The University of Adelaide

RANDOM PROCESSES III

APP MATHS 3016

Modelling the effects of the Brown Rat on a native bird population

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Executive Summary

The Black-throated Finch is native to Green Island in far north Queensland and is threatened by invasive Brown rats. This report details the extent of this threat to the native bird population, as well as exploring two possible control strategies for the rat population.

We begin by formulating a model for the population of Black-throated Finches on Green Island, the model used for this investigation was a continuous-time Markov chain. This Markov chain follows a birth-death process, with states ranging from 0 to a maximum of 1000 birds. This maximum population is derived from the availability of nesting sites on Green Island for the birds to occupy. A logistic growth model is used to determine the relevant birth and death rates for the bird population.

Following the development of the Black-throated Finch model, the introduction of Brown rats to Green Island is incorporated. To model two populations simultaneously, we use a 2-dimensional continuous-time Markov chain, based on a Susceptible-Infected-Recovery model. This Markov chain has four possible transitions based on how the birds and rats can die and reproduce. The bird and rat populations are linked together by one of these four transitions; when a rat eats a bird it is therefore able to reproduce. A range of rat birth rates and number of initially introduced rats are considered. The impact of these various birth rates and initial populations are then compared. It is clear from this analysis that the bird population is increasingly threatened as the birth rate of rats increases. For example, there is a 5% chance of survival for the birds after 50 years if the rat's birth rate coefficient is 1.5.

To prevent the potential extinction of the Black-throated Finch, we consider two strategies that will assist in controlling the rat population. The first method tested was to introduce contraceptives to the Brown rats which rapidly reduces the birth rate for the population. We find that the percentage of sterile rats increases the probability of rats dying (by reducing the chance of an equilibrium), but does not decrease the probability that rats kill all of the birds. The second control strategy is to introduce quolls to the island, since they prey on rats but not birds. The new model is then a 3-dimensional Markov chain, compared to the original which is 2-dimensions. Using either of these control strategies alone does not improve the Black-throated Finch's chance of survival on Green Island. The quoll strategy returned similar results to the contraceptives, again reducing the chance of an equilibrium, but not the probability of bird extinction.

Once both control strategies are combined, ie. we distribute contraceptives to the rats, as well as introduce Eastern quolls to the island. The results improve substantially with the control method being incredibly beneficial for the bird population. This is as the probability of extinction was dramatically reduced with the chance of equilibrium or survival increasing. We recommend that the Department of Energy and the Environment use a combination of both of the control strategies provided to have any chance of saving the Black-throated Finch population on Green Island.

1 Introduction

Australia has lost more animals to extinction than any other country in the world, totalling 29 species since colonisation [1]. The Black-throated Finch, native to a small island off the Australian mainland in far north Queensland, is yet another species that could face the same fate. Until recent years, the species has survived relatively well in its native habitat, but now a species of invasive rat threatens its potential extinction. In order to protect and conserve the environment on this Queensland island, the Department of the Environment and Energy will be provided with an in-depth analysis of the extent of the problem, as well as possible control strategies.

The evident facts of the situation are:

- There were approximately 500 birds living on the island prior to the introduction of the rats.
- The population of the birds is limited in particular by the availability of suitable nesting sites on the island.
- It is estimated that half of all possible (available and suitable) nesting sites are occupied at any one time
- The lifespan of the birds is difficult to ascertain, but is believed to be somewhere in the range of 2-5 years.
- The rats threaten the birds in two key ways; preying on the birds which are their primary food source, and competing for the limited nesting resources on the island.

We begin by modelling the physical problem at each stage in Section 2. While population dynamics frequently use Lotka-Volterra equations, having such a small population of the birds means we must instead develop a stochastic model. In Sections 3, 4 and 5 three models will be developed, which will explore the population dynamics of the Black-Throated Finch, the introduction of rats and any potential control strategies we may utilise to reduce the impact of the introduction of the foreign rats.

2 Black-throated Finch model

We begin by modelling the population of the Black-throated Finch on Green Island before the introduction of rats. The population of these birds at time t is given by the Continuous-Time Markov Chain $B(t)$. We know that the population of Black-throated Finches is approximately 500 before the introduction of rats and that half of the available nests are used (250). Since the birds are limited by the number of nesting sites on the island, of which they occupy approximately half, we assume that the upper bound on the bird population is 1000 (equivalent to 500 nests). Therefore we define the state space of the bird population as $S_{B(t)} = \{0, 1, 2, \ldots, 999, 1000\}$. We also define the rate at which a bird is born as λ_B and the rate at which a bird dies as μ_B. The state of things on the island before the introduction of rats is represented in Figure 1 below.

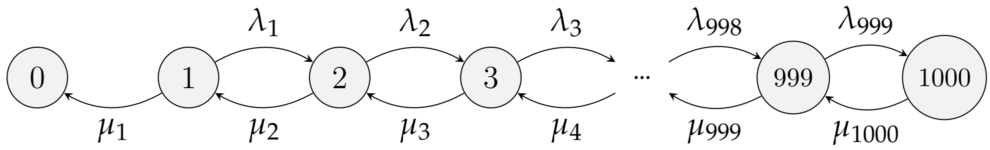


Figure 1: Population process $B(t)$

The death rate of birds, μ_B, is the simplest of the two rates to determine. We assume that the expected life span of Black-throated Finches is uniformly distributed over the provided interval of 2 to 5 years. Therefore, we expect the average life expectancy to be 3.5 years. Hence, we take the death rate to be:

$$\mu_B = \frac{1}{\text{mean life expectancy}} B(t) = \frac{1}{3.5} B(t) = \frac{2}{7} B(t)$$

where $B(t)$ is the bird population at time t. To determine the birth rate of birds, we first consider the growth rate of the Finch population. Black-throated Finches have broods of approximately 5 chicks, and have 1.5 broods per year on average. Meaning that the populations would grow exponentially. We know however that the population is limited by the number of nests, therefore a logistic growth model was deemed appropriate. This is as we have exponential growth limited by a maximum capacity. We define the birth rate as:

$$\lambda_B = \beta_B B(t) \frac{N - B(t)}{N}$$

where β_B is the per capita growth rate and N is the carrying capacity. That is, the limiting factor of the population. This occurs when there are two birds in each of the nests on Green Island. In order to determine β_B, we assume that the Black-throated Finch population is at equilibrium before the introduction of rats. We test a range of values for β_B, narrowing in until the population reaches equilibrium at 500 birds. These tests can be seen in Figure 2. The birth rate β_B that will cause the bird population to be somewhat constant under regular conditions will be found by analysing the population over the next 100 years, as well as the mean value of each population over this period.

Implementation

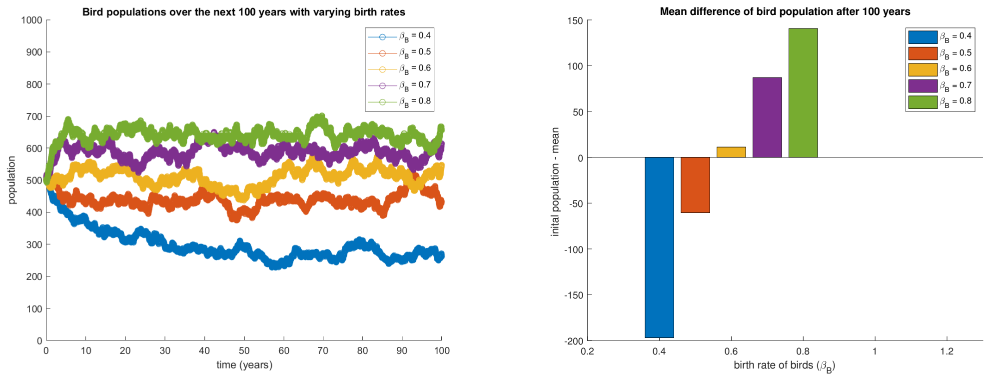
As the model is a Poisson process only one event can ever occur at a given time, and the time of each transition occurs with an exponential distribution. The time of the transitions these were simulated by transforming Matlab's inbuilt uniformly distributed random number generator. It can be determined that a time t_i will have a exponential distribution if:

$$t_i = t_{i-1} - \frac{\ln(r)}{\lambda}$$

where $\lambda = \lambda_B + \mu_B$, $r \sim U(0,1)$ and $t_0 = 0$.

A cumulative density function can be made over the interval $[0, \lambda_B + \mu_B]$. If a random number that is uniformly distributed over this interval falls within the interval $[0, \lambda_B]$ the bird population will increase by one. If not then the other transition $(B \to B - 1)$ will occur at this time instead. The transition changes the state of $B(t_i)$ resulting in a change in the bird's population, this is then recorded at the time of each of these transitions.

2.1 Results



(a) Bird populations with varying birth rates over 20 years.

(b) Mean difference of the bird population after 20 years.

Figure 2: A birth rate coefficient (β_B) of 0.6 appears to create an equilibrium in the population. The mean difference also agrees that 0.6 is the optimal birth rate for the birds, before the introduction of rats.

From comparing the population levels for 100 years as well as the mean of the population, it can be noted that $\beta_B = 0.6$ will return the best approximation of a population at equilibrium. As this is the desired result, from this point on wards, $\beta_B = 0.6$ will be used for all further models.

2.2 Assumptions

The Black-throated Finch model relies on a number of assumptions. While these may limit the reliability or accuracy of the results, they were necessary in order to create a working model. The first major assumption made for this model was that the population was stable when the data was recorded and will remain stable unless impacted upon by external factors. Using this, we began to solve for the birth rate for the finches. We assume that the life expectancy of the finches is uniformly distributed. This was because any other distribution would have required parameters which we have no way of solving for. We then assumed that the population follows a logistic growth pattern. This is since finches produce 7.5 offspring on average per year, so the unrestricted growth would be exponential. We assume that no additional nests can be created. Therefore, when the upper limit of 500 nests (with two birds per nest) is factored in, this exponential growth is restricted and thus we use a logistic model with a carrying capacity of 1000.

We assume that the finches give birth throughout the year at a consistent rate. Though this doesn't align with their known breeding seasons, by averaging the rate out to an annual rate, we expect the long-term effect to be approximately the same. However the short term results may be misleading. We also assume that chicks all hatch independently throughout the year, rather than the broods hatching over a couple days. In order to reproduce, we assume that finches can always find a mate, and so we do not track or differentiate between males and females. In addition, we assume that birth and death rates are the same regardless of gender and age i.e. older birds die at the same rate as younger birds. Finally, we do not consider the maturity of birds, and instead consider all birds 'mature' from birth.

3 Modelling the introduction of rats

We now consider the introduction of rats onto the island and model the population dynamics between the Black-Throated Finch and the invasive Brown Rat. The populations of the birds and the rats are modelled by utilising a Susceptible-Infected-Recovery (SIR) continuous-time Markov chain. Where birds are susceptible, rats are infectious and the death of the rats represent recovered. This particular model was utilised as we assume that birds are the primary food source for rats. We also assume that a rat needs to eat a bird in order to have enough energy to give birth and feed the litter. Thus, we can draw a relation between the death of birds (via rats) and the birth of rats. This process is defined as the Markov chain $P(t)$ with state space $S_{P(t)}$

$$P(t) = \{B(t), R(t)\}$$
$$S_{P(t)} = \{(0,0),(0,1),(1,0),(1,1),(2,1),\dots\}$$

where $R(t)$ is the rat population at time t. We define four possible transitions for this CTMC. These transitions are based off of five events; bird is born, bird dies, rat is born, rat dies and rat eats bird. Since we assume that rats can only give birth after having eaten a bird, we combine these two events into one transition for $P(t)$. We also assume that when a rat gives birth, it produces 6 offspring, which is reflected in its respective transition. The four transition rates are given in Table 1. It must be noted that birds and rats can only give birth if there is at least one of them alive ($B(t), R(t) \geq 1$). Hence, $(0,0)$ is an absorbing state (probability of leaving this state is 0).

