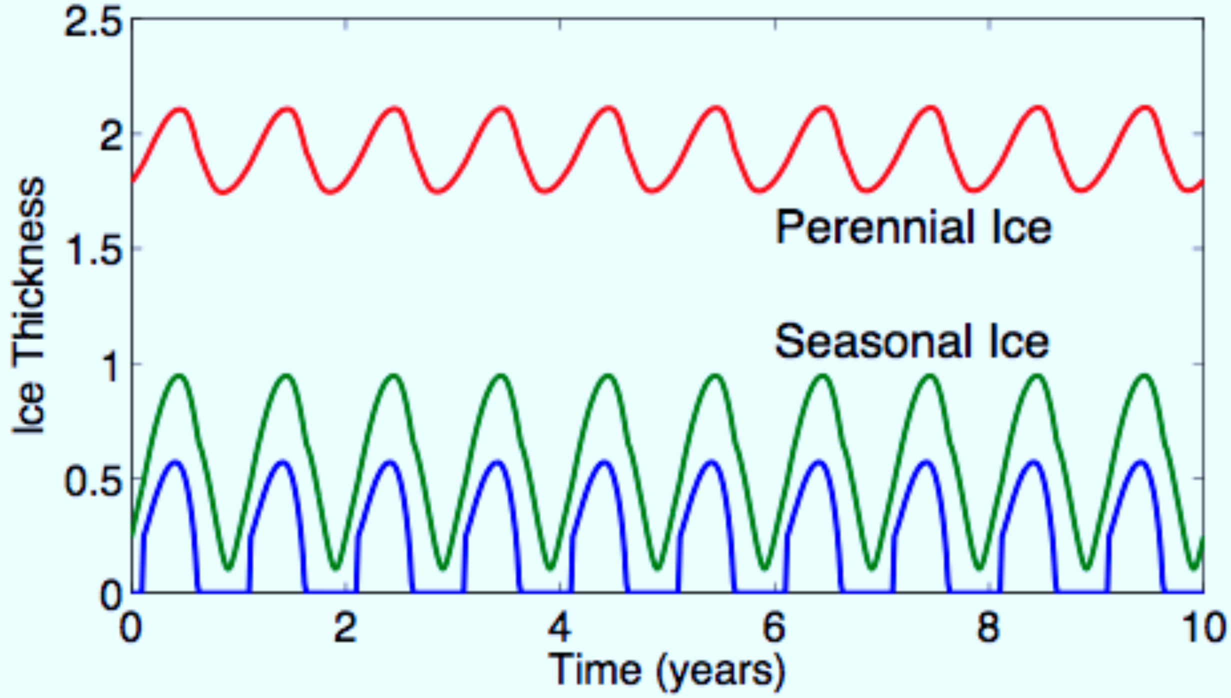
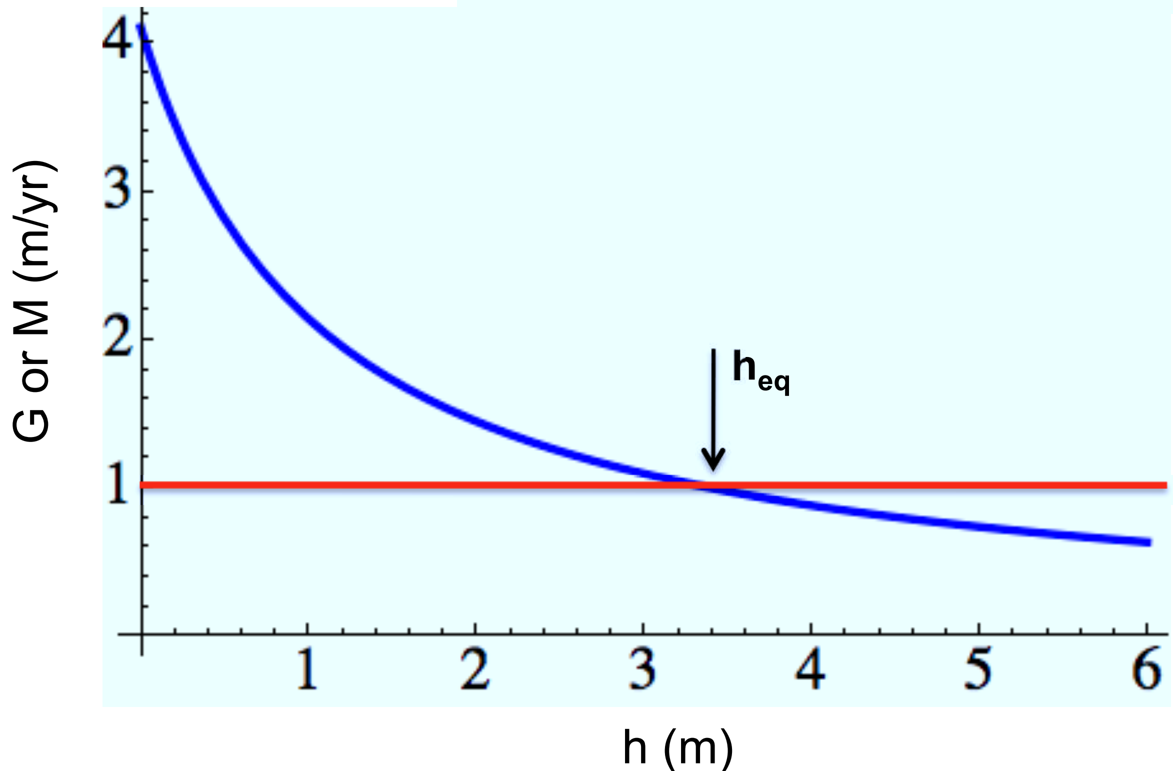
Seasonal cycle of ice thickness in a more complex model, like BL99



Equilibrium thickness defined by $G(h_{eq}) = M$

$$G(h) = \frac{\tau}{L}\left[\frac{A + BT_s(h)}{n_w} - \frac{D}{2} - F_W\right] \sim 1/h$$

$$M = \frac{\tau}{L}\left[-\frac{A}{n_s} + \frac{D}{2} + F_W + (1-\alpha)F_{SW}\right] \quad (30)$$



Bitz and Roe (2004)

