MATH221 Mathematics for Computer Science

Tutorial Sheet Week 9

Autumn 2017

1. How many permutations of 3 items, taken 5 at a time, are there? How many permutations of 5 items, taken 3 at a time, are there?

2. A "binary word" is a word constructed from the alphabet $A = \{0, 1\}$. How many binary words of length 5 are there? How many of these words contain at least one 0 and at least one 1?

3. A *function* from a set A to a set B is a rule which assignes to each $a \in A$ an element denoted $f(a)$, belonging to B. How many different functions from $\{a, b, c, d\}$ to $\{1, 2, 3\}$ are there?

4. How many car numberplates are there of the form $L_1L_2L_3n_1n_2n_3$ where $L_1, L_2, L3$ are letters and n_1, n_2, n_3 are numbers? If we restrict the possibilities by insisting that there are no repeats how many are there now? Repeat this question for numberplates of the form $L_1L_2n_1n_2L_3L_4$.

5. Ten points are marked on the circumference of a circle.

(i) How many chords can be formed by joining them in all possible ways?

(ii) How many triangles can be formed by joining them in all possible ways?

(iii) How many hexagons can be formed by joining them in all possible ways?

6. Determine the size of the smallest set having at least 100 proper, nonempty, subsets.

7. Simplify $\sum_{r=0}^{1} 3(-1)^r \binom{13}{r} 5^{13-r}$.

8. A political candidate runs for two offices, A and B. The probability that the candidate wins A is 0.70, wins B is 0.60, and wins both is 0.35. What is the probability that the candidate wins at least one office?

9. In a manufacturing plant, it is known that 12% of light bulbs produced are faulty on average. If three bulbs are tested, find the probability that

(a) all three are faulty;

(b) none are faulty;

(c) exactly one is faulty.