Event	Transition	Rate
bird gives birth	$B \to B+1$	$\lambda_B = \beta_B \frac{B(N-B)}{N}$
bird dies	$B \to B-1$	$\mu_B = \gamma_B B$
rat eats bird, and gives birth to litter	$B, R \to B-1, R+6$	$\lambda_R = \beta_R \frac{BR}{N}$
rat dies	$R \to R-1$	$\mu_R = \gamma_R R$

Table 1: Transitions of the model

Implementation

The same method as in section 3 was used to simulate the exponentially distributed times.

$$t_i = t_{i-1} - \frac{\ln(r)}{\lambda}$$

However, $\lambda = \lambda_B + \mu_B + \lambda_R + \mu_R$ as we now have four transitions. Now a similar cumulative density function can be made over the interval $[0, \lambda]$. With a bird being born if the random number r falls within $[0, \lambda_B]$, and a bird dying if $r \in (\lambda_B, \lambda_B + \mu_B]$, and so on.

To adapt an SIR model such that it was appropriate for a predator vs. prey scenario, we assumed that the rate at which rats eat birds is equivalent to the rate of infection. Thus, the transition rate in the form of $\beta\frac{SI}{N}$ was used. With recovery rate (death rate of rats) being of the form γI. Where S is the susceptible class, I is the infectious class, N is the capacity of the system and $\beta, \gamma \in \mathbb{R}$.

Summary of Parameters

For this updated model we define the following parameters:

$B(t)$: population of birds at time t months since the introduction of rats
$R(t)$: population of rats at time t months since the introduction of rats
μ_B : rate at which the bird population decreases
λ_B : rate at which the bird population increases
μ_R : rate at which the rat population decreases
λ_R : rate at which the rat population increases
β_B : birth rate of a bird
β_R : rate at which a bird is eaten by a rat, resulting in a litter being born
γ_B : death rate of a bird
γ_R : death rate of a rat

Outcomes

We will consider three distinct outcomes of the model.

- Birds survive : Birds survive the introduction of rats and rats become extinct on the island.
- Equilibrium : After time t, both populations still exist on the island.
- Birds become extinct : Rats cause the extinction of the native bird population.

3.1 Results

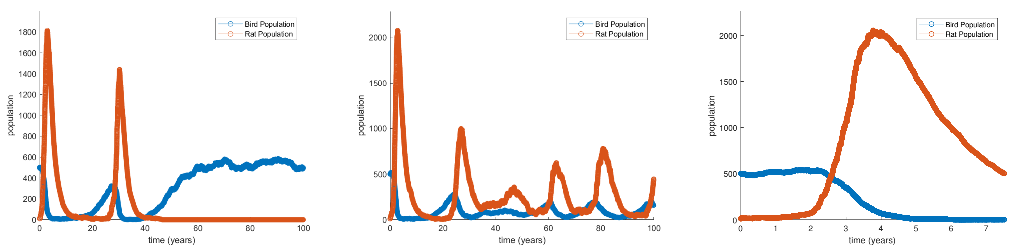


Figure 3: Example plot of each of the three outcomes after 100 years (birds survive, equilibrium and birds become extinct, respectively).

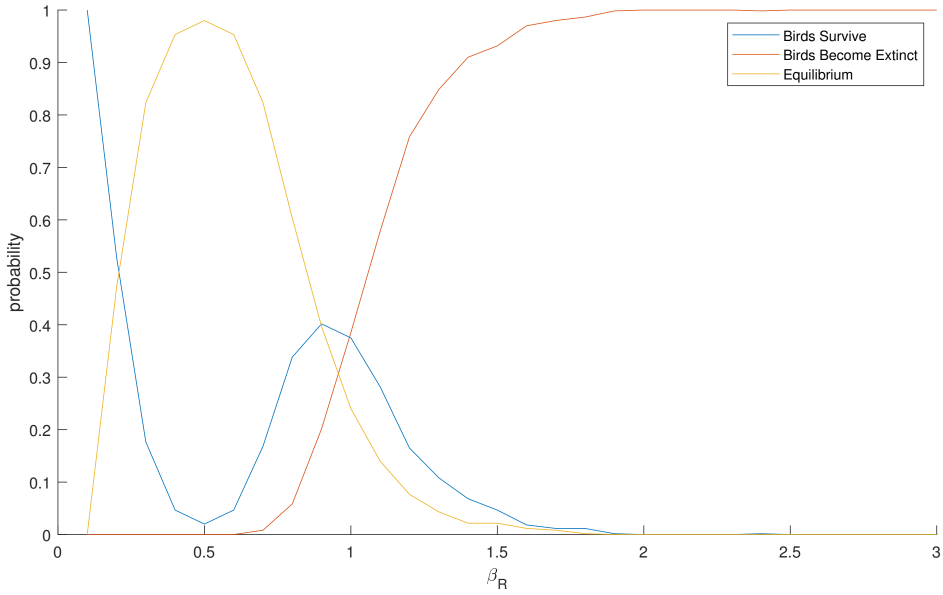


Figure 4: Probability of each outcome occurring after 50 years with varying rat birth rate. For values of β_R less than 0.5, birds are almost certain to survive, albeit rats are likely to still be present on the island. As β_R increases, birds are almost certain to go extinct.

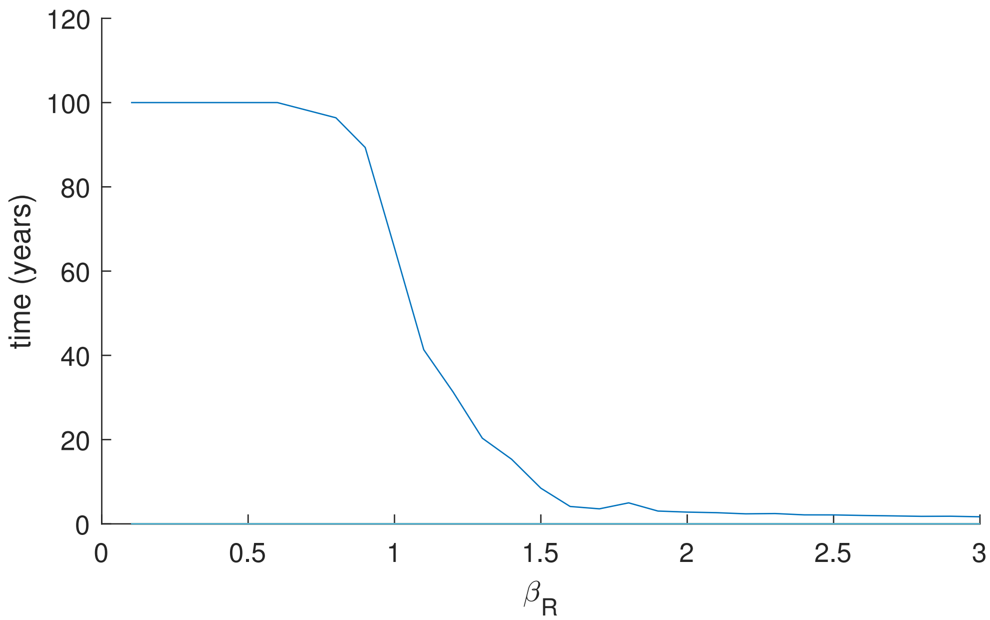


Figure 5: Expected life span of the bird population with varying rat birth rate. Increase in β_R corresponds with increased rate until extinction.

As the model has inherent inaccuracies, over a large amount of time these will be amplified. Hence, we chose a maximum time of survival of birds as 100 years. As we would expect, the average time until extinction of the bird population strictly decreases over β_R (with a minor outlier at $\beta_R \approx 1.8$). Moreover, values of β_R greater than 1.6 resulted in the extinction of birds within 5 years.

Three different initial sizes of the initial rat population was tested. The selected sample was 5, 10 and 20 rats (which are displayed respectively).

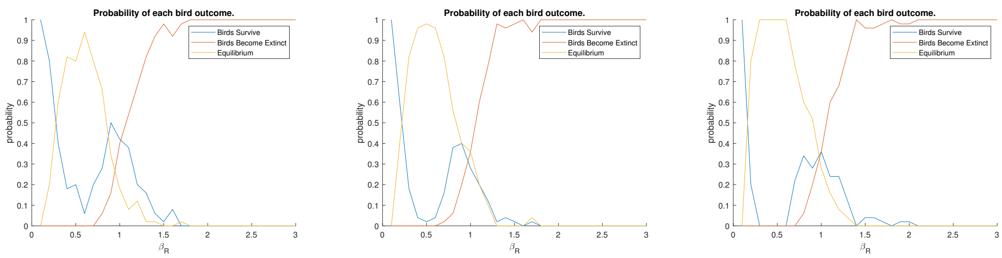


Figure 6: Example plot of each of the three outcomes after 50 years with varying amounts of initial rats ($\beta_R = 1.8$).

From Figure 6 it is evident that increasing the amount of rats appears to have a significantly negative effect on the survival chances of the birds. For all of the following results in the report it will be assumed that 10 rats were introduced to the island.

3.2 Analysis

For the example plots, 10 rats were introduced to the island with a birth rate β_R of 1. There appears to be a periodic pattern between the two populations, which can be seen on the first two plots in Figure 3. All three of these plots display exponential growth of the rat population which appears to reach a maximum after approximately 3 years. The rapid increase in rat population causes an abrupt decline in the bird population, thus leaving rats without this food source. This causes starvation among the rats, which is represented by the steep decline in rat population from 4-7 years. Hence, the number of predators for the birds is reduced and the bird population can recover. However, if the rats kill birds too rapidly the birds will become extinct, killing both species in the long term. Figure 3b displays what we expect from a predator versus prey model in equilibrium. This periodicity is consistent across data and differential equations models such as those derived from Lotka-Volterra equations. Therefore, we are inclined to believe there exists a long term equilibrium for the two populations.

As all three outcomes can occur with the same value for β_R, a probability density function was simulated to approximate the likelihood of each event occurring for a given β_R. It is apparent that birds are almost certain to survive for 50 years given a small value for β_R (< 0.3), with β_R values between 0.3 and 0.9 most likely causing an equilibrium outcome. With bird extinction being the most likely event for values of β_R greater than 1. The probability of survival appears to increase for $\beta_R \in (0.5, 1)$ before decreasing again, this corresponds with a decrease in the probability of equilibrium. This increase in the probability that birds survive causes there to be an interval of $\beta_R \in (0.9, 1)$ where each event is equally as likely to occur. The reasoning for this phenomena can be put down increased variance of the model. An increase in β_R corresponds to an increase in the rate of birth deaths, thus increasing the probability that the bird population reaches the absorbing state 0 within the time period. However, this sudden decrease in the Black-throated Finches drastically reduces the amount of food sources for the rats, causing mass starvation. Thus, a scenario occurs where if the birds can narrowly survive extinction for long enough the rat death rate causes the rat population to become extinct, allowing the birds to survive.

