profit each day by setting prices, determining whether to sell or not, and transferring passengers between classes or kicking them off the flight due to overbooking.

Strategy 1: Fixed Cap Strategy

In the fixed cap strategy, we allow ourselves to overbook coaches up to some fixed number. That is to say that, on each day, if the coach seats are not yet at the overbooking limit, we will sell coach seats at high or low prices. If, however, coach seats are fully booked up to the overbooking limit, no more seats can be sold. We rerun this model with many different overbooking caps (varying from 6-15), and select the overbooking cap that results in the greatest expected profits. The resulting plot is displayed below.

The **optimal overbooking cap** is **9**, with a profit of **$42134.62.**



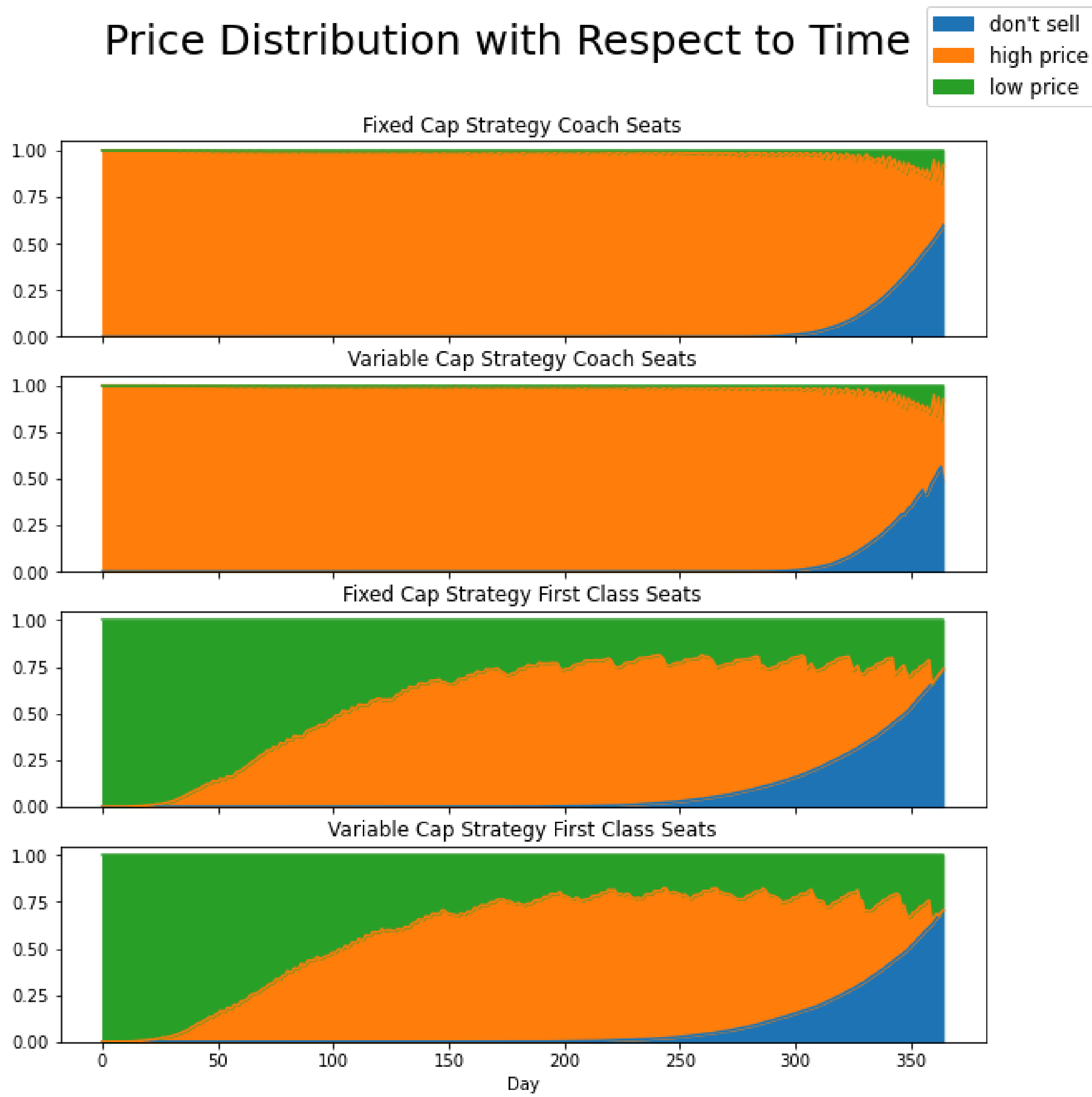
Strategy 2: Variable Cap Strategy (Alternate Strategy)

