School of Mathematics and Applied Statistics

**Student to complete:**

| | |
|---|---|
| Family name | |
| Other names | |
| Student number | |

# MATH221
# Mathematics for Computer Science
# Wollongong

## In-Session Test 1 (version A) - April 12th
## Autumn Session 2017

| | |
|---|---|
| Exam duration | 60 minutes |
| Aids supplied | None |

This exam is worth $20\%$ of your final grade for MATH221.

Answers should be written on the exam paper in the spaces provided.

If you require more paper, please raise your hand and request more. Write your name on all extra sheets of paper.

**This exam paper must not be removed from the exam venue**

**Questions 1–14 are from the Numbers Section.**

1. For $a, b \in \mathbb{R}$, define a binary operation on $\mathbb{R}$ by $a * b = a + ab + 1$. Then, for all $a, b, c \in \mathbb{R}$, $(a * b) * c$ equals

(a) $a + b + c + ab + ac + bc + 2$

(b) $2a + 2b + 2c + ab + ac + bc + 1$

(c) $a + ab + ac + abc + c + 2$

(d) $a + b + c + 2ab + 2ac + 2bc + abc + 2$

(e) $a + b + c + 2ab + 2ac + 2bc + 2$

2. Recall that we denote intervals of $\mathbb{R}$ using round and/or square brackets. A round bracket means the endpoint is not included, and a square bracket means the endpoint is included.

Which one of the following non-empty subsets of $\mathbb{R}$ is well-ordered.

(a) $(0, 1)$

(b) $[0, 1)$

(c) $(0, 1]$

(d) $[0, 1]$

(e) None of the above

3. Which of the following statements is **true**.

(a) Operations $+$, $\times$ and $-$ are closed operations on $\mathbb{N}$, and $\mathbb{N}$ is well-ordered

(b) Numbers $1$ and $-1$ are the only invertible elements in $\mathbb{Z}$ under $\times$

(c) Numbers $1$ and $-1$ are the only invertible elements in $\mathbb{Q}$ under $+$

(d) Every element of $\mathbb{R}$ is invertible under $\times$

(e) None of the above

4. The value of $\sum_{j=1}^{5}(j^2 - 4j - 1)$ is

(a) $-5$

(b) $12$

(c) $-14$

(d) $-18$

(e) $-10$

5. The sum $\sum_{j=1}^{4}(a_{j+1} - a_j)$ can be rewritten as

(a) $a_1 - a_5$

(b) $a_5 - a_1$

(c) $a_1 - a_2 + a_3 - a_4 + a_5$

(d) $-a_1 + a_2 - a_3 + a_4 - a_5$

(e) $a_1 + a_2 - a_3 - a_4$

6. Which one of the following most accurately describes Generalised Mathematical Induction?

(a) If $\mathrm{Claim}(n)$ is true for $n = a$, it is true for all $n \geq a$.

(b) If $\mathrm{Claim}(k)$ is true and $\mathrm{Claim}(k+1)$ is true, then $\mathrm{Claim}(n)$ is true for all $n \geq a$.

(c) If $\mathrm{Claim}(n)$ is true for $n = a$ and, for some $k \geq a$, $\mathrm{Claim}(k)$ implies $\mathrm{Claim}(k+1)$, then $\mathrm{Claim}(n)$ is true for all $n \geq a$.

(d) If $\mathrm{Claim}(n)$ is true for $n = a$ and, $\mathrm{Claim}(k)$ implies $\mathrm{Claim}(k+1)$ for all $k \geq a$, then $\mathrm{Claim}(n)$ is true for all $n \geq a$.

(e) If there exists an integer $a > 1$ such that $\mathrm{Claim}(n)$ is true for $n = 1, 2, \ldots, a$, then $\mathrm{Claim}(n)$ is true for all $n$.