3.3 Assumptions

This model incorporates rats into the original Black-throated Finch model and thus shares its assumptions. Due to the complexity of modelling the population dynamics between two species, additional assumptions have been made. Perhaps the biggest assumption made was that rats need to eat a bird in order to reproduce. This can be justified since the birds are the primary food source for rats, and animals clearly need to have eaten in order to have enough strength to reproduce and to be able to feed the offspring. In addition, this assumption is what essentially links the two populations together into one model, which is key to exploring the relationship between them. In relation to rat reproduction, we assume that it is acceptable to consider an average litter size of all rats at all times. In reality, the size of litters would vary largely due to external factors such as weather and food supply. Similarly to the Finches, we assume that the rats are not any more likely to die at an old age than a young age although this would be the case in the wild.

4 Control Strategies

As the brown rat is threatening the survival of the Black-throated Finch, the Department of Environment and Energy must consider enacting some population control strategies. We consider two main control options; decreasing the birthrate of the rats, and increasing the death rate of the rats. Each of these options are expected to impact the population of the Black-throated Finch in slightly different ways, but ultimately resulting in survival for the Finch.

4.1 Rat Contraception

One of the biggest contributors to the thriving population of rats is their extremely high birth rate. Decreasing this rate is expected to be an effective rat population control strategy. It is estimated that within a real life setting, a population of 2 rats can grow to over 15,000 in just one year [6]. It follows that being able to render two rats infertile is equivalent to decreasing the yearly population by 15,000 rats. To reduce the birth rate of rats, we consider distributing easily consumed contraception on Green Island. This method has already been proven successful in places such as New York City, which has seen a 43% reduction in underground rat activity since its introduction [8]. An Arizona-based company called SenesTech produces a world-first non-toxic fertility control product for rats. The bait contains a chemical known as 4-vinylcyclohexene diepoxide (VCD). VCD destroys female rats' ova as well as reducing the sperm production in males. It is thus assumed within our model that the contraceptive affects every rat that consumes it. In addition, it is assumed that the bait is accessible only to the rats and so it poses no threat to the Black-throated Finch.

4.1.1 Impact on population model

We adapt the original model to reflect the introduction of rat contraceptives. As previously mentioned, this means the birth rate for the rats, λ_R, is decreased. Since Green Island covers only 15 hectares, it is assumed that distributing the contraceptive bait across the island is relatively achievable. It must be noted, however, that it would be unrealistic to assume that the bait could be distributed to such a level that it is consumed by all of the rats.

The only transition modified for the contraceptive model is the number at which the rat population increases. Originally, when a rat gave birth the population would increase by the average litter size of 6. However, when a percentage of the rat population is given contraceptives, the average litter size will be reduced. This is how the effects of contraceptives will be averaged across the population. For example, if 50% of the rat population received contraceptives, we would expect that the average amount of offspring produced would also be halved, hence the new rat birth transition for the model would be: $(R \to R+3)$.

event	transition	rate
bird gives birth	$B \to B+1$	$\beta_B \frac{B(N-B)}{N}$
bird dies	$B \to B-1$	$\gamma_B B$
rat eats bird, and gives birth to litter	$(B,R) \to (B-1, R+6-k)$	$\beta_R \frac{BR}{N}$
rat dies	$R \to R-1$	$\gamma_R R$

Table 2: Transitions of the contraceptive model for some $k \in \{0,1,2,3,4,5,6\}$, where k represents the number in which average litter size is reduced due to contraceptives. The corresponding proportion of rats that require contraceptives is determined by $\frac{k}{6}$.

As it would not be feasible for the Australian Government Department of Environment and Energy to introduce contraceptives immediately to such a small rat population, we have allowed for a 5 year period before acting upon the problem. This timeline is likely a worst case scenario, as the rat population has already reached its highest (as seen in Figure 4). It should therefore display the effectiveness of contraceptives under any circumstances. Note that it is also assumed that the contraceptives are introduced and effective immediately after 5 years.

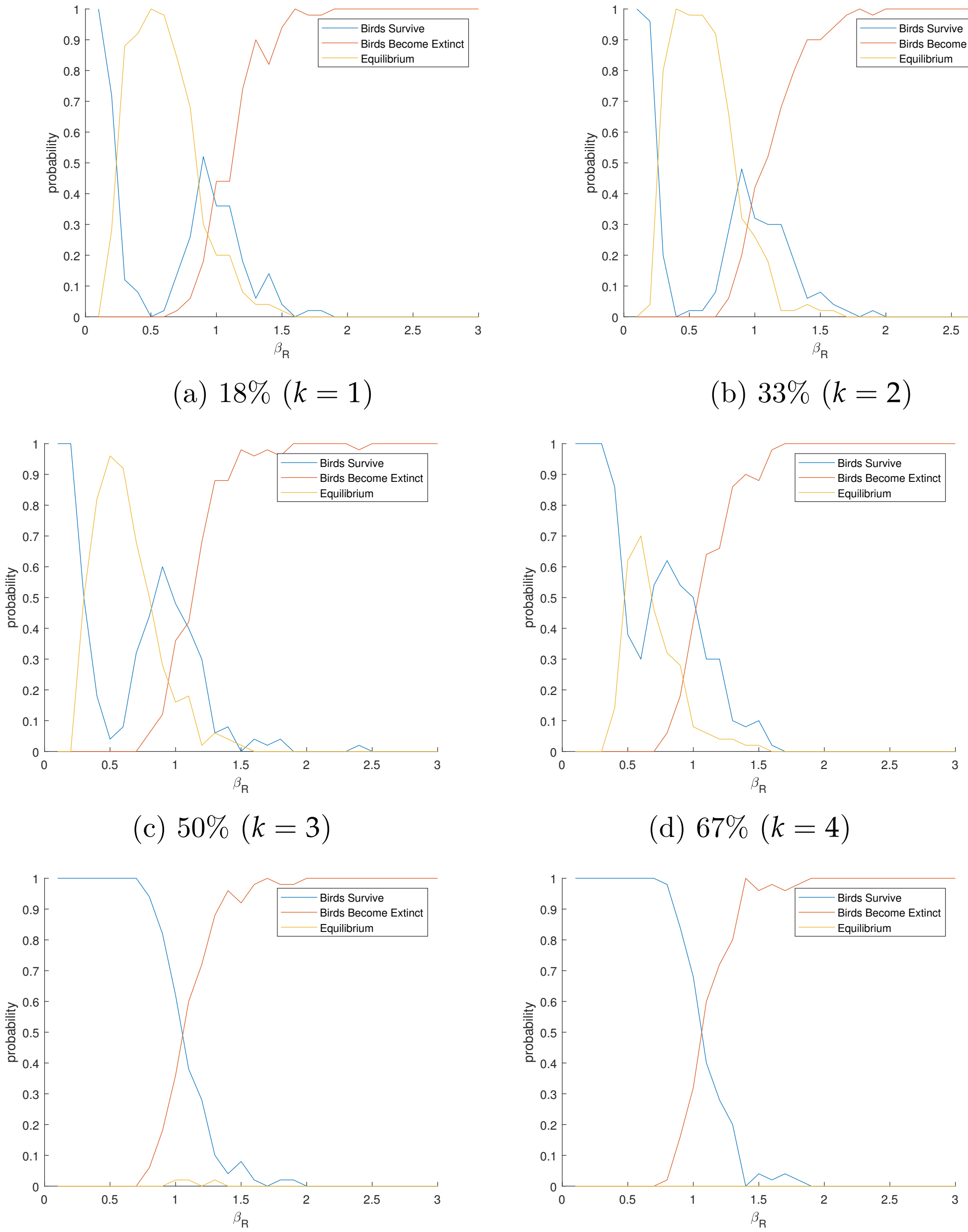
Another simulation was run to compare the effects of introducing contraceptive bait to the rat population. This simulation, once again, generates a probability density function for each of the three outcomes (bird survival, equilibrium and bird extinction) that can occur with varying rat birth rates (β_R).

4.1.2 Assumptions

This model which incorporates contraceptives relies on the same assumptions as the model from Section 3. In addition, we assume that the contraceptive is 100% effective on the rats whom consume it. We also assume that it is feasible to give contraceptives to relatively large proportions of the rat population. This is since the island is only 15 hectares in area, so we decided to at least explore a range for the percentage of rats consuming contraceptives. It will be up to the discretion of the Department of Energy and the Environment to determine what is actually a reasonable proportion of the rat population to which contraceptives are distributed. Another assumption that has been made is that giving contraceptives to some percentage, say 50% of the rat population would result in 50% of the females consuming it. In turn, this would halve the expected average litter size for the population. Since the contraceptive also reduces sperm production and maturity in males, the effective reduction in litter size would actually be slightly higher than the percentage of the rat population that receives the contraceptive. For example, giving 50% of the population contraceptives results in an expected 50% of females consuming it, which cuts the average litter size in half. When you factor in the reduced sperm in males too, the litter size would theoretically be even smaller than the originally expected half.

4.1.3 Results

The aim of this contraceptive solution is to increase the threshold of rat birth rate, β_R, for which birds can survive. Due to the stochastic nature of the model, the results were recorded over 50 trials and a likelihood percentage of each outcome was recorded. A sample probability density function could then be formed. Note that the case of $k = 0$ was not included, as when $k = 0$ the model is unchanged from the original.



(a) 18% ($k = 1$)
(b) 33% ($k = 2$)
(c) 50% ($k = 3$)
(d) 67% ($k = 4$)
(e) 83% ($k = 5$)
(f) 100% ($k = 6$)

Figure 7: Probability of each bird outcome with different prescriptions of rat contraceptives. The percentage of sterile rats will increase the probability of rats dying out, but will not decrease the probability that rats kill all of the birds.

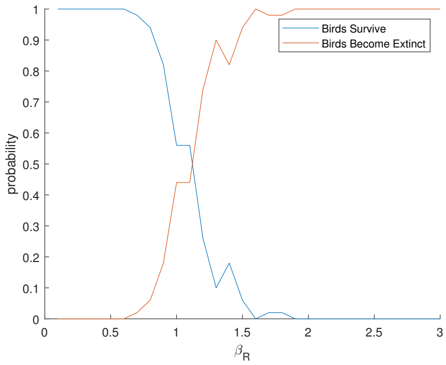
4.1.4 Discussion

It is noticed in Figure 7 that when 18%, 33%, 50% or 67% of the rats have ingested contraceptives, the birds and rats are able to survive simultaneously. That is, there is some probability that the populations reach equilibrium. However as seen again in Figure 7, it is almost certain that only one species will survive when 83% of rats are sterile. That is, as the contraceptive prescription increases, the

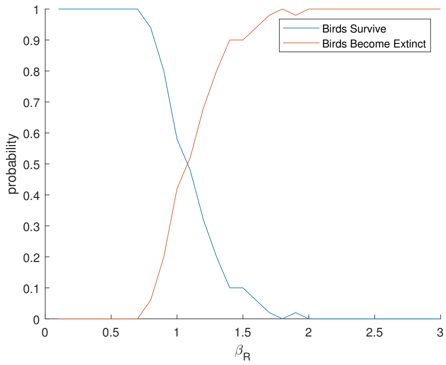
probability that birds survive and rats die out also increases. (Note that “birds surviving” means birds only, and rats become extinct). Interestingly, the probability that birds become extinct remains somewhat consistent regardless of the amount of rats that receive contraceptives. Hence, the probability increase of birds surviving comes directly from the probability of equilibrium. This means that an increase in contraceptive distribution results in rats being less likely to survive on the island after 50 years - a positive result for the bird population. However, as the probability of birds becoming extinct does not appear to change, there exists a significant likelihood that they die out if the rat birth rate exceeds 1.
To test the hypothesis that the contraceptives exclusively reduce that chance of population equilibrium, another simulation was run including only two outcomes: birds surviving or birds becoming extinct.