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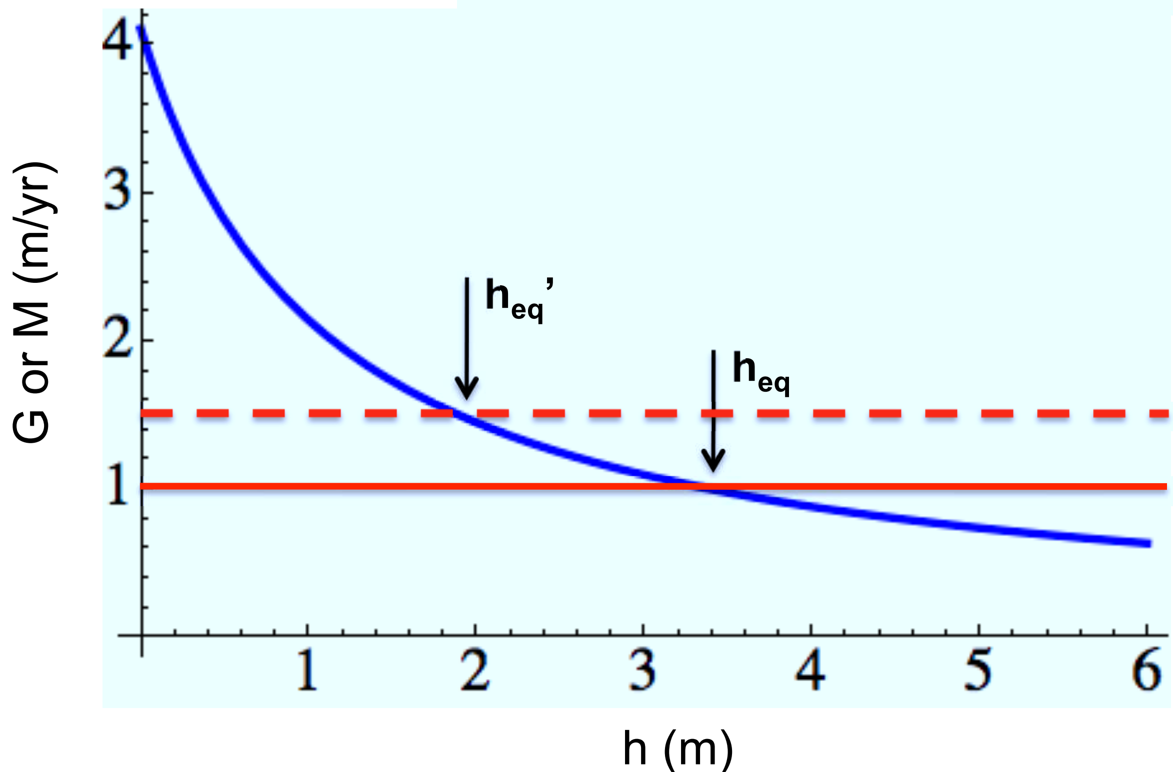

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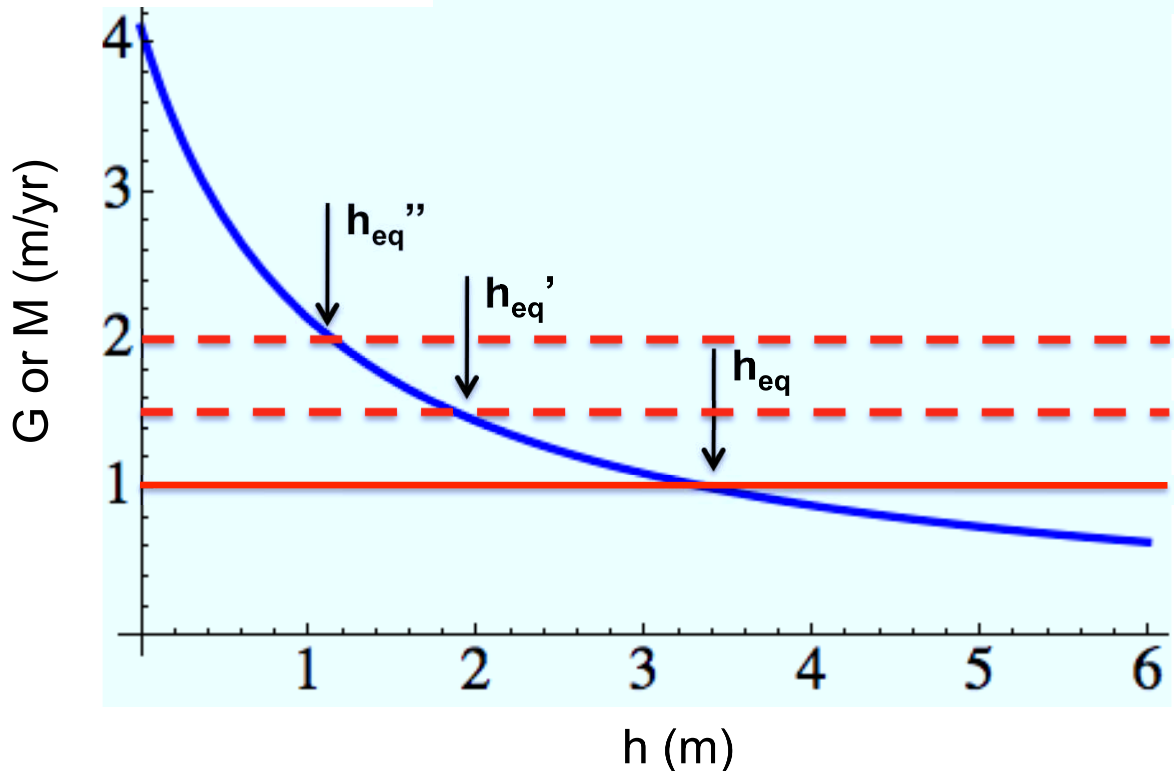
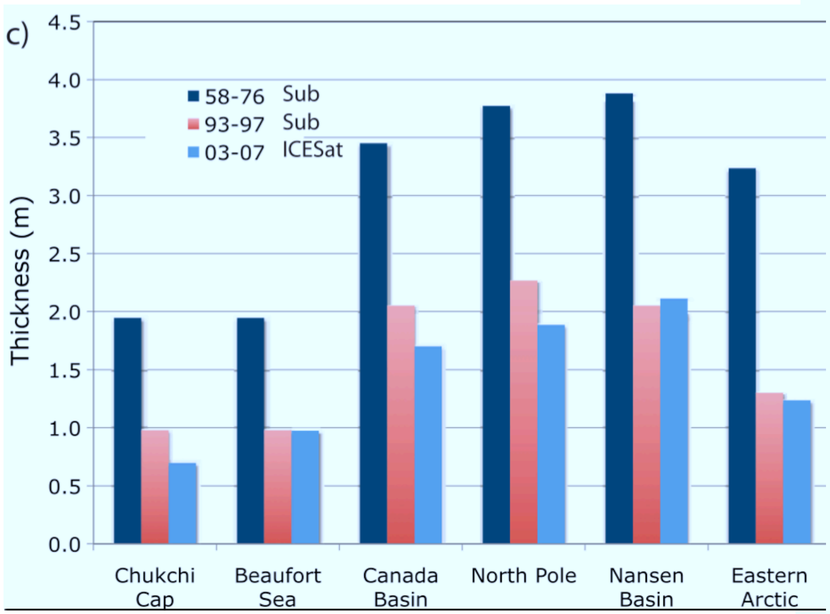


Bitz and Roe (2004)