7. If we use the Quotient-Remainder Theorem to write $-356$ as $-356 = 23q + r$, then

(a) $q = -16$ and $r = 12$

(b) $q = -15$ and $r = 11$

(c) $q = -15$ and $r = -11$

(d) $q = 16$ and $r = 12$

(e) $q = -12$ and $r = 16$

8. Which of the following lists consists of prime numbers **only**?

(a) $17, 31, 44, 71$

(b) $7, 21, 71, 123$

(c) $73, 97, 119, 67$

(d) $41, 43, 7, 31$

(e) $1, 3, 121, 7$

9. Given that $1863400 = 2^3 \times 5^2 \times 7 \times 11^3$ and $16500 = 2^2 \times 3 \times 5^3 \times 11$, then

(a) $\mathrm{gcd}(1863400, 16500) = 2^3 \times 3 \times 5^3 \times 7 \times 11^3$ and $\mathrm{lcm}(1863400, 16500) = 2^2 \times 5^2 \times 11$

(b) $\mathrm{gcd}(1863400, 16500) = 2^2 \times 3 \times 5^2 \times 7 \times 11$ and $\mathrm{lcm}(1863400, 16500) = 2^3 \times 3 \times 5^3 \times 7 \times 11^3$

(c) $\mathrm{gcd}(1863400, 16500) = 2^2 \times 5^2 \times 11$ and $\mathrm{lcm}(1863400, 16500) = 2^3 \times 3 \times 5^3 \times 7 \times 11^3$

(d) $\mathrm{gcd}(1863400, 16500) = 2^2 \times 5^2 \times 11$ and $\mathrm{lcm}(1863400, 16500) = 2^3 \times 5^3 \times 11^3$

(e) $\mathrm{gcd}(1863400, 16500) = 2 \times 5 \times 11$ and $\mathrm{lcm}(1863400, 16500) = 2 \times 3 \times 5 \times 7 \times 11$

10. The Euclidean algorithm is

(a) A method for deciding if two integers have a common divisor

(b) A statement about the existence of a greatest common divisor for two integers

(c) A way of determining if the square root of a number is rational

(d) A method for finding the greatest common divisor of two integers

(e) A way of calculating the divisors of a number

11. The remainder when $7^9$ is divided by 16. (i.e., $7^9 \pmod{16}$)

(a) $1$

(b) $7$

(c) $0$

(d) $9$

(e) None of the above.

12. Let $\mathbb{Z}_5 = \{[0], \ldots, [4]\}$ denote the equivalence classes modulo $5$. The value of $x$ which satifies the equation $[3][x] = [4]$ modulo $5$ is

(a) $0$

(b) $1$

(c) $2$

(d) $3$

(e) $4$.

13. An inverse for $5 \bmod 66$ is

(a) $1$

(b) $13$

(c) $53$

(d) $65$

(e) None of the above.

14. A certain type of basket can contain no more than 24 apples. Baskets of apples are brought into a room. What is the minimum number of baskets of apples you must have to ensure you have at least 5 baskets with the same number of apples in them. (Note that all baskets have at least one apple, so we are not considering empty baskets.)

(a) 96

(b) 25

(c) 121

(d) 120

(e) 97

**Questions 15–24 are from the Logic Section.**

15. In the table

| $P$ | $Q$ | $P$ **?** $(\sim Q)$ |
|---|---|---|
| T | T | F |
| T | F | T |
| F | T | F |
| F | F | F |

the question mark represents which of the following logical connectives

(a) $\Longleftrightarrow$

(b) $\wedge$

(c) $\Longrightarrow$

(d) $\vee$

(e) $\sim$

16. The statement $(\sim P \vee P) \Longrightarrow (\sim Q \wedge Q)$ is a

(a) tautology

(b) contradiction

(c) contingent statement

(d) all of the above

(e) none of the above

17. You discover a note signed by a pirate famous for his odd sense of humour and love of logic puzzles. In the note he says that he has hidden treasure somewhere on the property. He lists five true statements ((i)–(v) below) and challenges the reader to use them to locate the treasure.

(i) If this house is next to a lake, then the treasure is not in the kitchen.
(ii) If the tree in the front yard is a gum, then the treasure is in the kitchen.
(iii) This house is next to a lake.
(iv) The tree in the front yard is a gum or the treasure is buried under the flagpole.
(v) If the tree in the back yard is an oak, then the treasure is in the garage.