Outcomes

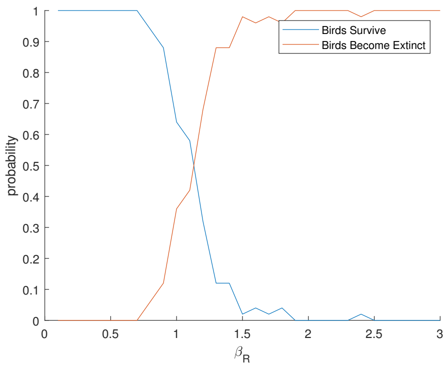
We will now consider two distinct outcomes of the model.
Birds survive : There exists a bird population after 50 years.
Birds become extinct : Rats cause the extinction of the native bird population.
The following plots were created considering only these two outcomes:



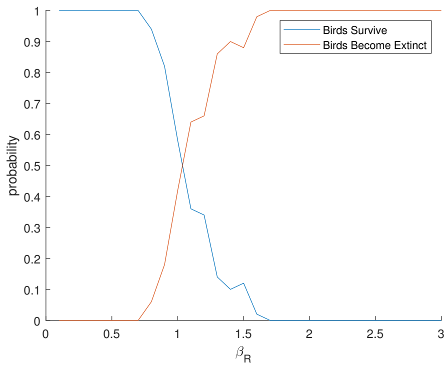
(a) 18% ($k = 1$)



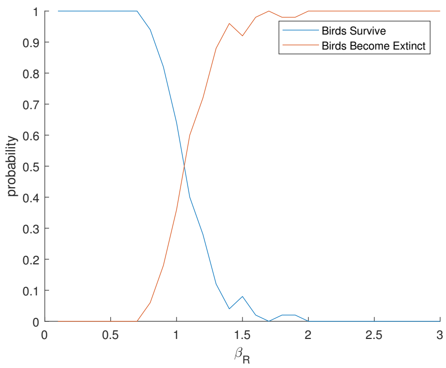
(b) 33% ($k = 2$)



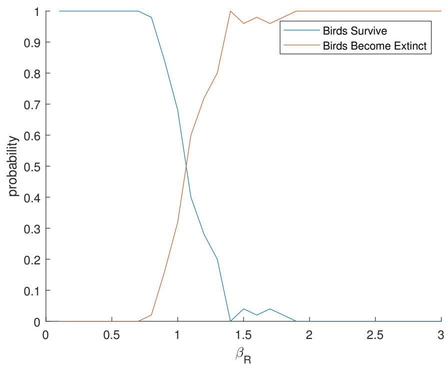
(c) 50% ($k = 3$)



(d) 67% ($k = 4$)



(e) 83% ($k = 5$)



(f) 100% ($k = 6$)

Figure 8: Survival vs Extinction. Both the probability of birds survivng and probability of birds becoming extinct remain constant regardless of the contraceptive level.

We can deduce from figure 8 that the level of contraception directly increases the probability of rats dying out in what was previously an equilibrium state. Contraceptives appear to be highly effective in large quantities, however in small quantities they do not lead to any significant improvement.

There also appears to be an intercept between the probability of birds surviving and the probability of birds becoming extinct at $\beta_R \approx 1$. Interestingly, this occurs whether we count one or two events, that is, if we include equilibrium in bird survival.

Within the derivation of this contraceptive model, the rate at which birds were eaten did not change. The only alteration from the original model was the expected number of rat offspring. We can thus deduce that the probability of birds becoming extinct does not change because the rate at which birds are being eaten is constant. Therefore, regardless of the amount of rat offspring, $\beta_R > 1.1$ will result in birds being more likely to become extinct than survive.

4.2 Introduction of Eastern Quolls

Our second control strategy to consider is to increase the death rate of the rats. While there are many existing methods to achieve this, common options including poison and traps are “generally short-term, and rodents will return if food and shelter are still available” [9]. We therefore aim to provide a new alternative for Green Island specifically. Our suggestion is to introduce Eastern Quolls to the island.

Eastern Quolls are small endangered carnivorous mammals that are native to Australia and prey on small mammals such as mice and rats. Though the Eastern Quoll will eat dead birds (or other dead animals), they can not climb trees and thus cannot actively prey on birds such as the Black-throated Finch. The Eastern Quoll is now considered extinct on mainland Australia because of introduced foxes but still survives in Tasmania [3]. We therefore propose the introduction of the Eastern Quoll to Green Island which would both assist in controlling the rampant rat population, thus aiding the survival of the Black-throated Finch, as well as conserving the endangered Eastern Quoll itself.

Since Green Island spans only 15 hectares [4] and is already a national park, it is a prime candidate for a wildlife sanctuary that could be home to endangered animals native to Australia. This would be the third time that Eastern Quolls are reintroduced to mainland Australia after two successful trials, in which some were released into a sanctuary and 20 were released into a national park in New South Wales [7].

4.2.1 Impact on population model

The birth/death model is now adapted to reflect the introduction of the Eastern Quoll. It is assumed that the Quoll does not compete with the Black-throated Finch for food or nesting materials, so the only adaptation to our model is the rat death rate, μ_r. Female Quolls birth only one litter per year and can give birth to up to 20 young, yet only about 6 of these pups will survive.

event	transition	rate
bird gives birth	$B \to B+1$	$\beta_B \frac{B(N-B)}{N}$
bird dies	$B \to B-1$	$\gamma_B B$
rat eats bird and gives birth to litter	$(B,R) \to (B-1, R+6)$	$\beta_R \frac{BR}{N}$
rat dies	$R \to R-1$	$\gamma_R R$
quoll eats rat and gives birth to litter	$(R,Q) \to (R-1, Q+6)$	$\beta_Q \frac{RQ}{N}$
quoll dies	$Q \to Q-1$	$\gamma_Q Q$

[2]

Further ecological investigation would need to take place to find the appropriate quoll birth rate, β_Q. We will hence demonstrate how a range of different hunting rates will effect the populations. Similarly to the contraceptives, the quolls will be introduced 5 years after the initial rat outbreak. The same amount of quolls are introduced as there were rats (10), for consistency.

4.2.2 Results

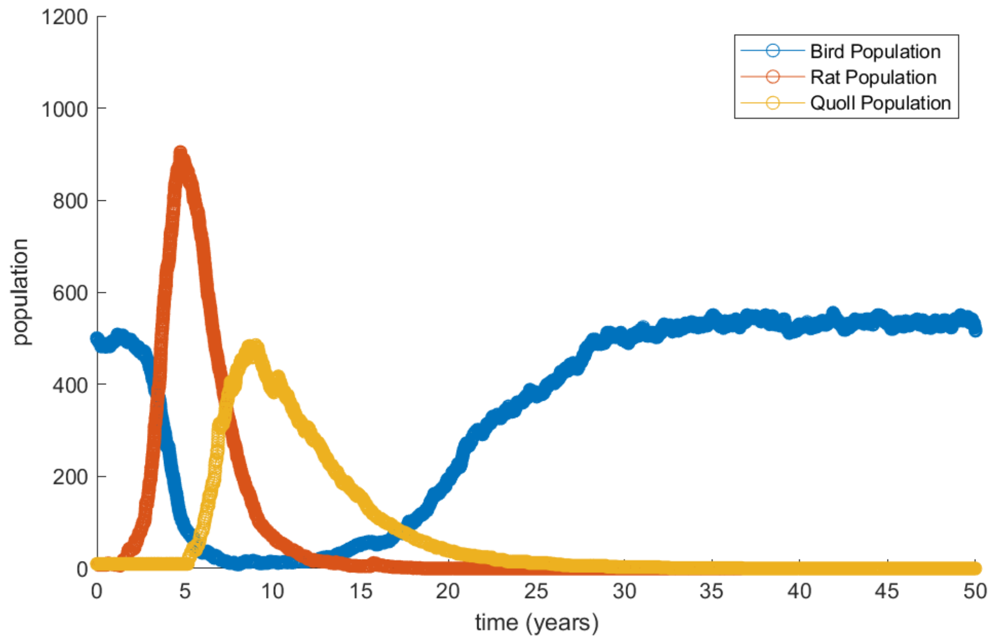
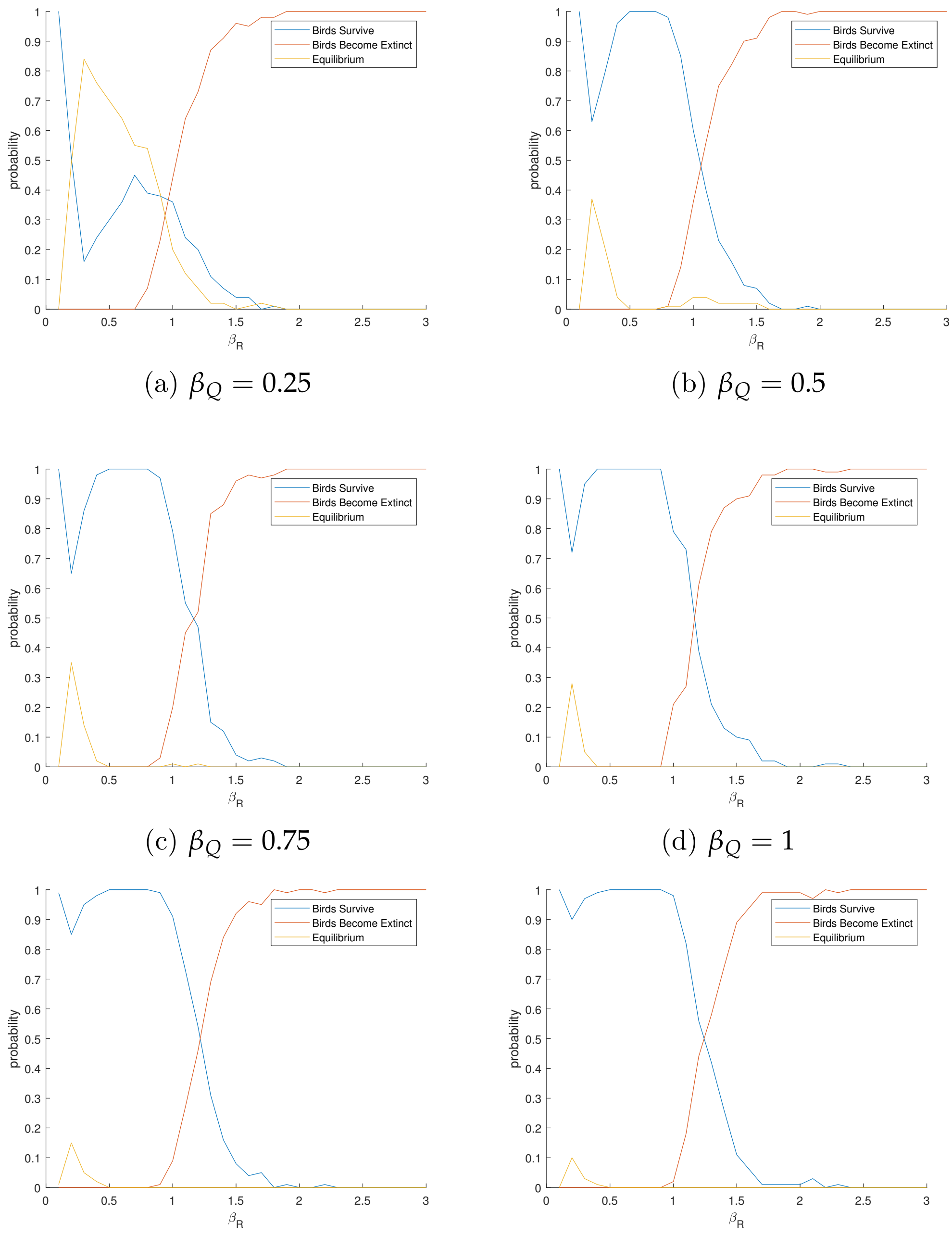


Figure 9: The effect of introducing 10 quolls to the island with $\beta_Q = 0.5$ and $\beta_R = 1.5$. The ecosystem returns to its natural balance after 15 years and the bird population eventually recovers.