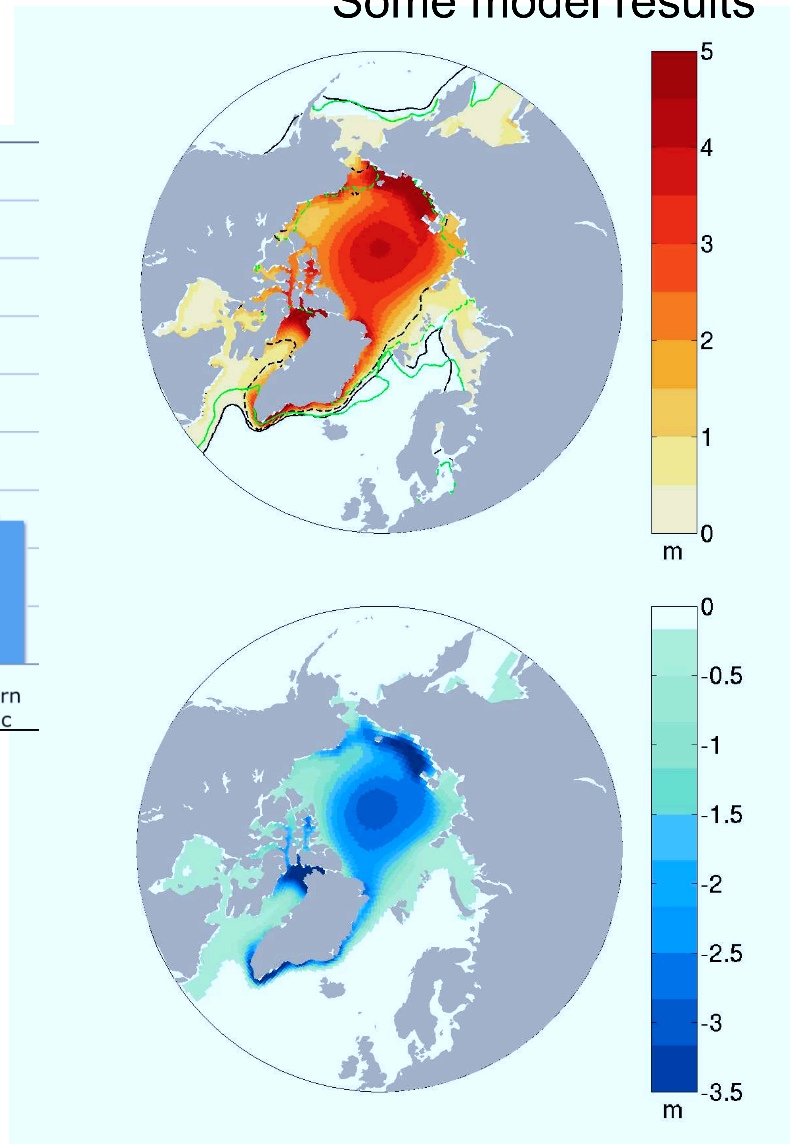
Thick ice thins more than thin ice thins

Observations of ice thickness



Kwok and Rothrock (2009)

Some model results

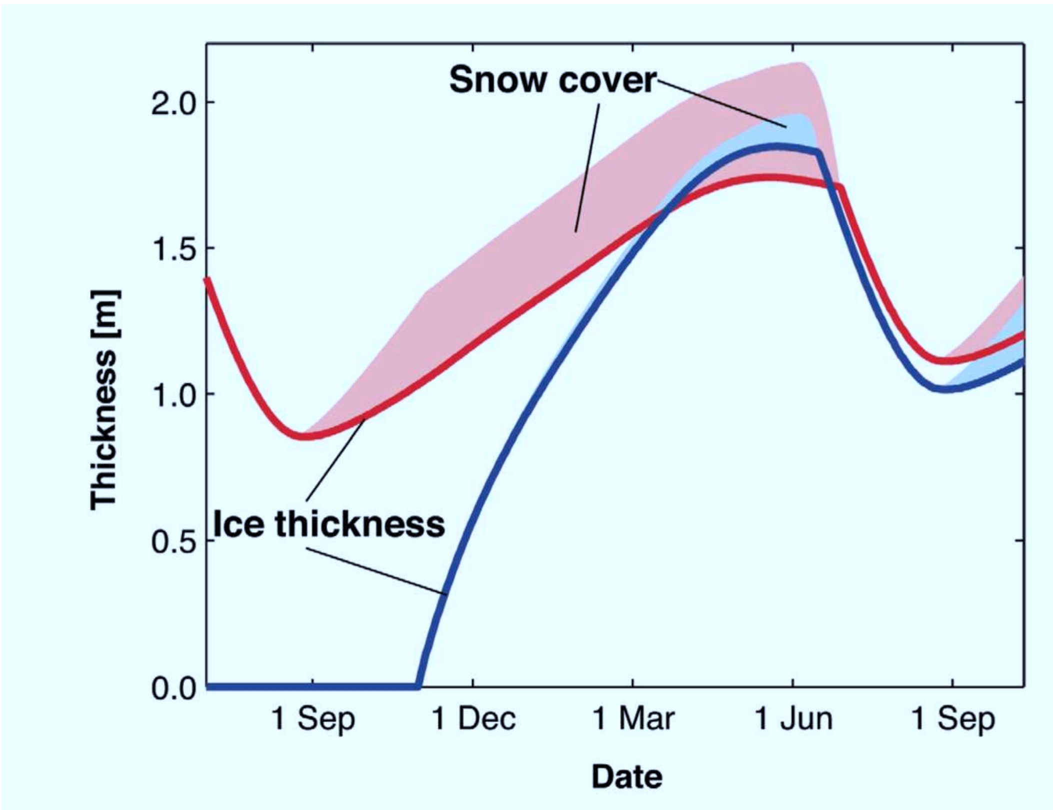


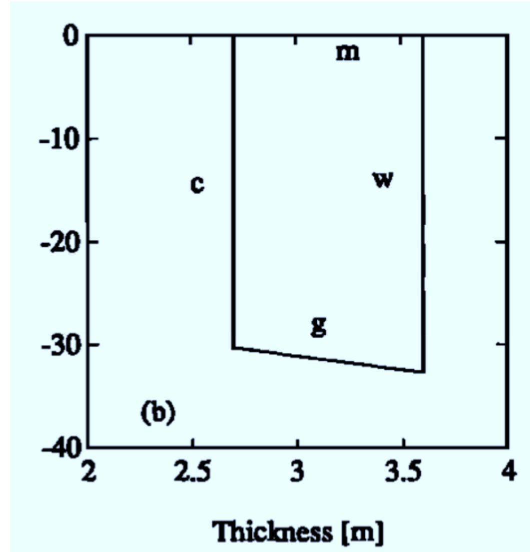
modern climate

thinning under $2xCO_2$

Bitz (2008)

