

MATH221 Mathematics for Computer Science

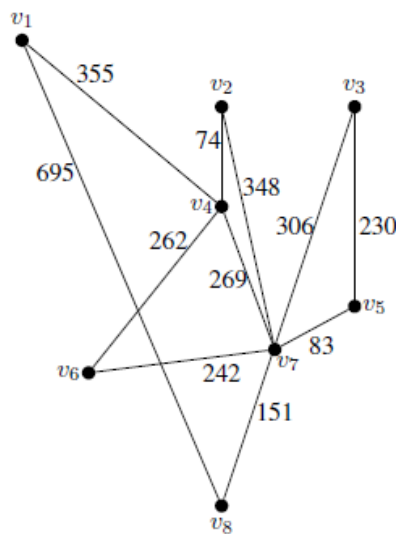
Tutorial Sheet Week 11

Autumn 2017

1.

- (i) How many subgraphs are there of K_n which have all n vertices? (Hint: Ideas about power sets are relevant.) Note that the question is not asking about the number of *non-isomorphic* subgraphs of K_n ; that is a harder problem.
- (ii) How many non-isomorphic subgraphs are there of K_3 which have all 3 vertices?

2. Use Kruskal's and Prim's Algorithms to find a minimum spanning tree for the following weighted graph. What is the total weight of the minimum spanning tree?



3. Let $f : \mathbb{N} \rightarrow \mathbb{N}$ be given by $f(n) = 1 + n/2$ if n is even, and $f(n) = 1 + (n - 1)/2$ if n is odd. Calculate the range of f and determine whether f is one-to-one.

4. Prove the following statements. (i) $f : [0, \infty) \rightarrow \mathbb{R}$, defined by $f(x) := x^2 + 1$, for $x \geq 0$, is one-to-one but not onto. (ii) $f : \mathbb{R} \rightarrow (0, \infty)$, defined by $f(x) := x^2$ for $x \in \mathbb{R}$, is onto but not one-to-one. (iii) $f : (0, 1) \rightarrow (0, \infty)$, defined by $f(x) := \frac{x}{1 - x}$ for $x \in (0, 1)$, is bijective.