Initially the population of birds diminish whilst the rat population grows exponentially. Soon after the Quolls are introduced, the rats die out rapidly. After about 10 years on the island, the Quolls have completely eradicated the rat outbreak and thus the bird population begins to recover. The Quolls eventually die out as their food source is lost, but the bird population thrives and reaches its initial size where it again equilibriates.

The effect of introducing 10 quolls to Green Island for varying rat birth rate, β_R, and quoll birth rates $\beta_Q = 0.25, 0.5, 0.75, 1, 1.5$ and 2, are shown below.



(a) $\beta_Q = 0.25$

(b) $\beta_Q = 0.5$

(c) $\beta_Q = 0.75$

(d) $\beta_Q = 1$

(e) $\beta_Q = 1.5$

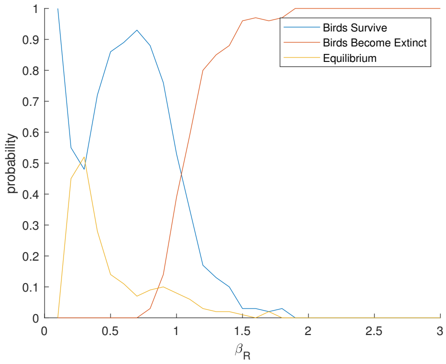
(f) $\beta_Q = 2$

Figure 10: Introduction of 10 quolls with varying birth rate (β_Q).

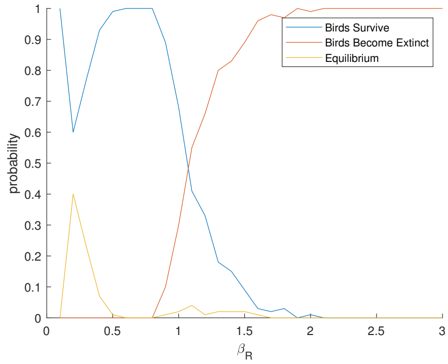
When $\beta_Q \geq 0.5$, we notice that the birds are more likely to survive for a larger range of rat birth rates. There also appears to be a drastic improvement in the probability of birds surviving. However, the rate of change decreases for greater values of β_Q.

We can therefore consider values $\beta_Q > 0.5$ as approximately equivalent, because the difference in results is not substantial.

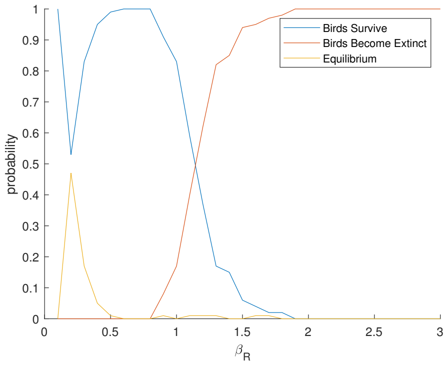
As birds tend to have on average 1.5 broods per year consisting of 5 eggs, and quolls have up to 6 surviving young per litter, it is assumed that $\beta_Q = \beta_B$ for the future experimentation. The impact of introducing quolls is tested for initial quoll populations of $2, 6, 10, 15, 20$ and 50 over a 50 year time period. Similarly to the previous simulations, these quolls were introduced 5 years after the 10 rats were initially introduced to the island.



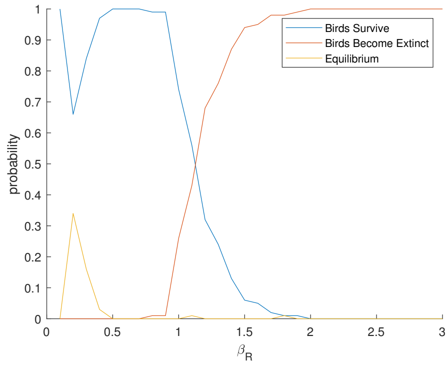
(a) 2



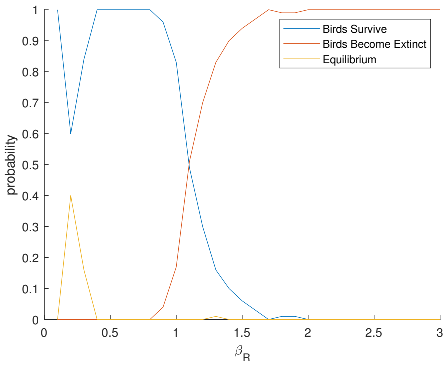
(b) 6



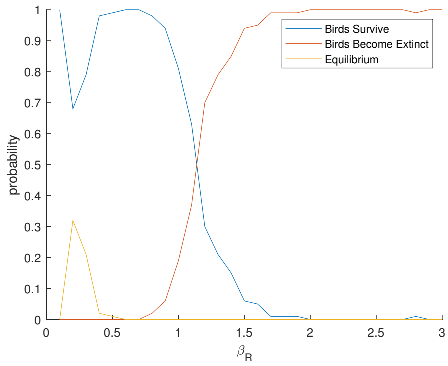
(c) 10



(d) 15



(e) 20



(f) 50

Figure 11: Introducing quolls ($\beta_Q = \beta_B = 0.6$) of various sizes over 100 trials.

4.2.3 Assumptions

As with the contraceptive model, the introduction of quolls involves the same assumptions as those from the model in Section 3. In addition to these, it is assumed firstly that the Department of Energy and the Environment are able to source this number of quolls to introduce to Green Island. Since quolls have already been reintroduced in small numbers to the Australian mainland after becoming extinct on it, we decided that it is reasonable to assume this could happen again. As with the birds and rats, it is assumed that it is acceptable to consider an annual average birth rate despite quolls having specific breeding seasons. This is as well as assuming that quolls die at the same rate regardless of their age.

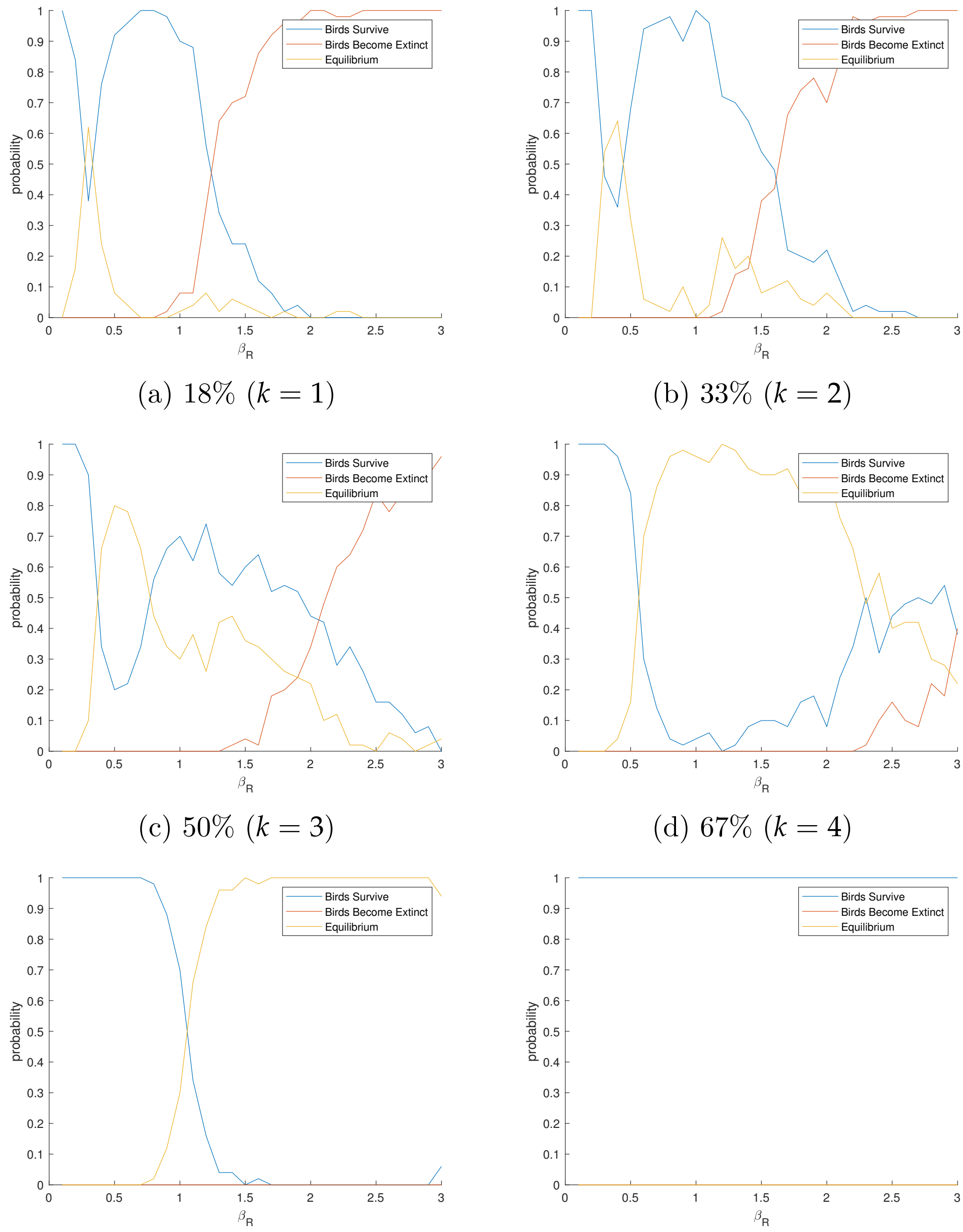
4.2.4 Discussion

The plots above demonstrate the effects of introducing different numbers of quolls to the existing bird and rat population for varying rat birth rate. We notice a slight increase in the probability of equilibrium when only 2 quolls are introduced. However, the results do not vary despite introducing a larger amount of quolls to the rat population for each simulation thereafter. Introducing 50 quolls results in very similar population dynamics to introducing just 6 of them. We thus hypothesise that a very large, possibly unattainable amount of quolls would be required to have any significant effect on the rat population for rat birth rates greater than 1. The idea of introducing more quolls to the population was therefore dismissed.

4.3 Quolls and Contraceptives

As neither the introduction of contraceptives or quolls result in a significant improvement for the Black-throated Finch for higher rat birth rates, the contraceptives and quolls were introducted together. Note that here we have $\beta_Q = 0.6$, following the previous assumption that $\beta_Q = \beta_B$.

4.3.1 Results



(a) 18% ($k = 1$)

(b) 33% ($k = 2$)

(c) 50% ($k = 3$)

(d) 67% ($k = 4$)

(e) 83% ($k = 5$)

(f) 100% ($k = 6$)

Figure 12: Introduction of 6 quolls ($\beta_Q = 0.6$) with varying contraceptive levels. An increase in rat control reduces the probability that the birds will become extinct in 50 years.

