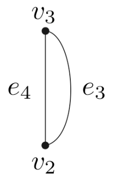
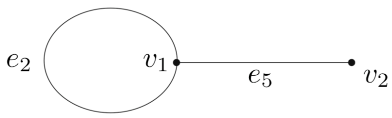
MATH221 Mathematics for Computer Science

Outline Solutions to Tutorial Sheet Week 12

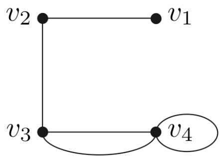
Autumn 2017

1. Here are two answers out of several:

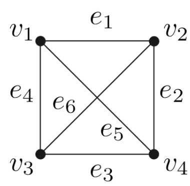




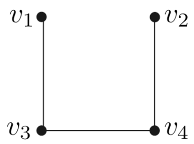
2. (i)



(ii)

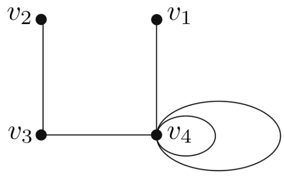


(iii)

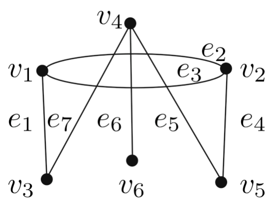


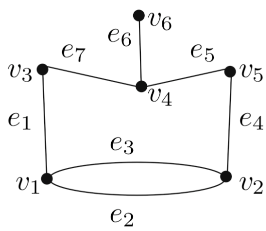
(iv) Such a graph cannot exist. A simple graph has neither parallel edges nor loops. There are only 5 vertices, so each vertex can only be joined to at most four other vertices. Hence, we cannot have a vertex of degree 5.

(v) Such a graph cannot exist. The sum of the given degrees is 8, so by the Handshake Lemma, G must have 4 edges. (vi)



3. We have used the same names for the vertices and edges in the two graphs to make the correspondence clear. An alternative (Nathan's preference, in fact) would be to use $v'_1, v'_2, \ldots$ for the vertices and $e'_1, e'_2, \ldots$ for the edges in the second graph, and have functions sending each v_i to v'_i and each e_i to e'_i.





4. (i) K_n has $\frac{1}{2}n(n-1)$ edges, and the subgraphs we want are formed by keeping all the vertices and picking an arbitrary subset of the edges. Therefore, the number of subgraphs is the number of elements in the power set of the set of edges, that is, $2^{\frac{1}{2}n(n-1)}$.

(ii) There are 4 non-isomorphic subgraphs of K_3 with all 3 vertices (compared to $2^3 = 8$ subgraphs with all 3 vertices in total).