MATH221 Mathematics for Computer Science

Outline Solutions to Tutorial Sheet Week 7

Autumn 2017

1.

(i) True	(ii) False	(iii) False	(iv) False
(v) True	(vi) False	(vii) False	(viii) True

2. We note that $D = \{0, 1, 2\}$ and $E = \{1, 2\}$. Hence $A = D$ but $A \neq E$ and $D \neq E$.

In fact, no other pairs of the sets are equal. We can see this by noting that A, D and E are finite while B and C are infinite, and that $-1 \in B$ while $-1 \notin C$.

3. $A \cup B = (0, 1] = \{x \in \mathbb{R} : 0 < x \leq 1\}$ $\quad$ $A \cap B = \emptyset$

$B \cap C = B$ $\quad$ $A \cup C = C$

$A \cap C = A$ $\quad$ $\overline{A} = \{x \in \mathbb{R} : x \neq 1\}$

$\overline{C} = (-\infty, 0) \cup (1, \infty) = \{x \in \mathbb{R} : x < 0 \vee x > 1\}$

$C - A = [0, 1) = \{x \in \mathbb{R} : 0 \leq x < 1\}$

$C - B = \{0, 1\}$ $\quad$ $A - C = \emptyset$

4. Prove that $A = \{0, 1\} = \left\{n \in \mathbb{Z} : \exists k \in \mathbb{Z}, \left(n = \frac{1-(-1)^k}{2}\right)\right\}$.

Step 1: Prove $A \subseteq B$. Let $x \in A$. Then $x = 0$ or $x = 1$. For this, we use a proof by cases.
Case 1: $x = 0 \Longrightarrow x = \frac{1-1}{2} = \frac{1-(-1)^2}{2}$. Therefore, $\exists k \in \mathbb{Z}$, $\left(x = \frac{1-(-1)^k}{2}\right)$, so $x \in B$.
Case 2: $x = 1 \Longrightarrow x = \dfrac{1-(-1)}{2} = \dfrac{1-(-1)^1}{2}$. Therefore, $\exists k \in \mathbb{Z}$, $\left(x = \frac{1-(-1)^k}{2}\right)$, so $x \in B$.
Therefore, $A \subseteq B$.

Step 2: Prove $B \subseteq A$. Let $y \in B$. Then $\exists k \in \mathbb{Z}$, $\left(y = \frac{1-(-1)^k}{2}\right)$. Now k can be an odd integer or an even integer, so again we consider cases.
Case 1: Let k be an odd integer. Then $y = \frac{1-(-1)^k}{2} = \frac{1-(-1)}{2} = \frac{2}{2} = 1$.
Case 2: Let k be an even integer. Then $y = \frac{1-(-1)^k}{2} = \frac{1-1}{2} = \frac{0}{2} = 0$.
Therefore, $y = 0$ or $y = 1$, so $y \in A$. Therefore, $B \subseteq A$.

Therefore, by Steps 1 and 2, $A = B$.

5. $A \cup B = \mathbb{N}$ $\quad$ $A \cap B = \emptyset$

$B \cap P = \{2\}$ $\quad$ $A \cup P = A \cup \{2\} = \{1, 2, 3, 5, 7, 9, 11, 13, \dots\}$

$A \cap P = P - \{2\} = \{3, 5, 7, 11, 13, \dots\}$ $\quad$ $\overline{A} = B$;

$\overline{P} = B - \{2\} \cup \{x : x \text{ is odd } \wedge x \text{ is not prime}\} = \{x \in \mathbb{N} : x \text{ is composite} \vee x = 1\}$

$P - A = \{2\}$

$B - P = B - \{2\}$ $\quad$ $A - B = A$

A and B are disjoint as $A \cap B = \emptyset$; P is not a subset of A, since $2 \in P$ but $2 \notin A$.

6. (i) $(C \cap U) \cup \overline{C} = (C \cup \overline{C}) \cap (U \cup \overline{C}) = U \cap U = U$

(ii) $\overline{(A \cap U)} \cup \overline{A} = (\overline{A} \cup \emptyset) \cup \overline{A} = \overline{A} \cup \overline{A} = \overline{A}$

(iii) $\overline{\overline{(C \cup \emptyset)} \cup C} = (C \cup \emptyset) \cap \overline{C} = C \cap \overline{C} = \emptyset$

(iv) $(A \cap B) \cap \overline{A} = A \cap B \cap \overline{A} = (A \cap \overline{A}) \cap B = \emptyset \cap B = \emptyset$

7. (i) The statement is true.

We have that for all x,

$$
\begin{aligned}
x \in \overline{A} - \overline{B} &\iff x \in \overline{A} \land x \notin \overline{B} && \textit{Definition of set difference} \\
&\iff x \in \overline{A} \land x \in B && \textit{Definition of complement} \\
&\iff x \in B \land x \in \overline{A} && \textit{Commutativity} \\
&\iff x \in B \land x \notin A && \textit{Definition of set difference} \\
&\iff x \in B - A && \textit{Definition of set difference}
\end{aligned}
$$

Hence $\overline{A} - \overline{B} = B - A$.

(ii) The statement is false.

Let $U = \mathbb{N}$, let $A = \{1\}$, let $B = \emptyset$ and let $C = \{1\}$. Then $A, B, C \in \mathcal{P}(U)$ and

$$
\begin{aligned}
A - (B - C) &= \{1\} - (\emptyset - \{1\}) = \{1\} - \emptyset = \{1\}\,, \\
(A - B) - C &= (\{1\} - \emptyset) - \{1\} = \{1\} - \{1\} = \emptyset\,.
\end{aligned}
$$

As $1 \in A - (B - C)$ and $1 \notin (A - B) - C$ we have that $A - (B - C) \neq (A - B) - C$.