MATH121: Discrete Mathematics

Outline Solutions to Tutorial Sheet Week 2

Autumn 2017

First Page.

At least 3 eyes means 3, 4, 5, ..., or more. (False).
At most 3 eyes means 0,1,2, or 3. (True). So, 2 should be circled.
No person has exactly 3 eyes, so a small triangle should be drawn at the top of the paper.

I think you are neither fish nor fowl, so the next sentence should have been omitted.(No folding!)

2 is the smallest prime number, so a circle should be drawn around your surname.

A square is not round, so the next sentence should have been omitted.(No 1 cm square!).
'whenever' means 'if', so "underline" should be underlined and since u is a vowel. The rectangle should be drawn around the word "sentence".

No tick unless you are VERY tall.

It is not true that all triangles are isosceles, so the underlined word is: isosceles .

The average rate of speed is $\dfrac{\text{distance}}{\text{time}}$. Distance = 8km.
Time = time at rate 1 + time at rate 2 $= \dfrac{1}{2} + \dfrac{2}{3} = \dfrac{7}{6}$. So, average speed $= 8\left(\dfrac{6}{7}\right) = 6\dfrac{6}{7} = 6.857$.

$(17)(19) = 323$. So, 323 has 2 different prime factors each less than 20.

Solutions to Problems.

1. (i) If $x = 3$, then $x < 2$.
- (a) Statement
- (b) False
- (c) $x = 3 \implies x < 2$

(ii) If $x = 0$ or $x = 1$, then $x^2 = x$.
- (a) Statement
- (b) True
- (c) $(x = 0 \ \vee \ x = 1) \implies x^2 = x$

(iii) $x^2 = x$ only if $x = 0$ or $x = 1$.
- (a) Statement
- (b) True
- (c) $x^2 = x \implies (x = 0 \ \vee \ x = 1)$

(iv) There exists a natural number x for which $x^2 = \frac{x}{2}$.
- (a) Statement
- (b) False

(v) If $x \in \mathbb{N}$ and $x > 0$, then if $\sqrt{x} > 1$, $x > 1$.
- (a) Statement
- (b) True
- (c) $(x \in \mathbb{N} \ \wedge \ x > 0) \implies (\sqrt{x} > 1 \implies x > 1)$

(vi) $xy = 5$ implies that either $x = 1$ and $y = 5$ or $x = 5$ and $y = 1$.
- (a) Statement
- (b) False
- (c) $xy = 5 \implies \big((x = 1 \ \wedge \ y = 5) \ \vee \ (x = 5 \ \wedge \ y = 1)\big)$

(vii) $xy = 0$ implies $x = 0$ or $y = 0$.
- (a) Statement
- (b) True
- (c) $xy = 0 \implies x = 0 \ \vee \ y = 0$

(viii) $xy = yx$.
- (a) Statement
- (b) True

(ix) There is a unique even prime number.
- (a) Statement
- (b) True

2. (i) $P:\ x$ is odd. $Q:\ y$ is odd. $R:\ x+y$ is even. Form: $P \wedge Q \Longrightarrow R$.

(ii) P : It is raining. Q : It is hot. Form: $\sim (P \wedge Q)$.

(iii) P : It is raining. Q : It is hot. Form: $P \wedge Q$.

(iv) P : It is raining. Q : It is hot. Form: $\sim P \wedge \sim Q$ or $\sim (P \vee Q)$.

(v) $P:\ x \geq -1$. $Q:\ x \leq 2$. Form: $(P \wedge Q)$.

3. $P \vee Q$: Mathematics is easy or I do not need to study.

$\sim Q$: It is not the case that I do not need to study. $\equiv$ I need to study.

$\sim\sim Q$: It is not the case that it is not the case that I do not need to study. $\equiv$ I do not need to study.

$\sim P$: Mathematics is not easy.

$\sim P \wedge Q$: Mathematics is not easy and I do not need to study.

$P \Longrightarrow Q$: If Mathematics is easy, then I do not need to study.

4. (i) The truth tables for $(\sim P \vee Q) \wedge Q$ and $(\sim P \wedge Q) \vee Q$.

P	Q	$(\sim$	$P \vee$	$Q) \wedge Q$	$(\sim$	$P \wedge$	$Q) \vee Q$
T	T	F	T	**T**	F	F	**T**
T	F	F	F	**F**	F	F	**F**
F	T	T	T	**T**	T	T	**T**
F	F	T	T	**F**	T	F	**F**

The tables are the same!

(ii) The truth tables for $(\sim P \vee Q) \wedge P$ and $(\sim P \wedge Q) \vee P$.

P	Q	$(\sim$	$P \vee$	$Q) \wedge P$	$(\sim$	$P \wedge$	$Q) \vee P$
T	T	F	T	**T**	F	F	**T**
T	F	F	F	**F**	F	F	**T**
F	T	T	T	**F**	T	T	**T**
F	F	T	T	**F**	T	F	**F**

The tables are not the same. The student's guess is false.

5. (i) The truth tables for $P \vee \sim P$ and $P \wedge \sim P$.

P	$P \vee$	$\sim P$	$P \wedge$	$\sim P$
T	**T**	F	**F**	T
F	**T**	T	**F**	F

(ii) $P \vee \sim P$ is a tautology; $P \wedge \sim P$ is a contradiction.

(iii) Use truth tables.

P	Q	$(P \vee \sim P)$	$\vee Q$	$(P \wedge \sim P)$	$\wedge Q$
T	T	T	**T**	F	**F**
T	F	T	**T**	F	**F**
F	T	T	**T**	F	**F**
F	F	T	**T**	F	**F**

Notice that "true $\vee$ anything" is true and "false $\wedge$ anything" is false (see Conclusion).

6. (i) Truth tables are as follows.

P	Q	R	$(P \vee \sim P)$	$\wedge$	$(Q \vee R)$	$Q \vee R$
T	T	T	T	**T**	T	**T**
T	T	F	T	**T**	T	**T**
T	F	T	T	**T**	T	**T**
T	F	F	T	**F**	F	**F**
F	T	T	T	**T**	T	**T**
F	T	F	T	**T**	T	**T**
F	F	T	T	**T**	T	**T**
F	F	F	T	**F**	F	**F**

Notice that the two statements are logically equivalent. In fact, the truth value of the first is dependent entirely on the second (see Conclusion.)

(ii) Truth tables are as follows.

P	Q	R	$(P \wedge \sim P)$	$\vee$	$(Q \wedge R)$	$Q \wedge R$
T	T	T	F	**T**	T	**T**
T	T	F	F	**F**	F	**F**
T	F	T	F	**F**	F	**F**
T	F	F	F	**F**	F	**F**
F	T	T	F	**T**	T	**T**
F	T	F	F	**F**	F	**F**
F	F	T	F	**F**	F	**F**
F	F	F	F	**F**	F	**F**

Notice that the two statements are logically equivalent. In fact, the truth value of the first is again dependent entirely on the second (see Conclusion.)