MATH221 Mathematics for Computer Science

Outline Solutions to Tutorial Sheet Week 10

Autumn 2017

1. $\binom{8}{5} = \frac{8!}{5! \times 3!} = 56.$

2.

(i) $\binom{10}{2} = \frac{10!}{8! \times 2!} = 45$ chords.
(ii) $\binom{10}{3} = \frac{10!}{7! \times 3!} = 120$ triangles.
(iii) $\binom{10}{6} = \frac{10!}{4! \times 6!} = 210$ hexagons.

3. Recall that if $|A| = n$, then A has 2^n subsets. To be proper and nonempty we exclude A and $\emptyset$; Solve $2^n - 2 > 100$; $n \geq 7$.

4. Recall $\sum_{k=0}^{n} \binom{n}{k} = 2^n$; We want $\binom{8}{1} + \binom{8}{2} + \ldots + \binom{8}{8}$, which equals $2^8 - 1 = 255$.

5. Apply the Binomial Theorem with $x = -1, y = 5, n = 13$ to show that $\sum_{r=0}^{13} (-1)^r \binom{13}{r} 5^{13-r} = (5 + (-1))^1 3$.

6. $A \times B = \{(1, a), (1, b), (2, a), (2, b)\}$
$B \times A = \{(a, 1), (a, 2), (b, 1), (b, 2)\}$
$(A \times B) \times B = \{((1, a), a), ((1, a), b), ((1, b), a), ((1, b), b), ((2, a), a), ((2, a), b), ((2, b), a), ((2, b), b)\}$

7. The elements $(x, x + 1) \in R$ for all $x \in \mathbb{R}$. Furthermore, $(2, 6), (2, 11), (3, 7), (3, 12) \in R$.

8. Since $x^2 + y^2 = 4$ is the equation of the circle of radius 2, and centered at the origin, dom $R =$ ran $R = [-2, 2]$.

9. Since $x \equiv x \pmod{p}$, $(x, x) \in R$, for each $x \in \mathbb{Z}$. Hence R is reflexive. For each pair $(x, y) \in R$, $x - y$ is divisible by p. This yields, $y - x$ is also divisible by p, and hence $(y, x) \in R$. Therefore, R is symmetric. Let $(x, y), (y, z) \in R$. Then there exists integers m and n such that

$$x - y = mp \quad \text{and} \quad y - z = np.$$

By adding the above two equations gives,

$$x - z = (m + n)p \quad \text{or equivalently} \quad x \equiv z \pmod{p}.$$

This proves that R is transitive.