In the variable cap strategy, we allow ourselves to overbook an arbitrary amount. However, each period, we can choose to not sell coach tickets that period, or not sell first class tickets that period. Formally, this is not difficult to formulate, as since kicking customers off the plane is so expensive, it will likely never be optimal to book more than 120 seats. Thus we can reformulate the problem with a cap of 120, and with some third possible price for both coach and first class, each which result in probability 0 of the ticket being sold. Here, we obtain an expected profit of **$42,139.89.**

Results

We use simulations (n=10,000) to estimate the results. Both strategies appear to result in similar behaviors. To visualize this, the below diagram is presented, displaying for each day what percentage of the time tickets are sold at each price.

1. **Price Distribution w.r.t to Time**

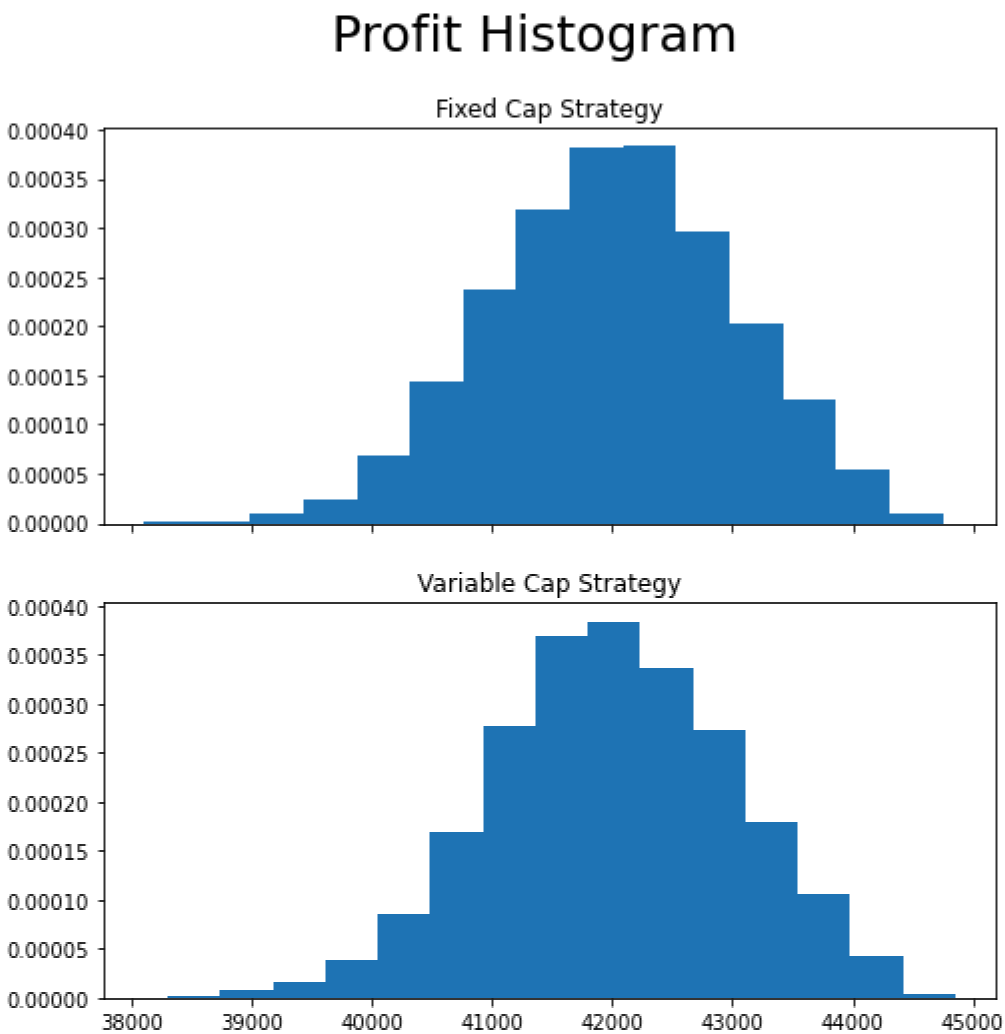


- Both strategies sell coach tickets almost exclusively at high prices, until the last 50ish days where tickets stop being sold or are sold at a lower price.

- Both strategies also behave similarly with fixed cap tickets, where first class tickets are sold at low prices on the earliest days, with higher prices being more common from day 50 to 300, and day 300 onward choosing generally to not sell more seats.

