MATH221 Mathematics for Computer Science

Tutorial Sheet Week 11

Autumn 2017

1. Let $A = \{1, 2, 3, 4\}$, $B = \{a, b, c, d, e\}$

(i) Write down non-empty $f \subseteq A \times B$ which is not a function

(ii) Write down $f \subseteq A \times B$ which is a function which is not one-to-one or onto

(iii) Write down $f \subseteq A \times B$ which is a one-to-one function

(iv) Write down $f \subseteq B \times A$ which is an onto function

(v) Write down $f \subseteq A \times A$ which is one-to-one and onto but is not the identity function (i.e. $f(x) \neq x$ for some $x \in A$.

(vi) Write down non-empty $R \subseteq A \times A$ which has none of the three properties of an equivalence relation.

(vii) Write down $R \subseteq A \times A$ which has one of the three properties of an equivalence relation (3 cases)

(viii) Write down $R \subseteq A \times A$ which has two of the three properties of an equivalence relation (3 cases)

(ix) Write down $R \subseteq A \times A$ which has all three of the properties of an equivalence relation. What are the equivalence classes?

2. Consider the following

(i) Let $f : [0, \infty) \to \mathbb{R}$, defined by $f(x) := x^2 + 1$, for $x \geq 0$. Show that f is a function, and that it is one-to-one but not onto. How can the range of f be changed to allow an inverse function f^{-1} to be defined?

(ii) Let $f : \mathbb{R} \to [0, \infty)$, defined by $f(x) := x^4$ for $x \in \mathbb{R}$. Show that f is a function, and that it is onto but not one-to-one. How can the domain of f be changed to allow an inverse function f^{-1} to be defined?

(iii) Let $f : \mathbb{R} \to \mathbb{R}$, defined by $f(x) := x^3$ for $x \in \mathbb{R}$. Show that f is a function, and that it is onto and one-to-one. Can an inverse function f^{-1} be defined?

(iv) Let $f : (0, 1) \to (0, \infty)$, defined by $f(x) := \dfrac{x}{1 - x}$ for $x \in (0, 1)$, is one-to-one and onto. Show further that $g : (0, \infty) \to (0, 1)$, defined by $g(t) := \dfrac{t}{t + 1}$ for $t > 0$ is the inverse of f.

3. Inverses of familiar functions.

(i) $\cos : \mathbb{R} \to \mathbb{R}$ is not one-to-one or onto. How can the domain of $\cos$ be changed to allow an inverse function $\arccos$ to be defined?

(ii) $\tan : \mathbb{R} \to \mathbb{R}$ is not one-to-one but is onto. How can the domain of $\tan$ be changed to allow an inverse function $\arctan$ to be defined?

(iii) $\exp : \mathbb{R} \to \mathbb{R}$ is not onto. How can the range of $\exp$ be changed to allow an inverse function $\ln$ to be defined?