e.g., snow cover





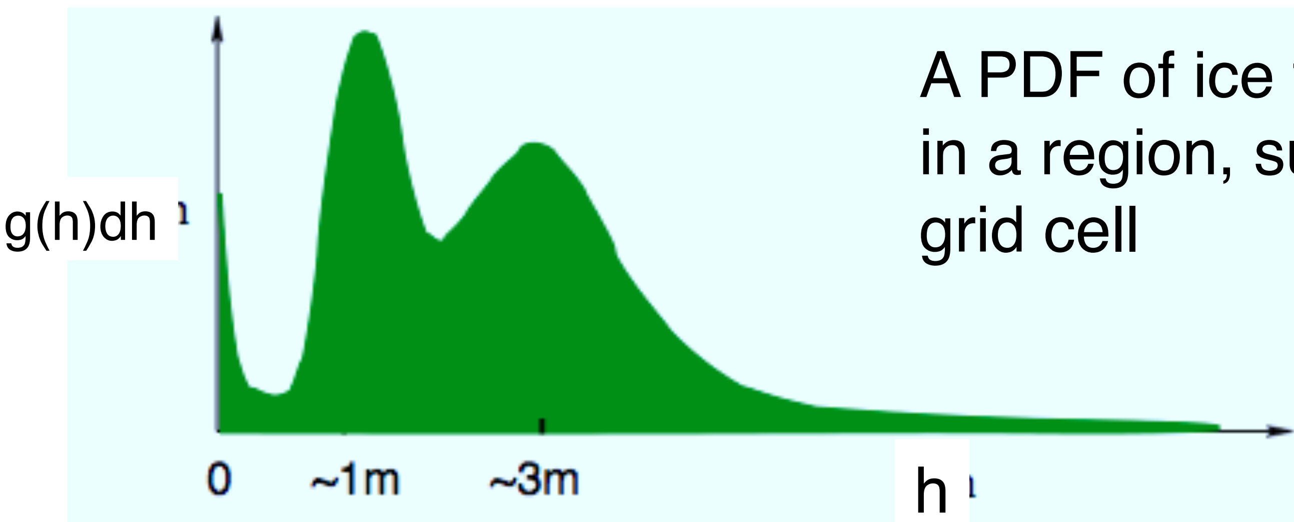
Notz (2009)

Next series of slides present the 4 governing equations for state of the art sea ice model used for climate studies (i.e., appropriate for basin scale or larger and for full seasonal cycle or longer)

1st Governing Equation

Ice thickness distribution $g(x,y,h,t)$ evolution equation from Thorndike et al. (1975)

$$\frac{\mathrm{Dg}}{\mathrm{Dt}} = -\mathrm{g}\nabla \cdot \mathbf{u} + \Psi - \frac{\partial}{\partial \mathrm{h}}(fg) + \mathcal{L}$$



A PDF of ice thickness h in a region, such as a grid cell

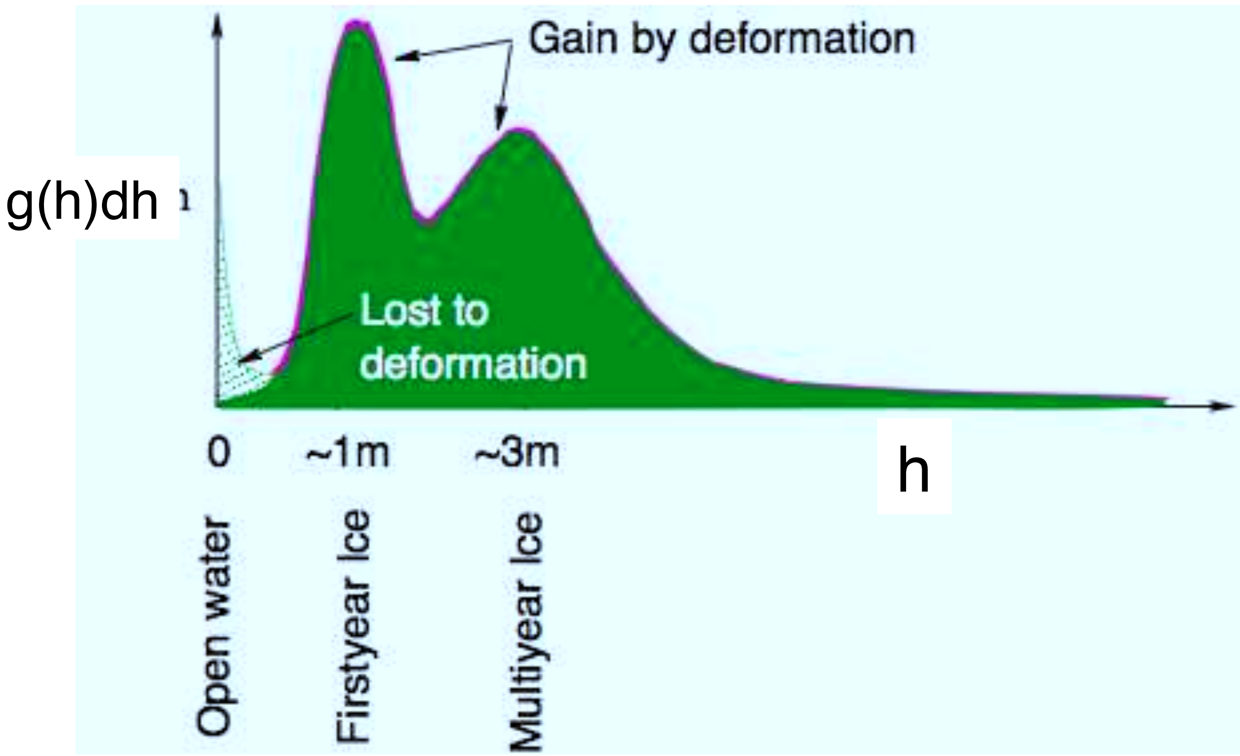
$$\frac{\mathrm{Dg}}{\mathrm{Dt}} = -\mathrm{g}\nabla \cdot \mathbf{u} + \Psi - \frac{\partial}{\partial \mathrm{h}}(fg) + \mathcal{L}$$