One thing this graphic does not show is how frequently prices for seats change. In simulations, both strategies have the price of coach seats change on average around 6.3 times, and the price of first class seats change on average 8 times. While this is optimal for our simplified model, it can create issues in the real world, in which customers may be incentivized to wait to buy tickets in hopes of a lower price.

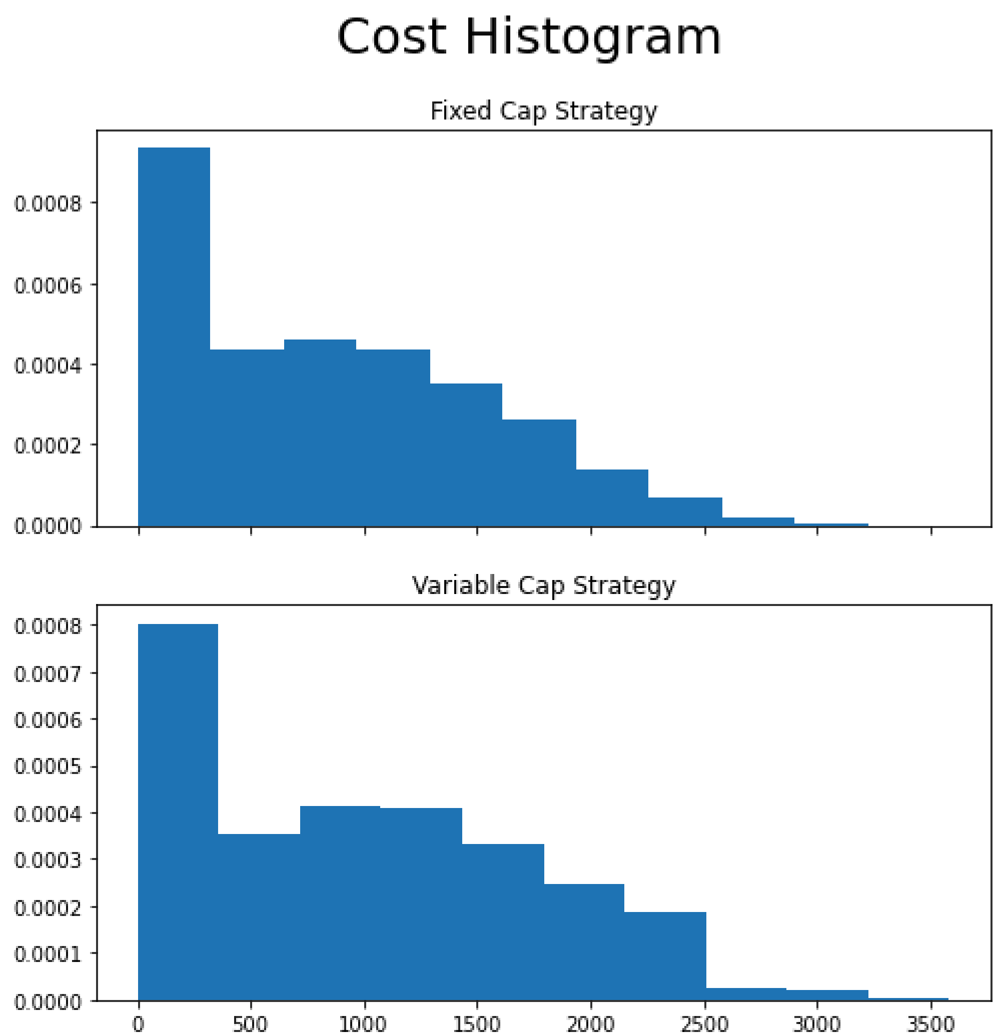
2. Profit Comparison



Upon first glance, the two strategies have roughly similar profits. The two however are slightly different. As with our theoretical means, the expected profits differ by about $5, with the variable cap strategy beating the fixed cap in this regard. However, the two also have differing standard deviations, with the fixed cap varying by about $930 on average as compared to $935 for the variable cap.