Where is the treasure hidden?

(a) in the garage

(b) in the lake

(c) in the kitchen

(d) on a pirate ship

(e) under the flagpole

18. Which of the following tables is the truth table for $(P \vee Q) \wedge (\sim (P \wedge Q))$?

(a)

| $P$ | $Q$ | $(P \vee Q) \wedge (\sim (P \wedge Q))$ |
|---|---|---|
| T | T | F |
| T | F | T |
| F | T | T |
| F | F | F |

(b)

| $P$ | $Q$ | $(P \vee Q) \wedge (\sim (P \wedge Q))$ |
|---|---|---|
| T | T | T |
| T | F | T |
| F | T | T |
| F | F | F |

(c)

| $P$ | $Q$ | $(P \vee Q) \wedge (\sim (P \wedge Q))$ |
|---|---|---|
| T | T | F |
| T | F | F |
| F | T | T |
| F | F | T |

(d)

| $P$ | $Q$ | $(P \vee Q) \wedge (\sim (P \wedge Q))$ |
|---|---|---|
| T | T | F |
| T | F | T |
| F | T | T |
| F | F | T |

(e)

| $P$ | $Q$ | $(P \vee Q) \wedge (\sim (P \wedge Q))$ |
|---|---|---|
| T | T | T |
| T | F | T |
| F | T | T |
| F | F | T |

19. The error in the truth table

| | $P$ | $Q$ | $(P \Longrightarrow Q) \Longrightarrow (P \vee Q)$ |
|---|---|---|---|
| **(1)** | T | T | T |
| **(2)** | T | F | T |
| **(3)** | F | T | F |
| **(4)** | F | F | F |

is in which row?

(a) **(1)**

(b) **(2)**

(c) **(3)**

(d) **(4)**

(e) there is no error!

20. Which of the following statements is **true**?

(a) $\forall x \in \mathbb{R}, \exists y \in \mathbb{R}, xy = 2$

(b) $\exists n \in \mathbb{N}, \forall y \in \mathbb{R}, n + y \in \mathbb{N}$

(c) $\exists x \in \mathbb{R}, \forall y \in \mathbb{Z}, xy \in \mathbb{Q}$

(d) $\forall x \in \mathbb{Q}, \forall y \in \mathbb{R}, xy \in \mathbb{N}$

(e) all of the above statements are false

21. The negation of the statement

$$\exists a > 0, \forall x \in \mathbb{R}, \frac{x+a}{a} > a$$

is which of the following?

(a) $\forall a > 0, \exists x \in \mathbb{R}, \frac{x+a}{a} > a$

(b) $\forall a < 0, \exists x \in \mathbb{R}, \frac{x+a}{a} < a$

(c) $\exists a < 0, \forall x \in \mathbb{R}, \frac{x+a}{a} > a$

(d) $\forall a > 0, \exists x \in \mathbb{R}, \frac{x+a}{a} \leq a$

(e) $\exists a > 0, \forall x \in \mathbb{R}, \frac{x+a}{a} \leq a$

22. Which law/rule can be used once to deduce the following logical equivalence:

$$P \implies (P \implies Q) \equiv P \implies (\sim P \vee Q)$$

(a) One of the distributive laws

(b) One of DeMorgans laws

(c) One of the associative laws

(d) Rule of substitution

(e) Rule of substitution of equivalence

23. Which of the following arguments uses the law of syllogism?

(a) If I need to drink, then I run for a tap. I am thirsty. Therefore I run for a tap.

(b) If I go for a walk, then I become tired. If I am tired, then I go to sleep. I just woke up from a sleep. I must have been walking.

(c) If I want to eat, then I buy food. If I buy food, then I become poor. I am hungry. Therefore I become poor.

(d) If I watch The Office, then I laugh. I laugh. Therefore I must have watched The Office.

(e) None of the above

24. The argument

$$(P \vee Q) \vee R.$$
$$\sim R.$$
Therefore $P \wedge Q$.

is which of the following?

(a) invalid

(b) a tautology

(c) valid

(d) a contradiction

(e) none of the above