MATH221: Mathematics for Computer Science

Tutorial Sheet Week 2

Autumn 2017

Logic - Lectures 1 & 2

Name: ____________________________________

If each person in this class has at least 3 eyes, then write the subjects you are currently enrolled in on the back of this paper. However, if each person in this class has at most 3 eyes, then circle the number of eyes you have: 0, 1, 2, 3, 4, 5, 6, or more. If no person in this class has exactly 3 eyes, then draw a small triangle at the top of this paper beside your name.

If you are neither a fish nor a fowl, then you must omit the next sentence. Fold this paper into two equal parts and draw a happy face on the largest of the two parts.

If a prime number must be greater than 1, draw a circle (approximately) around your surname at the top of this paper. However, if the statement that a prime number must be greater than 1 is false, draw a rectangle (approximately) around your surname at the top of this paper.

If a square is not round, you must omit the next sentence. In the margin of this paper draw a square approximately 1cm on a side. Now, whenever a square has two right angles underline the longest word in this sentence. However, if the underlined word begins with a vowel, then draw a rectangle around the second word having not less than 5 letters which follows the underlined word.

Now, if you are less than 40 years of age, then put a tick in the following box only if you are more than 2 metres tall.

☐

If it is not true that all triangles are isosceles, then underline all words of more than 2 letters beginning with the letter "i" in this sentence.

A student who walks 8 kilometres to attend MATH121 lectures, walks the first 4 km at a rate of 8 kph and the next 4 km at a rate of 6 kph. If the student's average rate of speed is 7 kph, then put your hand in the air until the tutor smiles at you. If the student's average speed is greater than 7 kph, then tap your pen/pencil on the desk top exactly 3 times. If neither of these answers is correct, write the correct answer on the blank line provided.

________________ kph.

If an integer which has exactly two different prime factors less than 20 can be greater than 320, you may stop working on this page and begin working on the next page. If not, then you should write out the multiplication table for the number 19 below, then begin working on the next page.

Note: Questions 5 and 6 on the next page contain important results. Make sure you complete them.

1. For each of the following collections of words:

(a) Determine if it is a statement.

(b) If it is a statement, determine if it is true or false.

(c) Where possible, translate the statement into symbols, using the connectives presented in lectures.

(i) If $x = 3$, then $x < 2$.

(ii) If $x = 0$ or $x = 1$, then $x^2 = x$.

(iii) $x^2 = x$ only if $x = 0$ or $x = 1$.

(iv) There exists a natural number x for which $x^2 = \frac{x}{2}$.

(v) If $x \in \mathbb{N}$ and $x > 0$, then $\sqrt{x} > 1 \Longrightarrow x > 1$.

(vi) $xy = 5 \Longrightarrow x = 1$ and $y = 5$ or $x = 5$ and $y = 1$.

(vii) $xy = 0 \Longrightarrow x = 0$ or $y = 0$.

(viii) $xy = yx$.

(ix) There is a unique even prime number.

2. Translate into symbols the following compound statements. In each case list the statements P, Q, R ... and give the form of the compound statement.

(i) If x is odd and y is odd then $x + y$ is even.

(ii) It is not both raining and hot.

(iii) It is raining but it is hot.

(iv) It is neither raining nor hot.

(v) $-1 \leq x \leq 2$.

3. Let P be the statement "Mathematics is easy." and Q be the statement "I do not need to study.". Write down in words the following statements, and simplify if possible.

$$P \vee Q, \quad \sim Q, \quad \sim\sim Q, \quad \sim P, \quad \sim P \wedge Q, \quad P \Longrightarrow Q.$$

4. Let P, Q, and R be statements.

(i) Write down the truth tables for $(\sim P \vee Q) \wedge Q$, and $(\sim P \wedge Q) \vee Q$. What do you notice about the truth tables?

Based on this result, a creative MATH121 student concludes that you can always interchange $\vee$ and $\wedge$ without changing the truth table.

(ii) Write down the truth tables for $(\sim P \vee Q) \wedge P$, and $(\sim P \wedge Q) \vee P$. What do you think of the rule formulated by the student in 4(i)?

5. (i) Construct truth tables for the compound statements $P \vee \sim P$ and $P \wedge \sim P$.

(ii) What do you notice about each of the statements in part (i)?

(iii) Determine the truth value of the compound statements $(P \vee \sim P) \vee Q$ and $(P \wedge \sim P) \wedge Q$. What do you notice?

Conclusion: If you have a compound statement R of the form "T $\vee P$", where T stands for a tautology (and P is any compound statement), then R is also a tautology. Similarly, if S is of the form "F $\wedge P$", where F stands for a contradiction, then S is also a contradiction.

6. (i) Construct truth tables for the compound statements $(P \vee \sim P) \wedge (Q \vee R)$ and $Q \vee R$. What do you notice?

(ii) Construct truth tables for the compound statements $(P \wedge \sim P) \vee (Q \wedge R)$ and $Q \wedge R$. What do you notice?

Conclusion: If you have a compound statement R of the form "T $\wedge P$", where T stands for a tautology (and P is any compound statement), then the truth value of R depends entirely on the truth value of P. Similarly, for S of the form "F $\vee P$", where F stands for a contradiction, S depends entirely on the truth value of P.