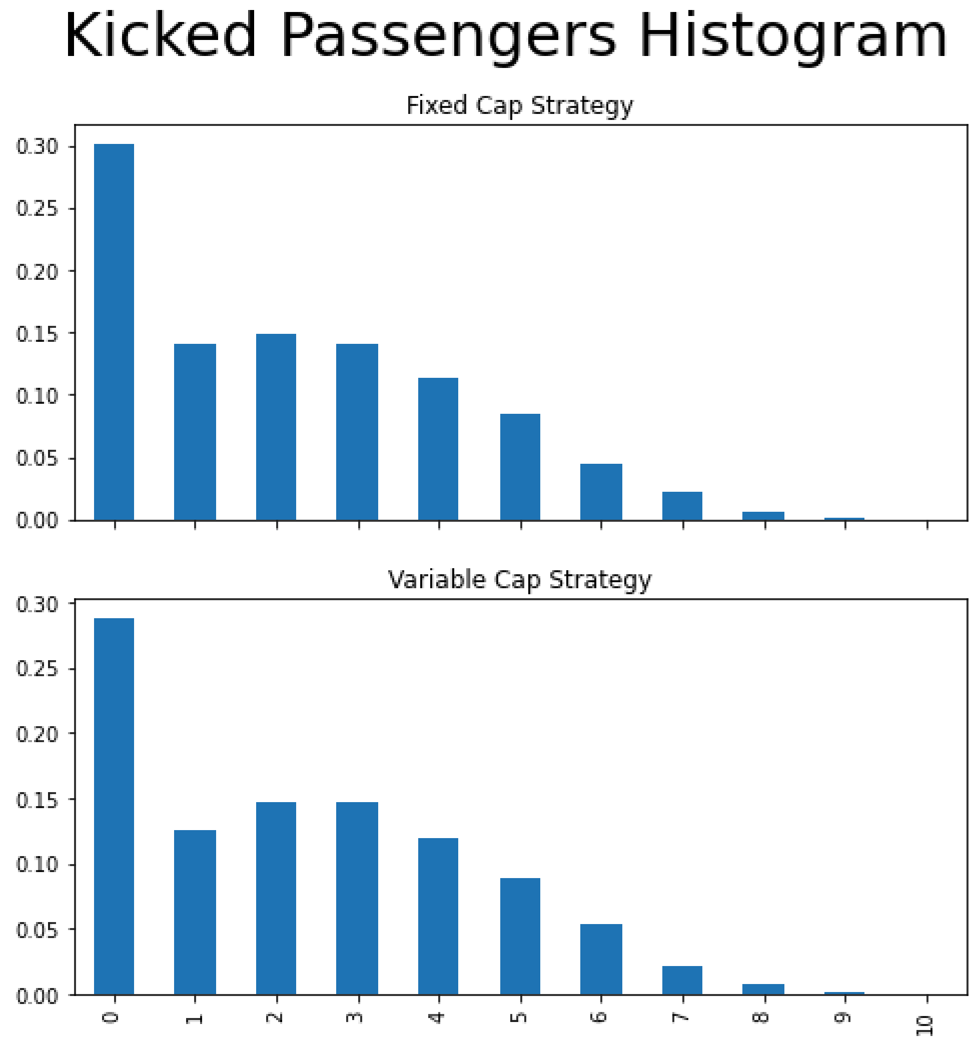
We also can compare the costs incurred by each.

