School of Mathematics and Applied Statistics

**Student to complete:**

| | |
|---|---|
| Family name | |
| Other names | |
| Student number | |

# MATH221
# Mathematics for Computer Science
# Wollongong

## In-Session Test 2-A: May 23rd
## Autumn Session 2018

| | |
|---|---|
| Exam duration | 60 minutes |
| Aids supplied | None |

This exam is worth $20\%$ of your final grade for MATH221.

Answers should be written on the exam paper in the spaces provided. SHOW ALL YOUR WORK.

**This exam paper must not be removed from the exam venue**

# Questions

1. Which of the following is possible?

   (a) A graph with 5 vertices, 5 edges and total degree 10.

   (b) A graph with 5 vertices of degrees 1, 1, 2, 2 and 3.

   (c) A simple graph with 6 vertices of degrees 1, 2, 2, 4, 6 and 6.

   (d) A simple graph with 5 vertices of degrees 2, 2, 2, 5 and 5.

   (e) None of the above.

2. Which of the following about a tree graph $G$ is false?

   (a) $G$ has no loops.

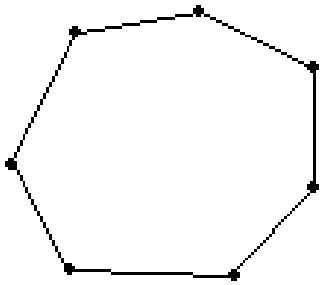
   (b) $G$ has no closed circuits.

   (c) The number of edges of $G$ exceeds the number of vertices of $G$ by one.

   (d) $G$ has no parallel edges.

   (e) The number of vertices of $G$ exceeds the number of edges of $G$ by one.

3. Draw a graph with 7 vertices that has an Eulerian circuit.



4. On $\mathbb{Z}$, let $A$ be the set of even integers and $B$ be the set of prime numbers less than 5. Write the set $\{3\}$ in terms of $A$ and $B$.

   $\{3\} = \overline{A} \cap B$.

5. Let $A = \{a, b, c, \{\emptyset\}\}$. Which is true?

   (a) $\{\emptyset\} \subseteq A$ and $\{a, b, c\} \in \mathcal{P}(A)$.

   (b) $a \in \mathcal{P}(A)$ and $\emptyset \in \mathcal{P}(A)$.

   (c) $\{\emptyset\} \subseteq \mathcal{P}(A)$ and $\{\{b\}\} \in \mathcal{P}(A)$.

   (d) $\emptyset \in A$ and $\{\{\emptyset\}\} \in \mathcal{P}(A)$.

   (e) $\{a, b, c\} \subseteq A$ and $\{\{\emptyset\}\} \in \mathcal{P}(A)$.

6. Which is true of $A = \{n^2 - 2n : n \in \mathbb{Z}\}$ and $B = \{m^2 + 2m : m \in \mathbb{Z}\} \cup \{1\}$?

   (a) $A \cap B = \emptyset$.

   (b) $A \cup B = \mathbb{Z}$.

   (c) $A \subseteq B$ and $A \neq B$.

   (d) $B \subseteq A$ and $B \neq A$.

(e) $A = B$.

7. Using the Binomial Theorm, we find that $\sum_{k=1}^{15} (-1)^k \binom{15}{k} 7^{15-k} =$

   (a) $6^{15}$
   (b) $7^{15}$
   (c) $7^{15} - 6^{15}$
   (d) $6^{15} - 7^{15}$
   (e) None of the above.

8. Consider the alphabet $A = \{a, b, c, d\}$. How many $A$-sequences of length $n \geq 2$ contain exactly two letters $a$?

   (a) $n(n-1)3^{n-2}$
   (b) $\binom{n}{2}4^{n-2}$
   (c) $3 \cdot 2^{n-2}$
   (d) $\binom{n}{2}3^{n-2}$
   (e) None of the above.

9. The value of $\binom{10}{0} + \binom{10}{1} + \cdots + \binom{10}{10}$ is

   (a) $10^2$
   (b) $2^{10}$
   (c) $10 \cdot 21$
   (d) $2^{11}$
   (e) None of the above.

10. Which is an onto function from $A = \{1, 2, 3\}$ to $B = \{a, b, c\}$?

    (a) $\{(a, 1), (b, 3), (c, 2)\}$
    (b) $\{(1, a), (2, b), (2, c)\}$
    (c) $\{(1, a), (3, b), (2, c)\}$
    (d) $\{(1, a), (2, b), (3, b)\}$
    (e) None of the above.

11. On $\mathbb{R}$, consider the equivalence relation $R = \{(x, y) : x - y \in \mathbb{Z}\}$. Which is false?

    (a) $[\pi] \cap [\sqrt{2}] = \emptyset$
    (b) $[\pi] \cup [2] = \mathbb{R}$
    (c) $[n] \cap \mathbb{Z} \neq \emptyset \ \forall n \in \mathbb{Z}$.
    (d) $[q] \subseteq \mathbb{Q} \ \forall q \in \mathbb{Q}$.