1 2 3 4 5

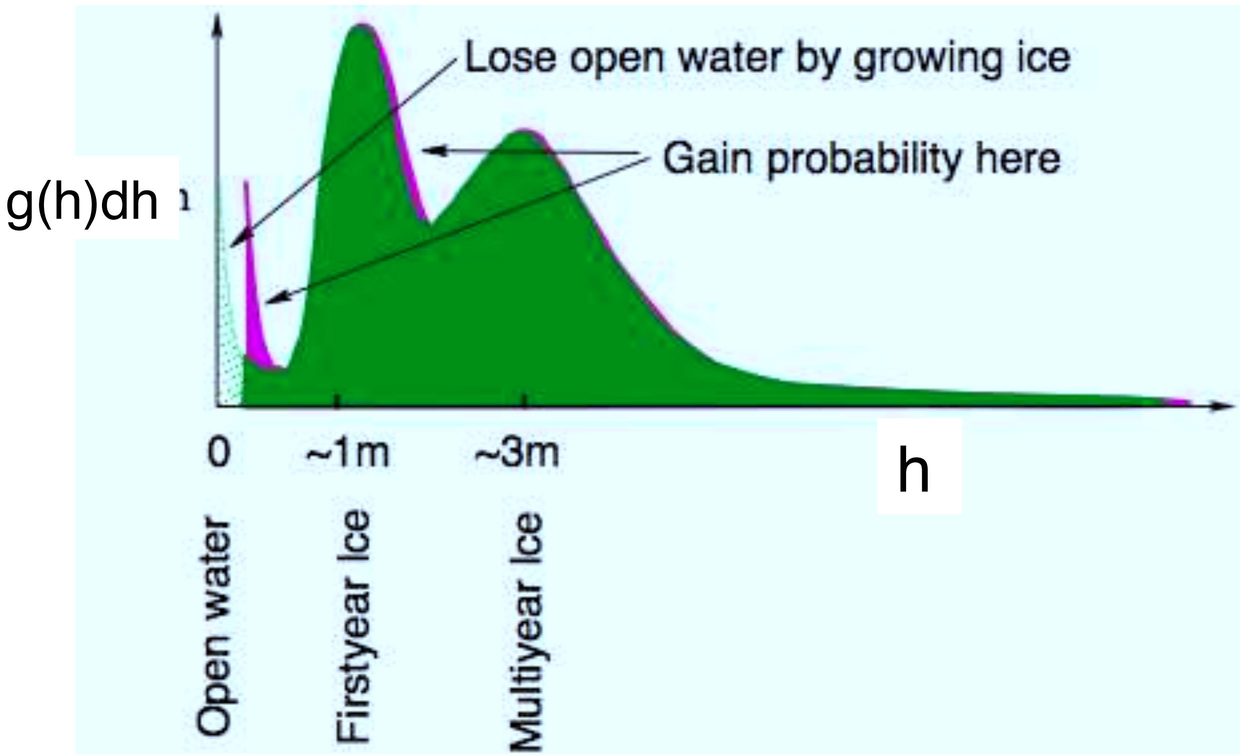
1. Lagrangian time derivative of g following "parcel"
2. Convergence of parcel
3. Ψ = Mechanical redistribution
4. Ice growth/melt results in "advection of g in thickness space"
5. $\mathcal{L}$ = Reduction of g from lateral melt

h = ice thickness
u = ice velocity
f = growth rate

Ψ = Mechanical redistribution



Advection in thickness space from growth



2nd Governing Equation

Conservation of momentum, see for example Hibler (1979)

$$\mathrm{m}\frac{\mathrm{D}\mathbf{u}}{\mathrm{D}t} = -\mathrm{mf}\mathbf{k}\times\mathbf{u} + \boldsymbol{\tau}_{\mathrm{a}} + \boldsymbol{\tau}_{\mathrm{w}} - \mathrm{mg}_{\mathrm{r}}\nabla\mathrm{Y} + \nabla\cdot\sigma$$

$$m\frac{D\mathbf{u}}{Dt} = -mf\mathbf{k}\times\mathbf{u} + \boldsymbol{\tau}_a + \boldsymbol{\tau}_w - mg_r\nabla Y + \nabla\cdot\sigma$$

1 2 3 4 5

1. Lagrangian time derivative of $\mathbf{u}$(x,y,t) following parcel
2. Coriolis force
3. τ_a, τ_w = air and water stresses
4. Ocean surface tilt
5. Ice interaction term

m = mass per unit area
f = Coriolis parameter
g_r = gravity
Y = Sea surface height
σ = ice stress

3rd Governing Equation

Conservation of Enthalpy E(x,y,z,t), the heat required to melt a unit area of sea ice or snow, see for example Bitz et al (2001)

$$\frac{\mathsf{DE}}{\mathsf{Dt}} = -\mathsf{E}\nabla \cdot \mathbf{u} + \Pi + \mathcal{E}$$

Models that neglect the heat capacity of ice, do not have this equation because in their case E is proportional to the ice volume

$$\frac{\mathsf{DE}}{\mathsf{Dt}} = -\mathsf{E}\nabla\cdot\mathbf{u} + \Pi + \mathcal{E}$$

1 2 3 4

1. Lagrangian time derivative of E(x,y,z,t) following parcel

2. Convergence of parcel

3. Π = Mechanical redistribution

4. $\mathcal{E}$ = contribution by thermodynamic processes

4th Governing Equation

Heat equation of sea ice and snow, Maykut and Untersteiner (1971)

$$\rho c \frac{\partial T}{\partial t} = \frac{\partial}{\partial z} k \frac{\partial T}{\partial z} + Q_{SW}(z)$$

Used to estimate last term in previous slide

$$\rho c \frac{\partial T}{\partial t} = \frac{\partial}{\partial z} k \frac{\partial T}{\partial z} + Q_{SW}(z)$$

1 2 3

1. Thermal energy change at a point

2. Gradient of the conductive flux

3. $Q_{SW}(z)$ Absorption of solar radiation

T = temperature
c = heat capacity
k = thermal conductivity
ρ= density

Caveats for this set of Governing Equation

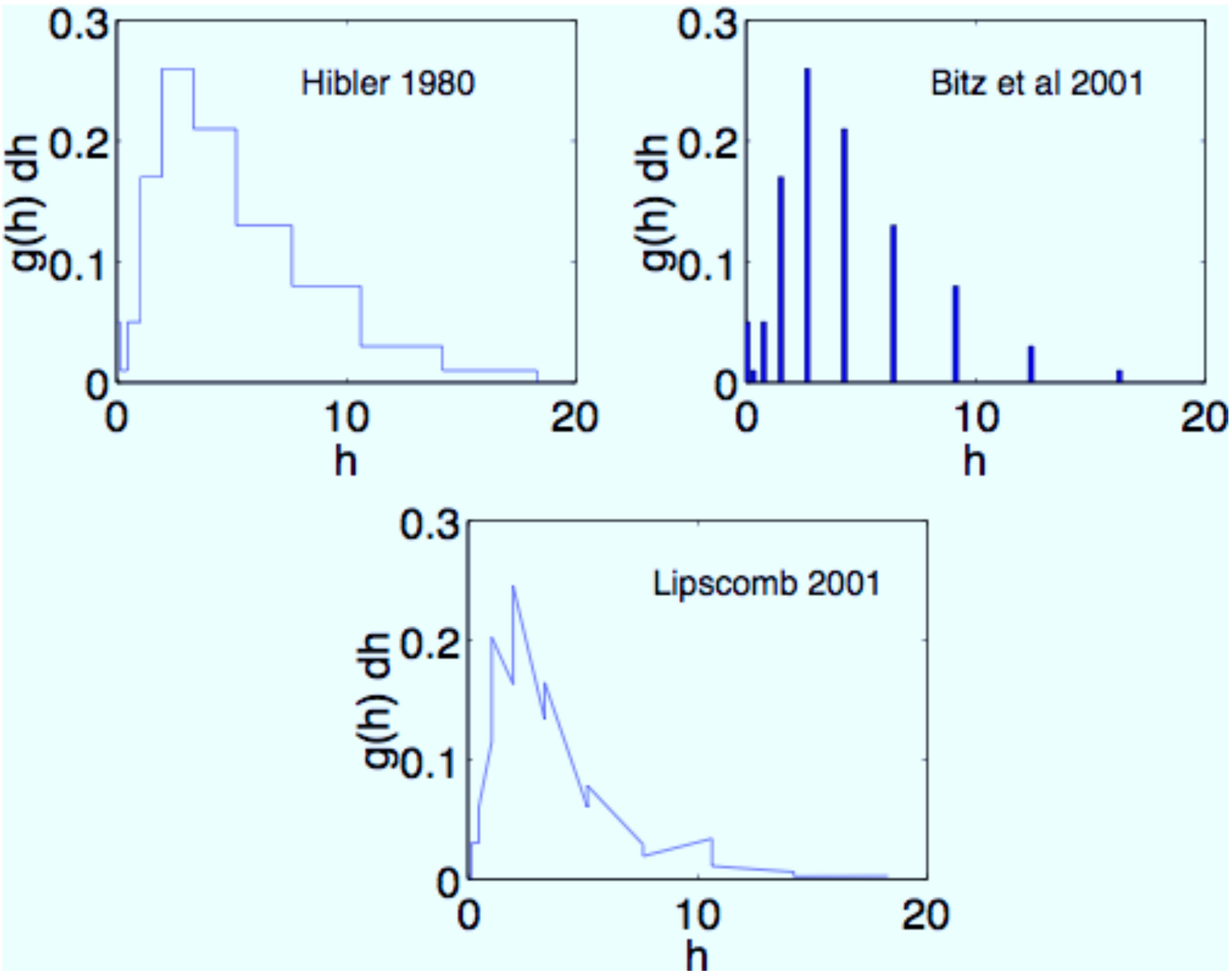
1. No explicit equation for the ice volume (or mass, yet), because conservation of volume is contained in the equation for g(h)

