MATH221: Mathematics for Computer Science

Tutorial Sheet Week 4

Autumn 2017

Validating arguments

1. Rewrite each of the following arguments into logical form, and then use a truth table to check the validity of the argument.

(i) If I go to the movies, then I will carry my phone or my 3D glasses. I am carrying my phone but not my 3D glasses. Therefore, I will go to the movies.

(ii) I will buy a new bike or a used car. If I buy both a new bike and a used car, I will need a loan. I bought a used car and I don't need a loan. Therefore, I didn't buy a new bike.

Methods of Proof

2. Each of the following demonstrates either the Rule of Modus Ponens or the Law of Syllogism. In each case, answer the question or complete the sentence and indicate which of the two logical rules is demonstrated.

(i) If Caz is unsure of an address then Caz will phone, and Caz is unsure of Graham's address. What does Caz do?

(ii) If $x^2 - 3x + 2 = 0$, then $(x-2)(x-1) = 0$. If $(x-2)(x-1) = 0$ then $x - 2 = 0$ or $x - 1 = 0$. If $x - 2 = 0$ or $x - 1 = 0$ then $x = 2$ or $x = 1$. Therefore, if $x^2 - 3x + 2 = 0$, then ...

(iii) We all know that if x is a real number, then its square is positive or zero. Now, when $\sqrt{y}$ is a real number what do we know about y?

3. Prove or disprove the following statements.

(i) For all $n \in \mathbb{N}$, the expression $n^2 + n + 29$ is prime, that is, has no factors other than 1 and itself.

(ii) $\exists x \in \mathbb{Q},\ \forall y \in \mathbb{Q},\ xy \neq 1$.

(iii) $\forall a, b \in \mathbb{R},\ (a+b)^2 = a^2 + b^2$.

(iv) The average of any two odd integers is odd.

4. Find the mistakes in the following "proofs".

(i) *Result:* $\forall k \in \mathbb{Z},\ (k > 0 \implies k^2 + 2k + 1 \text{ is not prime})$.

Proof: For $k = 2$, $k^2 + 2k + 1 = 9$, which is not prime. Therefore, the result is true.

(ii) *Result:* The difference between any odd integer and any even integer is odd.

Proof: Let n be any odd integer, and m any even integer. By definition of odd, $n = 2k+1$ for some $k \in \mathbb{Z}$, and by definition of even, $n = 2k$ for some $k \in \mathbb{Z}$. Then $n - m = (2k+1) - 2k = 1$. But 1 is odd. Therefore, the result holds.

5. Prove each of the following results using a direct proof.

(i) For $x \in \mathbb{R}$, $x^2 + 1 \geq 2x$.

(ii) For $n \in \mathbb{N}$, if n is odd, then n^2 is odd.

(iii) The sum of any two odd integers is even.

(iv) If the sum of two angles of a triangle is equal to the third angle, then the triangle is a right angled triangle.

6. In lectures, it has been mentioned that in a proof, you should avoid starting with what you are trying to prove as the steps in your proof may not be "reversible". Here is an example to demonstrate this!

Prove that if x is a negative real number, then $(x - 2)^2 > 4$.

7. Prove each of the following statements using a proof by contradiction.

(i) If n^2 is odd then n is odd.

(ii) There is no smallest positive real number.

8. Prove each of the following statements using a proof by cases.

(i) If $x = 4$, 5 or 6, then $x^2 - 3x + 21 \neq x$.

(ii) For all $x \in \mathbb{Z}$, if $x \neq 0$, then $2^x + 3 \neq 4$.

Numbers

9. For $a, b \in \mathbb{R}$ with $a \leq b$ we denote by (a, b) the set of all real number strictly between a and b. In set notation we write $(a, b) = \{x \in \mathbb{R} : a < x < b\}$. We denote by $[a, b]$ the set of all real numbers between a and b and including a and b. In set notation we write $[a, b] = \{x \in \mathbb{R} : a \leq x \leq b\}$. We let you figure out the sets $(a, b]$ and $[a, b)$. State which of the following sets have least elements and greatest elements, and what these elements are when they exist.

(i) $[0, 1]$

(ii) $[0, 1)$

(iii) $\left\{1 - \frac{1}{2}, 1 - \frac{1}{3}, 1 - \frac{1}{4}, \dots\right\}$

(iv) The set of all rational numbers (i.e., whole numbers and fractions) between 0 and 1 but excluding 0 and 1.

Which of the above sets are well-ordered?