When both control strategies were implemented, an increase of contraceptive percentage appears to be correlated with a decrease in the probability of birds becoming extinct. This is in contrast to the previous plots where the probability of birds becoming extinct remained untouched. An increase in contraceptive level also increases the probability of an equilibrium state. If 100% of the rats receive contraceptives it is certain that the rats will die out. The probability of 1 is likely caused by the limited number of trials, this is as we expect the probability of $B(t)$ hitting state 0 to still be non-zero.

4.3.2 Analysis

Previously, the introduction of either control strategy would cause a shift in probability density from the equilibrium state to bird survival. However, when both methods are implemented there appears to be a drastic reduction in the probability that the birds will become extinct. As the extinction of birds is irreversible, reducing the probability should be the priority of the solution. This implies that a combination of the two method will return more favourable results than either individual strategy. This is a result of decreasing the effectiveness λ_R as well as increasing μ_R, ie. reducing the number of births and life expectancy simultaneously. This weakens the relative strength of the rats, hence diminishing the threat to the birds.

The intersection between the three probability lines shifted, in particular the probability of birds becoming extinct is translated significantly to the right. The lines appear to hold the same shape as on the previous figures until drastic levels of contraception is administered. This causes the same shape present in Figure 8, however with only bird survival and equilibrium, meaning that the birds will always survive the next 50 years.

4.4 Prescription

We advise the introduction of 6 quolls in conjunction with contraceptive bait. Ideally the contraception will affect the largest proportion possible of the rat population. Realistically, the contraceptives will not be distributed to the entire population of rats. Even in this case, contraceptives still significantly reduce the probability that birds become extinct. As per Figure 12 the probability of the bird population surviving after 50 years is over 80% such that $\beta_R < 2.5$.

5 Limitations

Using mathematics to model real-world scenarios always involves making assumptions about the physical nature and dynamics of the chosen subject, thus increasing inaccuracies in the results. These limitations are typically directly related, but not limited to, the assumptions made. Further extensions incorporating external factors such as animals and phenomena would also significantly improve the simulation of Green Island's ecosystem.

5.1 Black-throated Finch Model

The introduction of a Poisson process that mimics the bird population, results in the assumption that the time between events is distributed exponentially. Another limitation that we may observe from our model follows from the assumption that the life span of the Black-throated Finch is uniformly distributed over the interval $[2, 5]$. Further investigation into the accurate life expectancy would increase the accuracy of the model. In addition to this, birds have an equal probability at dying at any age. Realistically young and elderly are significantly more likely to perish. Moreover, considering immature birds and breeding seasons will also greatly increase the accuracy of the model.

5.2 Black-throated Finch Model with Rats

To understand the interplay between the rats and the Black-throated Finches, it was vital to develop a mechanism that allowed us to quantify what impact an introduced rat population would have on the bird population. To achieve this an SIR epidemic Markovian model was used to simulate the population numbers. This however, relies on the assumption that the rats' diet primarily consists of the native finch. Furthermore, assuming that birds must be eaten in order for rats to breed. Implying that, the increase in the rat population is dependent on the population of bird, therefore, cannot survive on the island independently. In reality, while the lack of a viable food source would be a limiting factor in a given rat's propensity to procreate, rats tend to have diverse diet and would likely find alternate food sources.

To ensure simplicity, we have assumed that both rats and birds will breed consistently throughout the year. This removes the notion of a breeding season for both animals, and hence the dynamics of the populations of both predator and prey will vary from reality. As for example we expect an increase in population in spring. The implementation of this would again increase the accuracy of the results.

It should be noted that the birth and death rates are merely estimates of what may occur in reality. In order to obtain the actual birth and death rates, a survey would need to be carried out and a statistical analysis undertaken. Within our model we have also made the assumption that the litter and brood sizes are constant, that is, that rats give birth to 6 rats per litter. It may be noted that this does diverge slightly from what is observed in nature, as the size of a rat's litter is stochastic and does vary as well as that of a bird's brood.

5.3 Contraceptive Model

To help quell the invasive rat population, the introduction of a contraceptive to the rat population was modelled. It is also assumed that it was feasible to distribute the contraceptive across the island so that every rat may consume the oral birth control. Note that this may not be feasible in reality as the total population of rats who have ingested contraceptives at any given time is not known. It is unlikely that the desired prescription of birth control will be ingested by all of the rats.

The introduction of contraceptives reduces the rate at which the rat population increases. In its implementation we modelled this decrease by decreasing the average litter size of each rat. This does not allow for variance in litters sizes, which restricts the reliability of the model. Introducing this parameter as a random variable would improve the breeding simulation of the model. Furthermore, implementing the contraceptive effects on the male rats would also provide a more accurate representation of the treatment.

5.4 Quoll Model

The model incorporating the quolls inherits many assumptions and limitations from the Black-throated Finch and rat model. It has also been assumed that the quolls do not have a negative impact on the surrounding environment. Moreover, there is not certainty that the quolls will primarily prey on the rats. However, as there diet consists of small mammals this seems likely. Further research into quoll diet and habitat requirements would assist in determining whether they are a strong candidate for introduction.

6 Conclusion

The Black-throated Finch population on Green Island in north Queensland faces a substantial threat of extinction if rats are introduced to the island. Without any intervention, our model demonstrates that the bird population will become extinct in almost every case where 10 rats are introduced to the island. The only scenario in which this is not the case is if rats have a considerably low birth rate of 0.5.

To prevent the extinction of the Black-throated Finch population, two strategies to control the rat population were explored: to distribute contraceptives to the rats and to introduce quolls, a natural predator of rats, on to Green Island. Administering contraceptives alone did not prove beneficial to the bird population as the probability that rats kill all of the birds did not decrease. Introducing quolls to the population had a similar effect, and did not show any significant improvement in the Finches chance of survival. When quolls are introduced, the rat population is completely eradicated within the first decade after which point the Black-throated Finch population recovers.

A combination of both rat contraceptives and the introduction of quolls on to the island proves to be the most effective solution. When just 6 quolls are introduced, and half of the rat population consume contraceptives, the Black-throated Finches are able to survive on Green Island, despite a high rat birth rate.

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7 Appendices

7.1 Model

```
% Model simulates the interactions between the birds and introduced rats.
% This utilises a combination of a SIR and Birth/Death CTMC in order to
% achieve the simulation. Model will output a graph for both the birds and
% the rats over the determined time period. If birds become extinct the
% code will stop prior to the time limit in order to increase efficiency.


N = 1000; % number of nests available

```

```

% parameters
b_born = 0.6; % beta_B
b_death = 2/7; % 1/expected life (3.5 years)

r_born = 1; % beta_R
r_death = 0.5; % 1/expected life (2 years)

% initial conditions.
X = [500; 10];  % X(1) is bird pop, X(2) is rat pop
t = 0;


a = zeros(4,1);

X_out = X;
t_out = 0;
T = 50; % maximum time allowance for the model

while X(1) > 0


    % step 1. Calculate the rates of each event given the
        current state.

    a(1) = r_born*X(1)*X(2)/N; % rate at which rat eats bird
    a(2) = b_born*X(1)*(N-X(1))/N; % rate at which a bird is
        born
    a(3) = r_death*X(2); % rate at which a rat dies
    a(4) = b_death*X(1); % rate at which a bird dies




    a0 = a(1)+a(2)+a(3)+a(4); % total rate of events

    % step 2. Calculate the time to the next event.

    t = t - log(rand)/a0; % time of next event


    % step 3. Update the state.
    r = rand*a0;

    if r < a(1)
        % rat eats bird
        X(1) = X(1) - 1;
        X(2) = X(2) + 6;
    elseif r < a(1)+ a(2)
        % bird is born
```

```
            X(1) = X(1) + 1;
        elseif r < a(1)+a(2)+a(3)
            % rat dies
            X(2) = X(2) - 1;
        else
            % bird dies
            X(1) = X(1) -1;
        end

        if t_out(end) > T
            break
        end

        % record the time and state after each jump
        X_out = [X_out, X];
        t_out = [t_out, t];

end
clf
hold on
stairs(t_out,X_out(1,:),'-o')
stairs(t_out,X_out(2,:),'-o')
xlim([0 T])
ylim([0 1000])
txt = sprintf('Rats become extinct in month %f.', round(t_out(end),1));
legend ('Bird Population', 'Rat Population')
title(sprintf('Introduction of %g rats, with birth rate = %g', X_out(2,1), r_born))
xlabel('time (years)')
ylabel('population')
```

7.2 Bird Rates

```
% birdrate tests the basic bird model with varying birth rates, this was
% used to determine the birth rate that will create an equilibrium. With
% the population line that remains clostest to 500 over the trial length
% being selected as having the most appropriate corresponding birth rate.

N = 1000; % number of nests available
close all
clf
f1 = figure; % defining figures
f2 = figure;
for i = 0.4:0.1:0.8
    % parameters
```

```
    b_born = i; % testing different beta_B
    b_death = 2/7; % 1/expected life (3.5 years)
    % initial conditions.
    X = [500; 0];  % X(1) is bird pop, X(2) is rat pop
    % the rat population is 0 as we only consider birds present
        for this
    % case

    t = 0; % initialise time


    a = zeros(4,1);

    % initalising
    X_out = X;
    t_out = 0;
    T = 100;
    % time period the model will run for, if not extinction
        occurs prior

    while t_out(end) < T



        % step 1. Calculate the rates of each event given the
            current state.

        a(2) = b_born*X(1)*(1000-X(1))/N; % rate at which birds
            are born
        a(4) = b_death*X(1); % rate at which a bird dies

        a0 = a(2)+a(4); % total rate lambda

        % step 2. Calculate the time to the next event.

        t = t - log(rand)/a0;

        if X_out(1) == 0 % if birds become extinct end the
            script
             break
        end

        % step 3. Update the state.

        if rand*a0 < a(2)
            % bird is born
            X(1) = X(1) + 1;
        else
            % bird dies
            X(1) = X(1) -1;
```

```
        end


        % record the time and state after each jump
        X_out = [X_out, X];
        t_out = [t_out, t];

    end


    hold on
    stairs(t_out,X_out(1,:),'-o')
    xlim([0 T])
    ylim([0 1000])
    legend show
    legend('\beta_{B} = 0.4', '\beta_{B} = 0.5', '\beta_{B} =
        0.6', '\beta_{B} = 0.7', '\beta_{B} = 0.8')
    title('Bird populations over the next 100 years with varying
        birth rates')
    xlabel('time (years)')
    ylabel('population')
    figure(f1)


    hold on
    mean(X_out(1,:))
    bar(b_born,mean(X_out(1,:))-500, 0.08)
    xlim([0.2 1.3])
    legend show
    legend('\beta_{B} = 0.4', '\beta_{B} = 0.5', '\beta_{B} =
        0.6', '\beta_{B} = 0.7', '\beta_{B} = 0.8')
    title('Mean difference of bird population after 100 years')
    xlabel('birth rate of birds (\omega_B)')
    ylabel('inital population - mean ')
    figure(f2)
end
```

