Rechnungen HA5

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Spinor für Nele Blume:

Original-Spinor:

$$\phi = \begin{pmatrix} 14 e^{2i} \\ 5 e^{12i} \\ 12 e^{21i} \\ 5 e^{13i} \end{pmatrix}$$

zwei Komponenten ändern, um gültigen Spinor zu haben:

$$\phi' = \begin{pmatrix} 14 e^{2i} \\ 5 e^{12i} \\ 14 e^{2i} \\ -5 e^{12i} \end{pmatrix} \qquad \begin{aligned} a &= 14 e^{2i} \\ b &= 5 e^{12i} \end{aligned} \qquad \begin{aligned} |a| &= 14 \Rightarrow |a|^2 = 196 \\ |b| &= 5 \Rightarrow |b|^2 = 25 \end{aligned}$$

Energie: $2E = \phi'^{\dagger} \phi' = 2(|a|^2 + |b|^2)$

$$\Rightarrow E = |a|^2 + |b|^2 = 196 + 25 = 221$$

Masse: $2m = \phi'^{\dagger} \gamma^0 \phi' = (|a|^2 + |b|^2) - (|a|^2 + |b|^2) = 0$

$$\Rightarrow m = 0$$

Impuls: $2p^i = \phi'^{\dagger} \underbrace{\gamma^0 \gamma^i}_{= \begin{pmatrix} 0 & \sigma^i \\ \sigma^i & 0 \end{pmatrix}} \phi'$

$$\phi' = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \qquad u_1 = \begin{pmatrix} a \\ b \end{pmatrix} \qquad u_2 = \begin{pmatrix} a \\ -b \end{pmatrix}$$

$$\rightarrow 2p^i = u_1^{\dagger} \sigma^i u_2 + u_2^{\dagger} \sigma^i u_1$$

p_x: $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} a \\ -b \end{pmatrix} = \begin{pmatrix} -b \\ a \end{pmatrix}$, $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} b \\ a \end{pmatrix}$

$$(a^*, b^*) \begin{pmatrix} -b \\ a \end{pmatrix} + (a^*, -b^*) \begin{pmatrix} b \\ a \end{pmatrix} = \cancel{-a^* b} + \cancel{b^* a} + \cancel{a^* b} - \cancel{b^* a}$$

$$\Rightarrow p_x = 0$$

p_y: $\begin{pmatrix} 0 & -i \\ \cdot & \cdot \end{pmatrix} \begin{pmatrix} a \\ \cdot \end{pmatrix} = \begin{pmatrix} ib \\ \cdot \end{pmatrix}$ $\qquad \begin{pmatrix} 0 & -i \\ \cdot & \cdot \end{pmatrix} \begin{pmatrix} a \\ \cdot \end{pmatrix} = \begin{pmatrix} -ib \\ \cdot \end{pmatrix}$

p_y: $\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\begin{pmatrix} a \\ -b \end{pmatrix} = \begin{pmatrix} ib \\ ia \end{pmatrix}$, $\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} -ib \\ ia \end{pmatrix}$

$(a^*, b^*)\begin{pmatrix} ib \\ ia \end{pmatrix} + (a^*, -b^*)\begin{pmatrix} -ib \\ ia \end{pmatrix} = \cancel{a^* ib} + \cancel{b^* ia} - \cancel{a^* ib} - \cancel{b^* ia}$

$\Rightarrow p_y = 0$

p_z: $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} a \\ -b \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix}$, $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} a \\ -b \end{pmatrix}$

$(a^*, b^*)\begin{pmatrix} a \\ b \end{pmatrix} + (a^*, -b^*)\begin{pmatrix} a \\ -b \end{pmatrix} = a^*a + b^*b + a^*a + b^*b$

$\Rightarrow 2p_z = 2(a^*a + b^*b) \Rightarrow p_z = |a|^2 + |b|^2 = 221$

$\Rightarrow \vec{p} = (0, 0, 221) \Rightarrow |\vec{p}| = 221$

$\rightarrow E^2 = m^2 + |\vec{p}|^2$ passt ✓

Ruhespin: $\chi = \frac{1}{\sqrt{E}} u_1$

Spinrichtung: $\vec{n} = \chi^+ \vec{\sigma} \chi = (\chi^+ \sigma_x \chi, \chi^+ \sigma_y \chi, \chi^+ \sigma_z \chi)$

$\vec{S}_0 = \frac{1}{2}\vec{n}$

$n_x = \chi^+ \sigma_x \chi = \frac{1}{E}(a^*b + b^*a) = \frac{2\,\mathrm{Re}(a^*b)}{E}$

$n_y = \chi^+ \sigma_y \chi = \frac{1}{E}(-ia^*b + ib^*a) = \frac{2\,\mathrm{Im}(a^*b)}{E}$

