Truth Tables Cheat Sheet

Conjunction

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Disjunction

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Conditionals

p	q	$p => q$
T	T	T
T	F	F
F	T	T
F	F	T

Biconditionals

p	q	$p <=> q$
T	T	T
T	F	F
F	T	F
F	F	T

Logical Equivalence Laws

Commutative Laws

1. $p \vee q \equiv q \vee p$
2. $p \wedge q \equiv q \wedge p$
3. $p <=> q \equiv q <=> p$

Associative Laws

1. $(p \vee q) \vee r \equiv p \vee (q \vee r)$
2. $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$
3. $(p <=> q) <=> r \equiv p <=> (q <=> r)$

Distributive Laws

1. $p \vee (q \wedge r) \equiv (p \vee q) \wedge (q \vee r)$
2. $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
3. $p => (q \vee r) \equiv (p => q) \vee (p => r)$
4. $p => (q \wedge r) \equiv (p => q) \wedge (p => r)$

Double Negative Law

1. $\sim (\sim p) \equiv p$

De Morgan’s Laws

1. $\sim (p \vee q) \equiv \sim p \wedge \sim q$
2. $\sim (p \wedge q) \equiv \sim p \vee \sim q$

Implication Laws

1. $p <=> q \equiv (p => q) \wedge (q => p)$
2. $p => q \equiv \sim p \vee q$
3. $p => q \equiv \sim q => \sim p$
4. $\sim (p => q) \equiv p \wedge \sim q$

Set Algebra

Closed:
A binary operation $*$ is closed if:

$$\boldsymbol{x, y \in S \;\Rightarrow\; x * y \in S}$$

Identity:
An element $e \in S$ is called an identity if:

$$\boldsymbol{x * e = x \text{ } AND \text{ } e * x = x \;\forall\; x \in S}$$

Inverse:
If $\exists$ e identity of S, an element $x \in S$ is called invertible when $\exists\; y \in S \ni$:

$$\boldsymbol{x * y = e \text{ } AND \text{ } y * x = e}$$

Commutative:
A binary operation $*$ on S is commutative if:

$$\boldsymbol{x * y = y * x \;\forall\; x, y \in S}$$

Associative:
A binary operation $*$ on S is associative if:

$$\boldsymbol{(x * y) * z = x * (y * z) \;\forall\; x, y, z \in S}$$

Distributive:
A binary operation $*$ is distributive over another $\cdot$ if for all a, b, c ∈S.

$$\boldsymbol{a * (b \cdot c) = (a * b) \cdot (a * c)}$$
$$\boldsymbol{AND}$$
$$\boldsymbol{(a \cdot b) * c = (a * c) \cdot (b * c)}$$

Well-Ordered:
A set S with order $\leq$ is called well-ordered if every nonempty $\emptyset$ subset T of S has at least one smallest element.

That is, if $\boldsymbol{T \subseteq S, T \neq \emptyset}$**, then** $\boldsymbol{\exists\; \mathrm{s}_0 \leq \mathrm{s} \;\forall\; \mathrm{s} \in \mathrm{T}}$

Rules for $\mathbb{Z}$:
On $\mathbb{Z}$, $+$ and $\cdot$ are commutative and associative. On $\mathbb{Z}$, $-$ and $/$ are not commutative and associative. However, if we define $a - b = a + (-b)$ and $\frac{a}{b} = a \cdot (\frac{1}{b})$, then we have commutativity and associativity.

$\boldsymbol{a - b \neq b - a, BUT\; a + (-b) = -b + a}$ **(assoc.)**
$\boldsymbol{\frac{a}{b} \neq \frac{b}{a}, BUT\; a \cdot \frac{1}{b} = \frac{1}{b} \cdot a}$ **(distrib.)**

Multiplication distributes over addition and subtraction on $\mathbb{Z}$.

$$\boldsymbol{a \cdot (b \pm c) = (a \cdot b) \pm (a \cdot c)}$$
$$\boldsymbol{(a \pm b) \cdot c = (a \cdot c) \pm (b \cdot c)}$$

Common Rules:
An integer $m \in \mathbb{Z}$ is **even** if $m = 2k$ for some $k \;\in\; \mathbb{Z}$.

An integer $m \in \mathbb{Z}$ is **odd** if $m = 2k + 1$ for some $k \in \mathbb{Z}$

An integer $m > 1$ is **prime** if whenever $m = rs$ for $r, s \in \mathbb{N}$, either $r = 1$ or $s = 1$

An integer $m > 1$ is **composite** if it is not prime (i.e. $m = ab$ with $a, b > 1 \text{ } AND \text{ } a, b < m, a, b \in \mathbb{N}$)

Dedekind Cuts Properties:
A Dedekind Cut of $\mathbb{Q}$ is a pair of subsets (A,B) of $\mathbb{Q}$ that satisfy the following:

- A and B are nonempty
- $A \cup B = Q$
- A is closed downwards: If $q \in A$ and $r < q$, then $r \in A$
- B is closed upwards: if $q \in B$ and $r > q$, then $r \in B$
- A contains no greatest element: $\forall\; q \in A \;\exists\; r \in A \ni q < r$

Given $q \in \mathbb{Q}$, we can form a Dedekind Cut (A, B) where:

$A = \{x \in \mathbb{Q} : x < q\}$ AND $B = \{x \in \mathbb{Q} : x \geq q$