2. I don't have an equation for the salinity of the sea ice, which is time-independent in sea ice models used for climate (this may change soon). Must alter heat equation too for prognostic salinity.

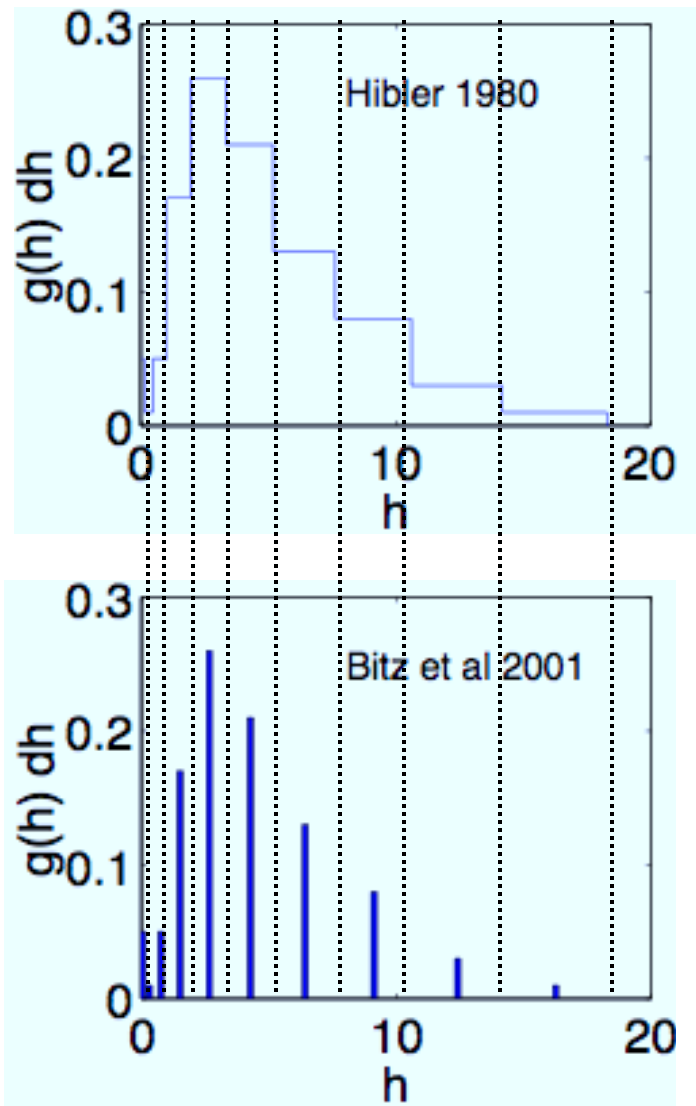
3. Brine pockets are accounted for in E

4. Radiative transfer - on your own.

Discretizations of g(H) for thickness advection



Discretizations of g(H) for thickness advection



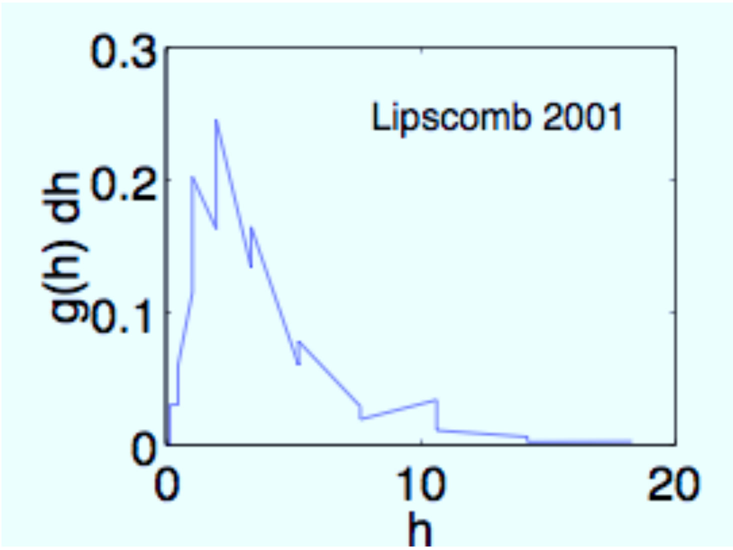
- Assumed uniform g within thickness bin, or category
- Mean thickness of ice in a category is midpoint of that category
- Eulerian

Simple but diffusive

- Delta functions move when ice grows/melts
- Thickness is a prognostic variable
- Lagrangian

Non-diffusive, bit less simple and categories empty abruptly

Just right



•g is a linear function of thickness in each bin
•Thickness is prognostic

Smooth and non-diffusive, but more complicated (though computationally efficient)

Area = sum of g for categories with finite thickness

How many people are actively developing sea ice models?