$n_z = \chi^+ \sigma_z \chi = \frac{1}{E}(|a|^2 - |b|^2)$

$a^*b = |a||b| e^{i(\beta-\alpha)} = 14 \cdot 5 \cdot e^{10i} = 70 e^{10i}$

$\mathrm{Re}(a^*b) \approx -58{,}735$

$\mathrm{Im}(a^*b) \approx -38{,}082$

$\rightarrow n_x \approx -0{,}5315 \;, n_y \approx -0{,}3446 \;, n_z \approx 0{,}7738$

$\Rightarrow S_x^0 \approx -0{,}268 \;, S_y^0 \approx -0{,}1723 \;, S_z^0 \approx 0{,}3869$

Spinor für Hanna Schulteisi

Original-Spinor: $\phi = \begin{pmatrix} 8e^{18i} \\ 1e^{3i} \\ 14e^{8i} \\ 14e^{21i} \end{pmatrix}$

zwei Komponenten ändern, um gültigen Spinor zu haben:

$\phi' = \begin{pmatrix} 8e^{18i} \\ e^{3i} \\ 8e^{18i} \\ e^{3i} \end{pmatrix}$ $\quad a = 8e^{18i}$, $b = e^{3i}$ $\quad |a| = 8 \Rightarrow |a|^2 = 64$, $|b| = 1 \Rightarrow |b|^2 = 1$

Energie: $2E = \phi'^{\dagger}\phi' = 2(|a|^2 + |b|^2)$

$\Rightarrow E = |a|^2 + |b|^2 = 65$

Masse: $2m = \phi'^{\dagger}\gamma^0\phi' = (|a|^2 + |b|^2) - (|a|^2 + |b|^2) = 0$

$\Rightarrow m = 0$

Impuls: analog zu Noble Blume, da gleiches Schema $\phi' = \begin{pmatrix} a \\ b \\ a \\ -b \end{pmatrix}$

$\Rightarrow p_x = 0$

$p_y = 0$

$p_z = |a|^2 + |b|^2 = 64 + 1 = 65$

$\Rightarrow \vec{p} = (0, 0, 65) \Rightarrow |\vec{p}| = 65$

$\rightarrow E^2 = m^2 + |\vec{p}|^2$ passt ✓

Ruhespin: $\chi = \frac{1}{\sqrt{65}} \begin{pmatrix} 8e^{19i} \\ e^{3i} \end{pmatrix}$

analog zu Nele Blume

$a^*b = 8e^{-19i} \cdot e^{3i} = 8e^{-16i}$

$Re(a^*b) \approx -7{,}661$

$Im(a^*b) \approx 2{,}303$

$n_x \approx \frac{2 \cdot (-7{,}661)}{65} \approx -0{,}2357$

$n_y \approx \frac{2 \cdot 2{,}303}{65} \approx 0{,}0709$

$n_z = \frac{64-1}{65} \approx 0{,}9692$

$\rightarrow \vec{S}^0 \approx (-0{,}118 ; 0{,}035 ; 0{,}485)$

Spinor für Noah Eichhorn:

Original-Spinor: $\phi = \begin{pmatrix} 14\,e^{5i} \\ 15\,e^{9i} \\ 1\,e^{3i} \\ 8\,e^{8i} \end{pmatrix}$

zwei Komponenten ändern, um gültigen Spinor zu haben:

$\phi' = \begin{pmatrix} 14e^{5i} \\ 15e^{9i} \\ 14e^{5i} \\ 8e^{8i} \end{pmatrix}$ $\quad a = 14e^{5i} \quad b = 15e^{9i}$ $\quad |a| = 14 \Rightarrow |a|^2 = 196 \quad |b| = 15 \Rightarrow |b|^2 = 15$

Energie: $2E = \phi'^{+}\phi' = 2(|a|^2 + |b|^2)$

$\Rightarrow E = (|a|^2 + |b|^2) = 421$

Masse: $2m = \phi'^{+} \gamma^0 \phi' = (|a|^2 + |b|^2) - (|a|^2 + |b|^2) = 0$

$\Rightarrow m = 0$

Impulsi: analog zu den anderen beiden Namen:

$\rightarrow p_x = 0$

$p_y = 0$

$p_z = |a|^2 + |b|^2 = 421$

$\Rightarrow \vec{p} = (0, 0, 421) \Rightarrow |\vec{p}|^2 = 421$

$\rightarrow E^2 = m^2 + p^2$ passt ✓

Ruhespin: analog zu den anderen Namen

$$\chi = \frac{1}{\sqrt{421}} \begin{pmatrix} 14 e^{5i} \\ 15 e^{9i} \end{pmatrix}$$

$$a^* b = 14 \cdot 15 e^{i(9-5)} = 210 e^{4i}$$

$$\mathrm{Re}(a^* b) \approx -137{,}26$$

$$\mathrm{Im}(a^* b) \approx -158{,}93$$

$$n_x \approx \frac{2 \cdot (-137{,}26)}{421} \approx -0{,}6521$$

$$n_y \approx \frac{2 (-158{,}93)}{421} \approx -0{,}7550$$

$$n_z = \frac{196 - 225}{421} \approx -0{,}0689$$

$$\vec{S}^0 \approx (-0{,}326;\ -0{,}378;\ -0{,}034)$$