7.3 Contraceptive

```
% contraceptive models the implementation of contraceptives to
    the island,
% with k determining the amount of contraceptives used. With the
% percentage of rats dosed corresponding to k/6. This will
    output an
% example of how the contraceptive affects the populations.

N = 1000; % number of nests available
clf
for i = 0:1:5
    % parameters
```

```
    b_born = 0.6; % beta_B
    b_death = 2/7; % 1/expected life (3.5 years)


    r_born = 1.5; % beta_R
    r_death = 0.5; % 1/expected life (2 years)

    % initial conditions.
    X = [500; 10];  % X(1) is bird pop, X(2) is rat pop
    t = 0;


    a = zeros(4,1);

    % initalising
    X_out = X;
    t_out = 0;
    T = 60; % time limit of the simulation

    while X(1) > 0


        % step 1. Calculate the rates of each event given the current state.

        a(1) = r_born*X(1)*X(2)/N; % rate at which a rat eats bird
        a(2) = b_born*X(1)*(N-X(1))/N; % rate at which a bird born
        a(3) = r_death*X(2);            % rate at which rat dies
        a(4) = b_death*X(1); % rate at which a bird dies




        a0 = a(1)+a(2)+a(3)+a(4);

        % step 2. Calculate the time to the next event.

        t = t - log(rand)/a0;


        % step 3. Update the state.
        r = rand*a0;

        if r < a(1)
            % rat eats bird
            X(1) = X(1) - 1;
            X(2) = X(2) + 6-i;
        elseif r < a(1)+ a(2)
```

```
                % bird is born
                X(1) = X(1) + 1;
            elseif r < a(1)+a(2)+a(3)
                % rat dies
                X(2) = X(2) - 1;
            else
                % bird dies
                X(1) = X(1) - 1;
            end

            if t_out(end) > T % if t surpasses the time limit break the loop
                break
            end

            % record the time and state after each jump
            X_out = [X_out, X];
            t_out = [t_out, t];

        end

        hold on
        stairs(t_out,X_out(1,:),'-o')
        xlim([0 100])
        ylim([0 max(max(X_out(1,:)), max(X_out(2,:)))+0.1*max(max(X_out(1,:)), max(X_out(2,:)))])
        txt = sprintf('Rats become extinct in month %f.', round(t_out(end),1));
        legend show
        legend('0%','17%', '33%', '50%', '66%', '83%')
        title(sprintf('Population of birds, with varying rat contraceptive prescriptions'))
        xlabel('time (months)')
        ylabel('population')
end
```

7.4 Contraceptive Rates

```
% contrarate builds on contraceptive by plotting the
% probability of each of the bird outcomes for given k.

clear all
close all


N = 1000; % number of nests available
outcome = zeros(30,100);
X_outcome = [];
f1 = figure;
f2 = figure;
```

```
count_survive = zeros(1,30);
count_die = zeros(1,30);
count_equilibrium = zeros(1,30);
k = 5; %percentage receiving contraception
    (1=17%,2=33%,3=50%,...)
for i = 1:30
    for j = 1:50  % Number of trials for each rate



        % parameters
        b_born = 0.6;
        b_death = 2/7; % 1/expected life (3.5 years)


        r_born = 0.1*i;
        r_death = 0.5; % 1/expected life (2 years)

        % initial conditions.
        X = [500; 10];  % X(1) is bird pop, X(2) is rat pop
        t = 0;


        a = zeros(4,1);

        X_out = X;
        t_out = 0;
        T = 50;

        while X(1) > 0


            % step 1. Calculate the rates of each event given
                the current state.

            a(1) = r_born*X(1)*X(2)/N; % rate at which rat eats
                bird
            a(2) = b_born*X(1)*(N-X(1))/N; % rate at which bird
                born
            a(3) = r_death*X(2);             % rate at which rat
                dies
            a(4) = b_death*X(1); % rate at which bird dies




            a0 = a(1)+a(2)+a(3)+a(4);

            % step 2. Calculate the time to the next event.
```

```
        t = t - log(rand)/a0;


        % step 3. Update the state.
        r = rand*a0;

        if t<5
            if r < a(1)
                % rat eats bird
                X(1) = X(1) - 1;
                X(2) = X(2) + 6;
            elseif r < a(1)+ a(2)
                % bird is born
                X(1) = X(1) + 1;
            elseif r < a(1)+a(2)+a(3)
                % rat dies
                X(2) = X(2) - 1;
            else
                % bird dies
                X(1) = X(1) -1;
            end
        else
            if r < a(1)
                % rat eats bird
                X(1) = X(1) - 1;
                X(2) = X(2) + 6-k;
            elseif r < a(1)+ a(2)
                % bird is born
                X(1) = X(1) + 1;
            elseif r < a(1)+a(2)+a(3)
                % rat dies
                X(2) = X(2) - 1;
            else
                % bird dies
                X(1) = X(1) -1;
            end
        end

        if t_out(end) > T
            break
        end

        % record the time and state after each jump
        X_out = [X_out, X];
        t_out = [t_out, t];

    end

    %counting the number of times each outcome occurs
```

```

        if X_out(2,end) == 0
            outcome(i,j) = 1; % birds survive
            count_survive(1,i) = sum(outcome(i,:)==1); % stores the number of trials(/30) in which birds survive for each rat birthrate

        elseif X_out(1,end) == 0
            outcome(i,j) = 3; % birds become extinct
            count_die(1,i) = sum(outcome(i,:)==3); % stores the number of trials(/30) in which birds die for each rat birth rate

        else
            outcome(i,j) = 2; % equilibrium
            count_equilibrium(1,i) = sum(outcome(i,:)==2); % stores number of trials (/30) in which birds and rat population equilibriate for each rat birth rate
        end
        X_outcome = [X_outcome X];
    end
    fprintf('.');
end

% each outcome
count_survive;
count_die;
count_equilibrium;

% probability of each outcome
probability_survive = count_survive/j;          % j = number of trials
probability_die = count_die/j;
probability_equilibrium = count_equilibrium/j;

rates=0.1:0.1:3;

hold on
plot(rates,probability_survive)
plot(rates,probability_die)
plot(rates,probability_equilibrium)
legend('Birds Survive', 'Birds Become Extinct', 'Equilibrium')
title('Probability of each bird outcome when 50% of the rat population receives contraception.')
xlabel('\beta_{R}')
ylabel('probability')
hold off
figure(f1)

```

```

prob_s = (count_survive + count_equilibrium)/j;
hold on
plot(rates,prob_s)
plot(rates,probability_die)
legend('Birds Survive', 'Birds Become Extinct')
title('Probability of each bird outcome when 50% of the rat population receives contraception.')
xlabel('\beta_{R}')
ylabel('probability')
hold off
figure(f2)
```

7.5 Mean Time

```
%meantime simulates the model over multiple trials and records the time of
%extinction of each, if birds survive and return to equilibrium the time
%will be infinite and hence a maximum constraint is set, currently it is
%100 years. This will output the averages times of each bird populations
%survival given a beta_R.

clear all
close all
m=30; % number of beta_R tested
n=50; % number of trials for each beta_R
N = 1000; % number of nests available
outcome = zeros(m,n);
X_outcome = [];
f1 = figure;
meanval = zeros(m);

for i = 1:m
    for j = 1:n  % Number of trials for each rate

        % parameters
        b_born = 0.6; % beta_B
        b_death = 2/7; % 1/expected life (3.5 years)

        r_born = 0.1*i; % beta_R
        r_death = 0.5; % 1/expected life (2 years)

        % initial conditions.
        X = [500; 10];  % X(1) is bird pop, X(2) is rat pop
        t = 0;


```

```
    a = zeros(4,1);

    % initialise
    X_out = X;
    t_out = 0;
    T = 50; % maximum time allowed for the simulation

    while X_out(1,end) > 0


        % step 1. Calculate the rates of each event given the current state.

        a(1) = r_born*X(1)*X(2)/N; % rate at which rat eats bird
        a(2) = b_born*X(1)*(N-X(1))/N; % rate at which bird born
        a(3) = r_death*X(2); % rate at which rat dies
        a(4) = b_death*X(1); % rate at which bird dies



        if t > T % if time restriction is broken break the loop
            break
        end

        a0 = a(1)+a(2)+a(3)+a(4); % total rate of events

        % step 2. Calculate the time to the next event.

        t = t - log(rand)/a0;


        % step 3. Update the state.
        r = rand*a0;

        if r < a(1)
            % rat eats bird
            X(1) = X(1) - 1;
            X(2) = X(2) + 6;
        elseif r < a(1)+ a(2)
            % bird is born
            X(1) = X(1) + 1;
        elseif r < a(1)+a(2)+a(3)
            % rat dies
            X(2) = X(2) - 1;
        else
            % bird dies
            X(1) = X(1) - 1;
```

```
            end

            % record the time and state after each jump
            X_out = [X_out, X];
            t_out = [t_out, t];

        end

                    outcome(i,j) = t;

    end

end

for k = 1:30
    meanval(k) = mean(outcome(k,:)); % averaging the times of each trial
end

rates=0.1:0.1:3;

hold on
plot(rates,meanval)
xlabel('\beta_{R}')
ylabel('time (years)')
hold off
```

7.6 Quolls and Contraceptives

```
% quollcontra simulates the populations of rats, birds and quolls after the
% introduction of contraceptives to the rat population. Built off of
% contraceptive, k behaves the same (determining the percentage affected).
% this will output an example plot of the animal populations over the
% desired time period.

N = 1000; % number of nests available


% parameters
b_born = 0.6; % beta_B
b_death = 2/7; % 1/expected life (3.5 years)
k = 1;
r_born = 1.5; % beta_R
r_death = 0.5; % 1/expected life (2 years)

q_born = 0.6; % beta_Q
```

```
q_death = 2/7; % 1/expected life (2-5 years)
% initial conditions.
X = [500; 10; 6];  % X(1) is bird pop, X(2) is rat pop, X(3) is
    quoll pop
t = 0;


a = zeros(6,1);

X_out = X;
t_out = 0;
T = 50; % maximum time period

while X(1) > 0


    % step 1. Calculate the rates of each event given the
        current state.

    a(1) = r_born*X(1)*X(2)/N; % rate at which a rat eats a bird
    a(2) = b_born*X(1)*(N-X(1))/N; % rate at which a bird is
        born
    a(3) = r_death*X(2);% rate at which a rat dies
    a(4) = b_death*X(1); % rate at which a bird dies

    if t < 5 % quolls arent introduced until after 5 years
        a(5) = 0;
        a(6) = 0;
    else
        a(5) = q_born*X(2)*X(3)/N; % rate at which a quoll eats
            a rat
        a(6) = q_death*X(3); % rate at which a quoll dies
    end




    a0 = a(1)+a(2)+a(3)+a(4)+a(5)+a(6); % total rate

    % step 2. Calculate the time to the next event.

    t = t - log(rand)/a0;


    % step 3. Update the state.
    r = rand*a0;

    if r < a(1)
        % rat eats bird
        X(1) = X(1) - 1;
```

```
            X(2) = X(2) + 6 - k;
        elseif r < a(1)+ a(2)
            % bird is born
            X(1) = X(1) + 1;
        elseif r < a(1)+a(2)+a(3)
            % rat dies
            X(2) = X(2) - 1;
        elseif r < a(1)+a(2)+a(3)+a(4)
            % bird dies
            X(1) = X(1) -1;
        elseif r < a(1)+a(2)+a(3)+a(4)+a(5)
            % quoll eats rat
            X(2) = X(2) - 1;
            X(3) = X(3) + 6;
        else
            %quoll dies
            X(3) = X(3) - 1;
        end

        if t_out(end) > T
            break
        end

        % record the time and state after each jump
        X_out = [X_out, X];
        t_out = [t_out, t];

end

clf
hold on
stairs(t_out,X_out(1,:),'-o')
stairs(t_out,X_out(2,:),'-o')
stairs(t_out,X_out(3,:),'-o')
xlim([0 50])
ylim([0 1200])
legend ('Bird Population', 'Rat Population', 'Quoll Population')
title(sprintf('Quoll introduction with %g intially, with birth rate = %g', X_out(3,1), q_born))
xlabel('time (years)')
ylabel('population')
```