Sea ice models tend to be broken up numerically into three pieces

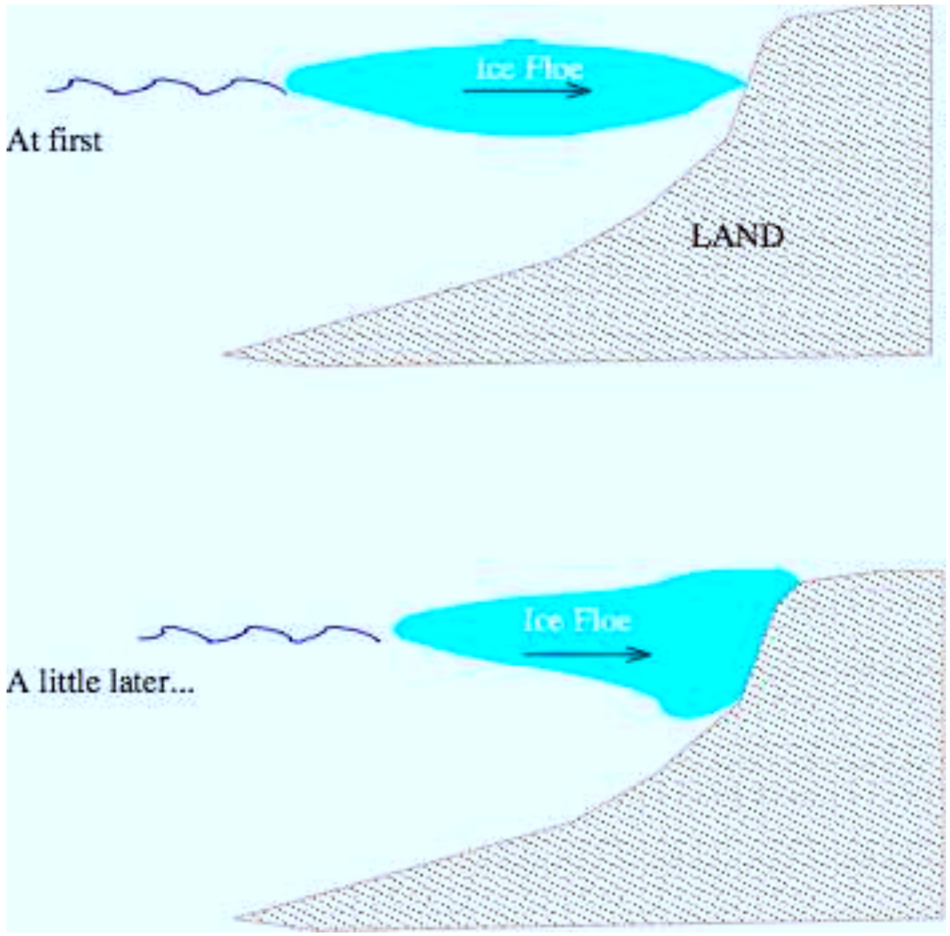
1. Dynamics including advection
2. Thermodynamics
3. Ice thickness distribution

Where more than one of these pieces influences a single equation, time splitting is employed, e.g.:

$$A^{n+1/2} = A^{n} + \Delta t\,(\text{term one})$$
$$A^{n+1} = A^{n+1/2} + \Delta t\,(\text{term two})$$

So the following slides break up governing equations

Sea Ice Dynamics in climate models



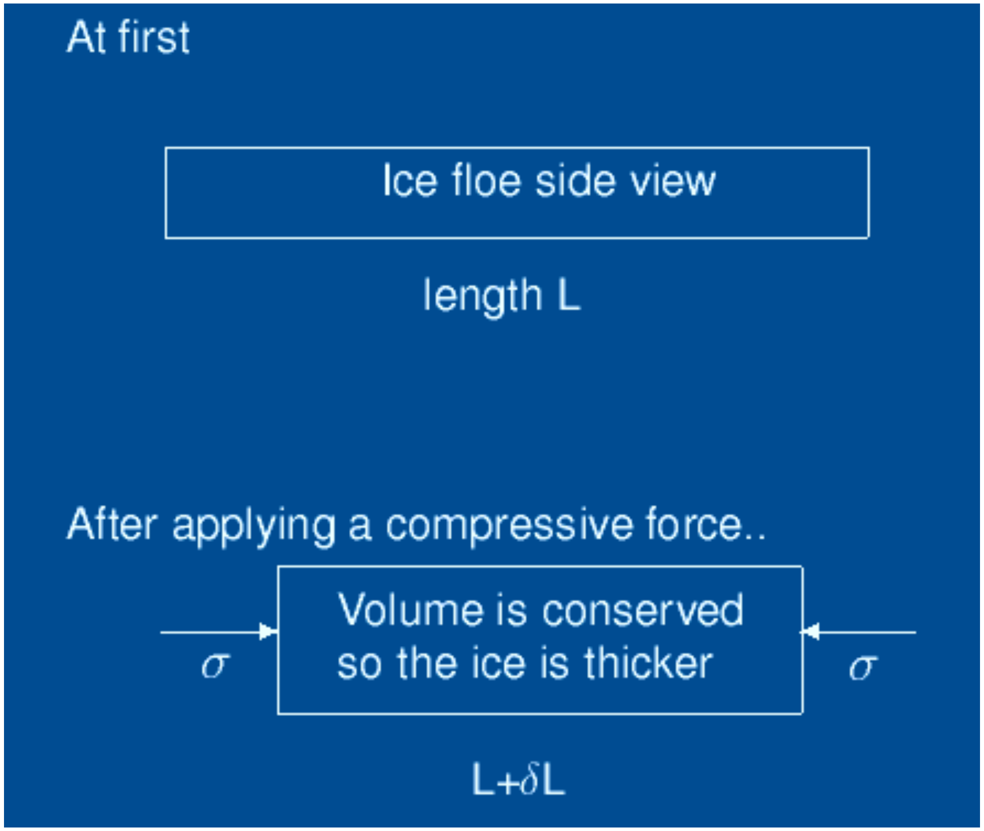
Past ad hoc method was to stop ice from moving at a critical thickness, sometimes called stopage

Sea Ice Dynamics Constitutive Law

A constitutive law characterizes the relationship between stress σ_{ij} and strain rate $\dot{\varepsilon}_{ij} = \partial u_i/\partial x_j$ defining the nature of the ice interaction.

The rheology used in most models is from Hibler 1979, where the the sea ice is treated as a continuum that is plastic at normal strain rates and viscous at very small strain rates.