3. **Cost Comparison**



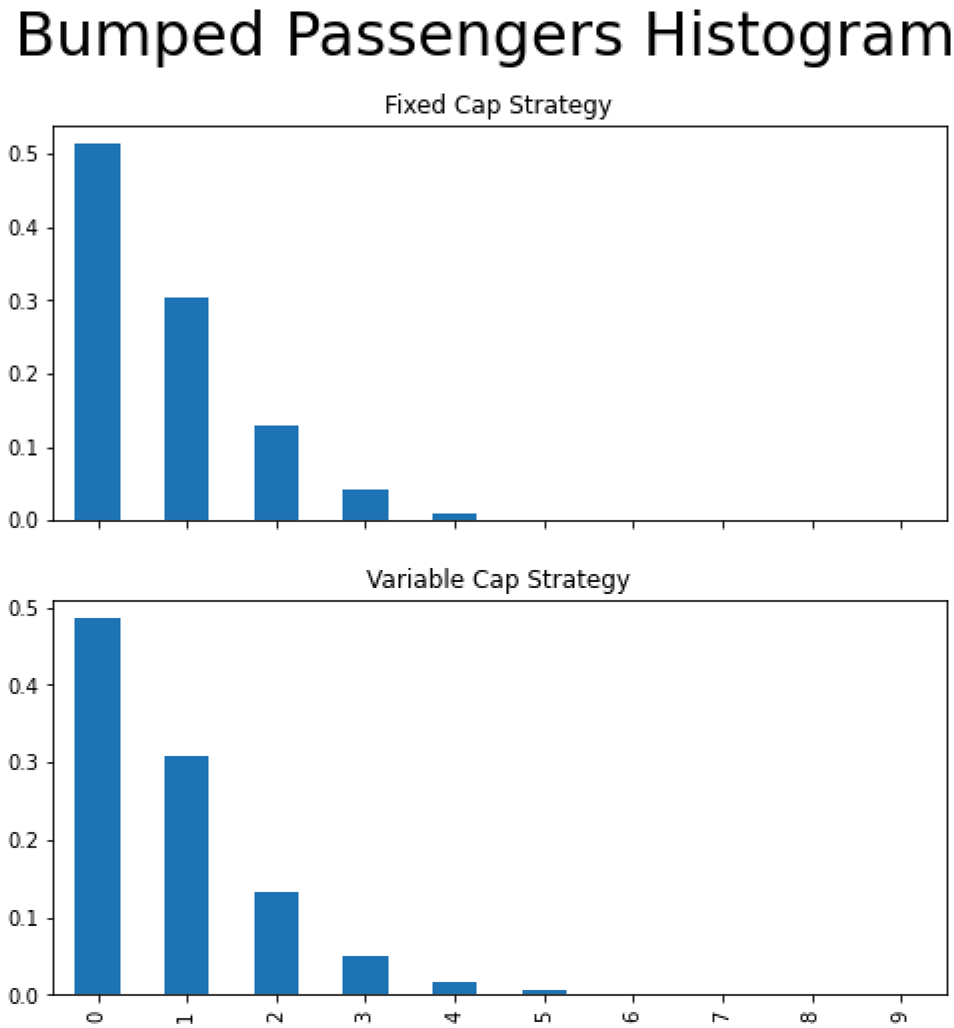
Notably, both often incur little to no costs, with 18.8% and 17.6% of simulated scenarios never incurring any cost for the fixed and variable cap strategies respectively. On average also, the alternate strategy incurs $50 more in costs than the fixed strategy, with averages being $868.91 and $819.95 respectively.

4. **Number of People Kicked Off**



Here we display a chart showing the number of passengers kicked off the plane for each strategy. Again, the two histograms appear identical, with the number of passengers kicked differing very minimally, with 2.328 kicked on average for the variable strategy, and 2.2 for the fixed. Comparatively, it is perhaps worth noting that the variable strategy has sometimes 10 passengers kicked, however, the histogram makes clear that the likelihood of this event occurring is negligible.

5. **Bumped Passengers**



We have found that the average number of passengers who have to be bumped off of a flight using the fixed cap strategy is roughly equal, with on average 0.737 bumped up in the fixed cap strategy, and 0.829 kicked up in the variable.

Comparison of the Two Strategies

Metric	Fixed Cap Strategy	Variable Cap Strategy
Average Profits	**$42134.62**	**$42,139.89**
Average Revenue	$42978.284	$43033.106
Average Overbooking Cost	$819.95	$868.91
Volatility of Profits	930.149	935.141
Overbooked Passengers	2.937	3.155
Bumped Up Passengers	0.737	0.829
Kicked Off Passengers	2.2	2.328

- The mean profits from the Fixed Cap Strategy ($42134.62) is marginally lower compared to the Variable Cap Strategy ($42,139.89)
- Average overbooking costs for the Variable Cap Strategy ($868.91) is higher than the one obtained from the fixed cap strategy ($819.85)
- The mean overbooked passengers, the passengers bumped up and the passengers kicked off are roughly the same from both the approaches
- The volatility of profits is also similar for both the strategies

Conclusion

Variable Cap Strategy (Alternate Strategy) only leads to a slightly higher profit (**~$5**) compared to the Fixed Cap Strategy. Also, both the strategies perform similarly when it comes to customer experience (similar kickoff and transfer rates). However, the Variable Cap Strategy runs faster compared to the Fixed Cap Strategy where we had to decide upon the optimal overbooking cap with multiple loops and hence we recommend choosing it over the Fixed Cap Strategy. While this is not a major issue for our simulations of one flight, it can become an issue when this must be done for every flight, and needing to be updated with the number of no-shows.

Also worth noting is the variable cap strategy can be more difficult for employees to understand. It is easier for staff to understand that we can book up to a set limit, as opposed to having black box output tell them about booking decisions, in which we might choose to leave available first class seats unbooked for the purposes of bumping up.

Lastly, both strategies should be understood as being optimal given our model. This is important as both approaches involve having greatly fluctuating prices. If passengers are rational rather than random, they might wait out high prices in hopes of lower prices to come. Overall however, between the two strategies, we recommend the variable cap strategy.