7.7 Quoll and Contraceptive Probabilities

```
%quollcontraprob will return a probability plot of each of the bird
%outcomes given a pecentage of contraception that the rats received. This
%model incorporates the addition of quolls after 5 years with a birth rate
```

```
%of 0.6.


clear all
close all


N = 1000; % number of nests available
outcome = zeros(30,50);
X_outcome = [];
f1 = figure;

count_survive = zeros(1,30);
count_die = zeros(1,30);
count_equilibrium = zeros(1,30);


for i = 1:30
for j = 1:50  % Number of trials for each rate

% parameters
b_born = 0.6; % beta_B
b_death = 2/7; % 1/expected life (3.5 years)
k = 6; % contraceptive level
r_born = 0.1*i; % beta_R
r_death = 0.5; % 1/expected life (2 years)

q_born = 0.6; % beta_Q
q_death = 2/7; % 1/expected life (2-5 years)
% initial conditions.
X = [500; 10; 6];  % X(1) is bird pop, X(2) is rat pop, X(3) is
    quoll pop
t = 0;


a = zeros(6,1);

X_out = X;
t_out = 0;
T = 50; % time restriction

while X(1) > 0


    % step 1. Calculate the rates of each event given the
        current state.

    a(1) = r_born*X(1)*X(2)/N; % rate at which rat eats bird
    a(2) = b_born*X(1)*(N-X(1))/N; % rate at which bird born
    a(3) = r_death*X(2);   % rate at which a rat dies
```

```
    a(4) = b_death*X(1); % rate at which a bird dies

    if t < 5 % quolls are introduced after 5 years
        a(5) = 0;
        a(6) = 0;
    else
        a(5) = q_born*X(2)*X(3)/N; % quoll eats rat
        a(6) = q_death*X(3); % quoll dies
    end




    a0 = a(1)+a(2)+a(3)+a(4)+a(5)+a(6); % total rate

    % step 2. Calculate the time to the next event.

    t = t - log(rand)/a0;


    % step 3. Update the state.
    r = rand*a0;

    if r < a(1)
        % rat eats bird
        X(1) = X(1) - 1;
        X(2) = X(2) + 6 - k;
    elseif r < a(1)+ a(2)
        % bird is born
        X(1) = X(1) + 1;
    elseif r < a(1)+a(2)+a(3)
        % rat dies
        X(2) = X(2) - 1;
    elseif r < a(1)+a(2)+a(3)+a(4)
        % bird dies
        X(1) = X(1) -1;
    elseif r < a(1)+a(2)+a(3)+a(4)+a(5)
        % quoll eats rat
        X(2) = X(2) - 1;
        X(3) = X(3) + 6;
    else
        %quoll dies
        X(3) = X(3) - 1;
    end

    if t_out(end) > T
        break
    end

    % record the time and state after each jump
```

```
    X_out = [X_out, X];
    t_out = [t_out, t];

end

if X_out(2,end) == 0
    outcome(i,j) = 1; % birds survive
    count_survive(1,i) = sum(outcome(i,:)==1); % stores the number of trials(/30) in which birds survive for each rat birthrate

elseif X_out(1,end) == 0
    outcome(i,j) = 3; % birds become extinct
   count_die(1,i) = sum(outcome(i,:)==3); % stores the number of trials(/30) in which birds die for each rat birth rate

else
    outcome(i,j) = 2; % equilibrium
    count_equilibrium(1,i) = sum(outcome(i,:)==2); % stores number of trials (/30) in which birds and rat population equilibriate for each rat birth rate
end
X_outcome = [X_outcome X];
end

end

%each outcome
count_survive;
count_die;
count_equilibrium;

%probability of each outcome
probability_survive = count_survive/j;          % j = number of trials
probability_die = count_die/j;
probability_equilibrium = count_equilibrium/j;

rates=0.1:0.1:3;

hold on
plot(rates,probability_survive)
plot(rates,probability_die)
plot(rates,probability_equilibrium)
legend('Birds Survive', 'Birds Become Extinct', 'Equilibrium')
title('Probability of each bird outcome when 10 quolls are introduced after 5 years.')
xlabel('\beta_{R}')
ylabel('probability')
hold off
```

7.8 Quoll Probabilities

```
%quollprob models the introduction of quolls after 5 years with
    a birth rate
%of 0.6 and how this effects the probability of each bird
    outcome.


clear all
close all


N = 1000; % number of nests available
outcome = zeros(30,100);
X_outcome = [];
f1 = figure;

% defining vectors
count_survive = zeros(1,30);
count_die = zeros(1,30);
count_equilibrium = zeros(1,30);


for i = 1:30 % number of beta_Rs tested
for j = 1:100  % Number of trials for each rate


% parameters
b_born = 0.6; % beta_B
b_death = 2/7; % 1/expected life (3.5 years)

r_born = 0.1*i; % beta_R
r_death = 0.5; % 1/expected life (2 years)

q_born = 0.6; % beta_Q
q_death = 2/7; % 1/expected life (2-5 years)
% initial conditions.
X = [500; 10; 50];  % X(1) is bird pop, X(2) is rat pop, X(3) is
    quoll pop
t = 0;


a = zeros(6,1);

X_out = X;
t_out = 0;
T = 50;

while X(1) > 0

```

```

    % step 1. Calculate the rates of each event given the current state.

    a(1) = r_born*X(1)*X(2)/N; % rate at which rat eats bird
    a(2) = b_born*X(1)*(1000-X(1))/N; % rate at which bird born
        a(3) = r_death*X(2);   % rate at which rat dies
    a(4) = b_death*X(1); % rate at which bird dies

    if t < 5 % as quolls are introduced after 5 years
        a(5) = 0;
        a(6) = 0;
    else
        a(5) = q_born*X(2)*X(3)/N; % quoll eats rat
        a(6) = q_death*X(3); % quoll dies
    end



    a0 = a(1)+a(2)+a(3)+a(4)+a(5)+a(6);

    % step 2. Calculate the time to the next event.

    t = t - log(rand)/a0;


    % step 3. Update the state.
    r = rand*a0;

    if r < a(1)
        % rat eats bird
        X(1) = X(1) - 1;
        X(2) = X(2) + 6;
    elseif r < a(1)+ a(2)
        % bird is born
        X(1) = X(1) + 1;
    elseif r < a(1)+a(2)+a(3)
        % rat dies
        X(2) = X(2) - 1;
    elseif r < a(1)+a(2)+a(3)+a(4)
        % bird dies
        X(1) = X(1) -1;
    elseif r < a(1)+a(2)+a(3)+a(4)+a(5)
        % quoll eats rat
        X(2) = X(2) - 1;
        X(3) = X(3) + 6;
    else
        %quoll dies
        X(3) = X(3) - 1;
    end
```

```

    if t_out(end) > T % if time restriction is violated break the loop
        break
    end

    % record the time and state after each jump
    X_out = [X_out, X];
    t_out = [t_out, t];

end


if X_out(2,end) == 0
    outcome(i,j) = 1; % birds survive
    count_survive(1,i) = sum(outcome(i,:)==1); % stores the number of trials(/30) in which birds survive for each rat birthrate

elseif X_out(1,end) == 0
    outcome(i,j) = 3; % birds become extinct
   count_die(1,i) = sum(outcome(i,:)==3); % stores the number of trials(/30) in which birds die for each rat birth rate

else
    outcome(i,j) = 2; % equilibrium
    count_equilibrium(1,i) = sum(outcome(i,:)==2); % stores number of trials (/30) in which birds and rat population equilibriate for each rat birth rate
end
X_outcome = [X_outcome X];
end

end
% outcomes
count_survive;
count_die;
count_equilibrium;

% probability of each outcome
probability_survive = count_survive/j;              % j = number of trials
probability_die = count_die/j;
probability_equilibrium = count_equilibrium/j;

rates=0.1:0.1:3;

hold on
plot(rates,probability_survive)
plot(rates,probability_die)
```

```
plot(rates,probability_equilibrium)
legend('Birds Survive', 'Birds Become Extinct', 'Equilibrium')
xlabel('\beta_{R}')
ylabel('probability')
hold off
```

7.9 Rat Probabilities

```
% ratprob determines the probability of each bird outcome with a
    variety of
% \beta_R values (from 0,3). These are then plotted on the same
    graph such
% that they can be compared.

clear all
close all


N = 1000; % number of nests available
outcome = zeros(30,100);
X_outcome = [];
f1 = figure;

% defining vectors
count_survive = zeros(1,30);
count_die = zeros(1,30);
count_equilibrium = zeros(1,30);


for i = 1:30
    for j = 1:50  % Number of trials for each rate

        % parameters
        b_born = 0.6; % beta_B
        b_death = 2/7; % 1/expected life (3.5 years)

        r_born = 0.1*i; % beta_R
        r_death = 0.5; % 1/expected life (2 years)

        % initial conditions.
        X = [500; 10];  % X(1) is bird pop, X(2) is rat pop
        t = 0;


        a = zeros(4,1);

        X_out = X;
        t_out = 0;
        T = 50;

```

```
    while t_out(end) < T


        % step 1. Calculate the rates of each event given the current state.

        a(1) = r_born*X(1)*X(2)/N; % rate at which rat eats bird
        a(2) = b_born*X(1)*(1000-X(1))/N; % rate at which bird born
        a(3) = r_death*X(2);% rate at which rat dies
        a(4) = b_death*X(1); % rate at which bird dies




        a0 = a(1)+a(2)+a(3)+a(4); % total rate

        % step 2. Calculate the time to the next event.

        t = t - log(rand)/a0;


        % step 3. Update the state.
        r = rand*a0;

        if r < a(1)
            % rat eats bird
            X(1) = X(1) - 1;
            X(2) = X(2) + 6;
        elseif r < a(1)+ a(2)
            % bird is born
            X(1) = X(1) + 1;
        elseif r < a(1)+a(2)+a(3)
            % rat dies
            X(2) = X(2) - 1;
        else
            % bird dies
            X(1) = X(1) -1;
        end
        %if either animal becomes extinct break
        if X_out(1,end) == 0
            break
        end

        if X_out(2,end) == 0
            break
        end

        % record the time and state after each jump
```

```
                X_out = [X_out, X];
                t_out = [t_out, t];

            end


            if X_out(2,end) == 0
               % bird survives
                outcome(i,j) = 1;
                count_survive(1,i) = sum(outcome(i,:)==1); % stores the number of trials(/30) in which birds survive for each rat birthrate

            elseif X_out(1,end) == 0
                % birds becomes extinct
                outcome(i,j) = 3;
                count_die(1,i) = sum(outcome(i,:)==3); % stores the number of trials(/30) in which birds die for each rat birth rate

            else
               % equilibrium
                outcome(i,j) = 2;
                count_equilibrium(1,i) = sum(outcome(i,:)==2); % stores number of trials (/30) in which birds and rat population equilibriate for each rat birth rate
            end
            X_outcome = [X_outcome X];
        end

end

% outcomes
count_survive;
count_die;
count_equilibrium;

% probability of each outcome
probability_survive = count_survive/j              % j = number of trials
probability_die = count_die/j
probability_equilibrium = count_equilibrium/j

rates=0.1:0.1:3;

hold on
plot(rates,probability_survive)
plot(rates,probability_die)
plot(rates,probability_equilibrium)
```

```
legend('Birds Survive', 'Birds Become Extinct', 'Equilibrium')
title('Probability of each bird outcome.')
xlabel('\beta_{R}')
ylabel('probability')
hold off
```