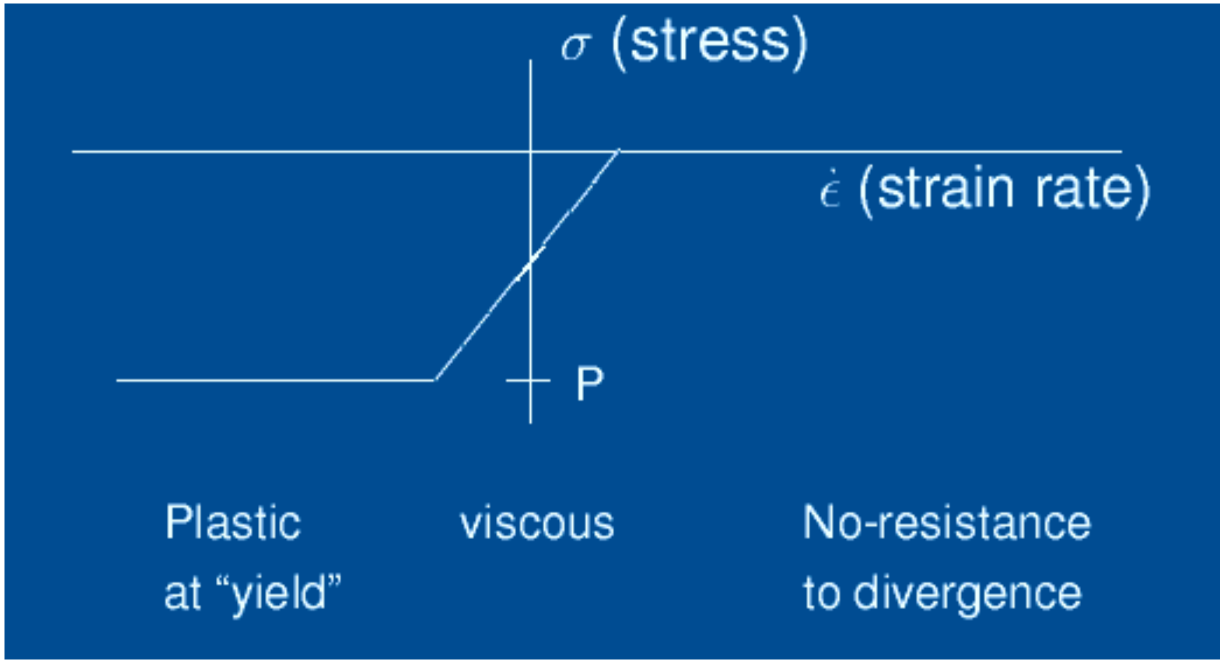
Engineering Compressive Stress Test



$$\text{Strain } \epsilon = \frac{\delta \mathrm{L}}{\mathrm{L}}$$

$$\text{Strain rate } \dot{\epsilon} = \frac{\delta L}{L \delta t}$$

VP Constitutive Law 1-D Representation



P=Ice compressive Strength

•when viscous, the stress state is maintained by a non-recoverable dissipation of energy
•when plastic, the ice yields and the strain energy goes into ridge building

VP Constitutive Law in 2-D

Invariants of stress (σ_I and σ_{II}) and strain rate ($\dot{\varepsilon}_I$ and $\dot{\varepsilon}_{II}$) are related by

$$\sigma_I = \zeta \dot{\epsilon}_I - P/2$$

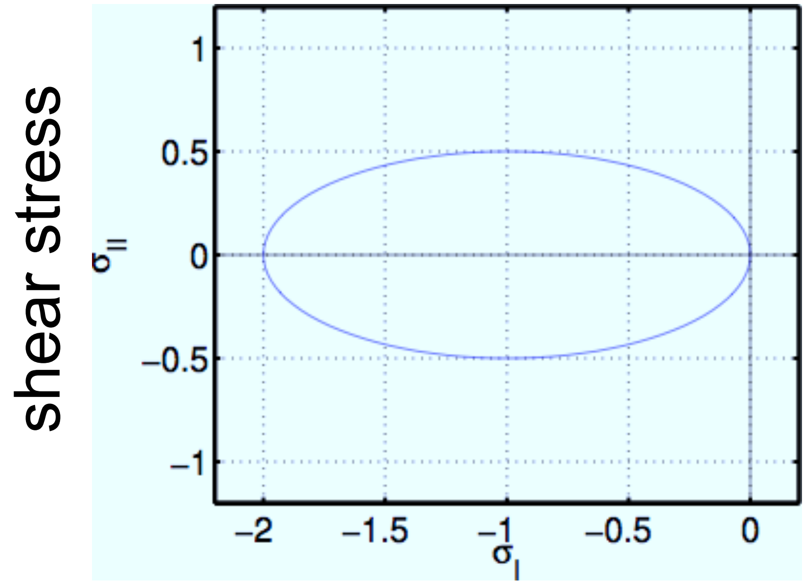
$$\sigma_{II} = \eta \dot{\epsilon}_{II}$$

ζ , η = bulk and shear viscosities:

$$\zeta = \frac{P}{2\Delta}, \quad \Delta = \left(\dot{\epsilon}_I^2 + \dot{\epsilon}_{II}^2 e^{-2}\right)^{1/2}$$

$$\eta = \frac{\zeta}{e^2}$$

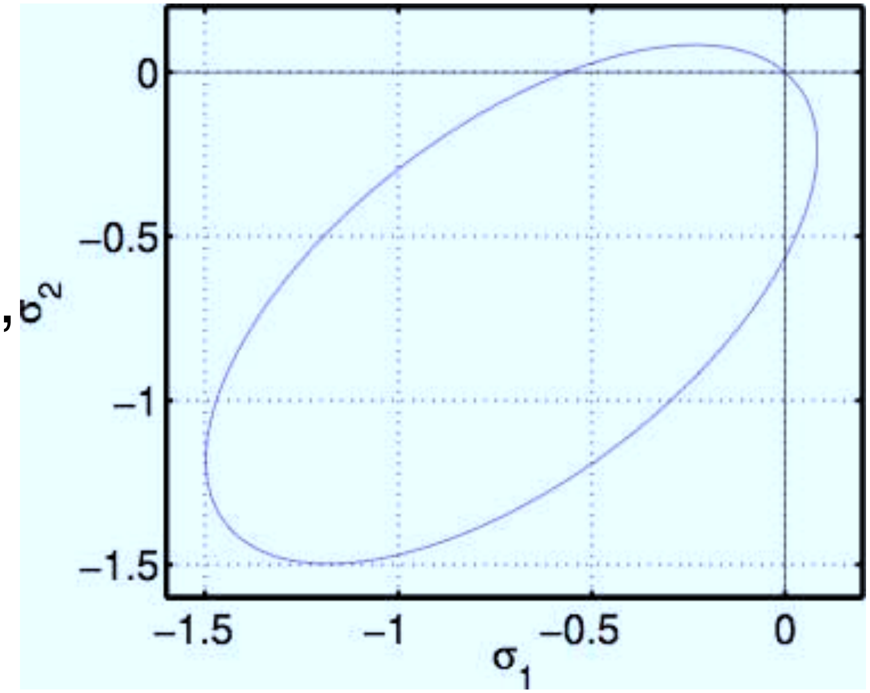
Are chosen so that the stress state for plastic flow lies on an elliptical yield curve with ratio of principal axes e=2



-1x compression

Some like to to rotate

Same yield curve but plotted against principal stress states, rather than stress invariants



Dynamics summary:

Must solve momentum equation, **u** in terms of σ, simultaneous with constitutive law, σ in terms of **u**

EVP model uses explicit time stepping by adding elastic waves to constitutive law, see Hunke and Dukowicz (1997)