MATH221 – Mathematics for Computer Science – Autumn 2018

Assignment One Answers

Question 1:

Given $\forall n, r \in \mathbb{N}, 0 \leq r < 3 \ \exists q \in \mathbb{N} \ni n = 3q + r$.

If $n = 5721970$, $r = 1 \in \mathbb{N}$, then

$$5721970 = 3q + 1$$
$$3q = 5721969$$
$$q = \frac{5721969}{3}$$
$$q = 1,907,323 \in \mathbb{N} \quad \square$$

Question 2. (1)

The truth table for $\sim Q \Rightarrow P \vee (P \wedge \sim Q)$

Row	P	Q	$\sim Q$	$P \wedge \sim Q$	$P \vee (P \wedge \sim Q)$	$\sim Q \Rightarrow P \vee (P \wedge \sim Q)$
1	T	T	F	F	T	F
2	T	F	T	T	T	T
3	F	T	F	F	F	T
4	F	F	T	F	F	T

The truth table for the statement $\sim Q \Rightarrow P \vee (P \wedge \sim Q)$ shows that this is a compound statement.

Question 3.

The truth table below will test if the syllogy:

$P \Rightarrow Q$,

$\sim P$

$\therefore \sim Q$

is valid or not valid.

	1	2	3	4	5
1	P	Q	$P \Rightarrow Q$	$\sim P$	$\sim Q$
2	T	T	T	F	F
3	T	F	F	F	T
4	F	T	T	T	F
5	F	F	T	T	T

This truth table shows that in row 4, the premise $P \Rightarrow Q$ and $\sim P$ are both True but the conclusion is False. Therefore, this syllogy is NOT VALID.

Question 4.

CLAIM(n): $1^2 + 3^2 + 5^2 + \dots + (2n-1)^2 = \frac{4n^3 - n}{3} \quad \forall n \in \mathbb{N}$

CLAIM(1): $\frac{4(1)^3 - (1)}{3} = 1$ is true.

CLAIM(k): Suppose $1^2 + 3^2 + 5^2 + \dots + (2k-1)^2 = \frac{4k^3 - k}{3}$

CLAIM(k+1): Prove $1^2 + 3^2 + 5^2 + \dots + (2k-1)^2 + (2k+1) = \frac{4(k+1)^3 - (k+1)}{3}$

$$\frac{4(k+1)^3 - (k+1)}{3} = \frac{4(k^3 + 3k^2 + 3k + 1) - (k+1)}{3}$$

$$= \frac{(4k^3 + 12k^2 + 12k + 4) - (k+1)}{3}$$

$$= \frac{4k^3 + 12k^2 + 11k + 3}{3}$$

By the supposition, $\frac{4k^3 - k}{3} + (2k+1)^2 = \frac{4k^3 - k}{3} + 4k^2 + 4k + 1$

$$= \frac{4k^3 + 12k^2 + 11k + 3}{3} \quad \square$$

$\therefore$ The claim $1^2 + 3^2 + 5^2 + \dots + (2n-1)^2 = \frac{4n^3 - n}{3}$ is true for all $n \in \mathbb{N}$.