MATH221 Mathematics for Computer Science

Tutorial Sheet Week 6

Autumn 2017

1. A penny collection contains twelve 1967 pennies, seven 1968 pennies, and eleven 1971 pennies. If you are to pick some pennies without looking at the dates, how many must you pick to be sure of getting at least five pennies from the same year?

2.

(i) How many integers must you pick in order to be sure that at least two of them have the same remainder when divided by 15?

(ii) How many integers between 100 and 999 must you pick in order to be sure that at least two of them have a digit in common?

3. (i) Find values for $m, n \in \mathbb{Z}$ such that $\gcd(98, 85) = 98m + 85n$.

(ii) Find values for $m, n \in \mathbb{Z}$ such that $\gcd(224, 1715) = 224m + 1715n$.

4. Let $a, b \in \mathbb{N}$. Show that a and b are relatively prime (that is, $\gcd(a, b) = 1$) if and only if no prime p divides both a and b (that is, a and b have no common prime factor).

5. Write down three pairs of integers, all with at least two digits, that are relatively prime. Make sure that some of the numbers you pick are not primes. Explain how you know that they are relatively prime. (Question 3 may be useful; don't use the Euclidean algorithm.)

6. Find numbers in the range $0, 1, \ldots, 11$ to which the following numbers are congruent modulo 12.

$$-17, \quad 3, \quad 43, \quad 15, \quad 37, \quad -11, \quad -21$$

7. Find all solutions $m \in \mathbb{Z}$ of the equations $6 \equiv m \pmod 3$ and $125 \equiv m \pmod 7$.

8. Find $2^{100} \bmod 5$ and $3^{100} \bmod 7$ using the methods discussed in lectures.

9. Let $n \in \mathbb{N}$. Prove that for all $a, b \in \mathbb{Z}$, $[a] = [b] \iff a \equiv b \pmod n$.

10. Write out the addition and multiplication tables for $\mathbb{Z}_5$.

11. In this question, we work modulo $n = 7$.

(i) For $m \in \mathbb{Z}$, is $[m]$ a number, a set of numbers or a set of sets?
Is $\mathbb{Z}_7$ a number, a set of numbers or a set of sets?

(ii) Describe $[2]$, $[5]$ and $[7]$ in $\mathbb{Z}_7$ by listing eight elements in each set.
Write down the set $\mathbb{Z}_7$.

(iii) Are the following statements true or false (or perhaps meaningless)?

(a) $[2] = [9]$ (b) $[5] = 5 \pmod 7$ (c) $5 \in [19]$

(d) $3 \in \mathbb{Z}_7$ (e) $[10] \in \mathbb{Z}_7$ (f) $[a] \in [b]$ whenever $a \equiv b \pmod 7$