School of Mathematics and Applied Statistics

Student to complete:

Family name	
Other names	
Student number	
Table number	

MATH221
Mathematics for Computer Science
Wollongong

Examination Paper
Autumn Session 2017

Exam duration	3 hours
Items permitted by examiner	UOW approved calculator
Aids supplied	None
Directions to students	38 questions to be answered. Each question has equal value. The exam paper is printed on both sides. All notation is as given in lectures. Answer each question on the exam paper.

This exam paper must not be removed from the exam venue

1. Let $A = \{-5, -3, -1, 1, 3, 5, \dots\}$ and $B = \{x : x \in \mathbb{R}$ and $-2 \leq x < 9\}$. Which one of the following is **true**?

(a) A does not have a greatest element and B has a least element.

(b) A does not have a greatest element and B has a greatest element.

(c) A does not have a least element and B does not have a least element.

(d) A has a greatest element and B has a greatest element.

(e) Neither A nor B has a smallest element.

2. Which of the following is **true**.

(a) Numbers 1 and -1 are the only invertible elements in $\mathbb{R}$ under $+$

(b) Every element of $\mathbb{Q}$ is invertible under $+$

(c) Operations $+$, $\times$ and $-$ are closed operations on $\mathbb{N}$, and $\mathbb{N}$ is well-ordered

(d) Numbers 1 and 0 are the only invertible elements in $\mathbb{Z}$ under $\times$

(e) None of the above

3. The sum of all multiples of 3 from 3 to 99 can be written

(a) None of the below

(b) $\sum_{j=0}^{33}(3j)$

(c) $\sum_{j=1}^{33} 3^j$

(d) $\sum_{j=0}^{33} 3^j$

(e) $\sum_{j=1}^{33}(3j)$

4. The sum $\sum_{j=1}^{4}(a_{j+2} - 2a_{j+1} + a_j)$ can be rewritten as

(a) $a_1 - a_5$

(b) $a_1 + a_5$

(c) $a_5 - a_1$

(d) $a_1 - a_2 - a_5 + a_6$

(e) None of the above

5. Which of the following **cannot** be proved by any version of mathematical induction

(a) $a_n \leq 3^n$ for all $n \in \mathbb{N}$ when $a_n = a_{n-1} + a_{n-2} + a_{n-1}$ for $n \geq 4$ and $a_1 = 1$, $a_2 = 2$, $a_3 = 28$

(b) $\sum_{j=1}^{n}(4j - 2) = 2n^2$ for all $n \in \mathbb{N}$

(c) $3^n - 1$ is even for all $n \in \mathbb{N}$

(d) $\sum_{j=1}^{n} 2j = n(n+1)$ for all $n \in \mathbb{N}$

(e) $5 \mid a_n$ for all $n \in \mathbb{N}$ when $a_n = a_{n-2} - 2a_{n-1}$ for $n \geq 3$ where $a_1 = 5$, $a_2 = -10$.

6. Let $m, n \in \mathbb{N}$. If we use the Quotient-Remainder Theorem to write m as $m = n \times 9 + 7$, then

(a) gcd$(m, n) = 9$

(b) gcd$(m, n) = 7$

(c) gcd$(m, n) = 1$

(d) it is not possible to compute gcd(m, n) from this data

(e) none of the above are correct

7. Which of the following lists consists of **coprime** numbers only?

(a) $17, 31, 34, 71$

(b) $7, 21, 84, 123$

(c) $3, 97, 119, 69$

(d) $41, 43, 7, 31$

(e) $11, 3, 121, 7$

8. The value of $5^{65} \bmod 24$ is

(a) 0

(b) 5

(c) 65

(d) 1

(e) None of the above

9. The numbers a, b are **congruent** modulo n, that is $a \equiv b \bmod n$ if and only if

(a) ab is a multiple of n

(b) None of the other options

(c) $a + b$ is a multiple of n

(d) $a - b$ is a multiple of n

(e) $a - b$ is a divisor of n

10. After writing down the multiplication table for $\mathbb{Z}_4 = \{[0], [1], [2], [3]\}$ we find that the set of **all** solutions to equation $[2][x] = [1]$ is

(a) $\{1\}$

(b) $\{2\}$

(c) $\{3\}$

(d) $\{0\}$

(e) None of the above

11. Suppose $n \in \mathbb{N}$ is a number with $n \geq 2$ and p a prime number such that p does not divide n. Suppose that $xp + yn = 1$ for some integers x, y. Then an inverse of p modulo n is

(a) 1

(b) n

(c) y

(d) x

(e) None of the above

12. Suppose that there are 365 days in every year. A crowd of 2200 students are divided into groups according to their birthday (e.g. June 21). Then there must be a group whose size is bigger than

(a) 7

(b) 8

(c) 6

(d) 2200

(e) None of the above

13. In the table

P	Q	$(\sim P)$ **?** Q
T	T	T
T	F	T
F	T	T
F	F	F

the question mark represents which of the following?

(a) $\iff$

(b) $\wedge$

(c) $\implies$

(d) $\vee$

(e) None of the above

14. The statement $(\sim P \wedge P) \implies (\sim Q \vee Q)$ is a

(a) tautology

(b) contradiction

(c) contingent statement

(d) all of the above

(e) none of the above

15. The error in the truth table

	P	Q	$(P \implies Q) \implies (P \wedge Q)$
(1)	T	T	T
(2)	T	F	T
(3)	F	T	T
(4)	F	F	F

is in which row?

(a) **(1)**

(b) **(2)**

(c) **(3)**

(d) **(4)**

(e) there is no error!

16. Which of the following tables is the truth table for $(P \wedge Q) \vee (\sim (P \vee Q))$?

(a)

P	Q	$(P \wedge Q) \vee (\sim (P \vee Q))$
T	T	F
T	F	T
F	T	T
F	F	F

(b)

P	Q	$(P \wedge Q) \vee (\sim (P \vee Q))$
T	T	T
T	F	T
F	T	T
F	F	F

(c)

P	Q	$(P \wedge Q) \vee (\sim (P \vee Q))$
T	T	F
T	F	F
F	T	T
F	F	T

(d)

P	Q	$(P \wedge Q) \vee (\sim (P \vee Q))$
T	T	T
T	F	F
F	T	F
F	F	T

(e)

P	Q	$(P \wedge Q) \vee (\sim (P \vee Q))$
T	T	T
T	F	T
F	T	T
F	F	T

17. Which of the following statements is **true**?

(a) $\exists x \in \mathbb{R}, \forall y \in \mathbb{Z}, xy \in \mathbb{N}$

(b) $\exists n \in \mathbb{N}, \forall y \in \mathbb{R}, n - y \in \mathbb{N}$

(c) $\forall x \in \mathbb{R}, \exists y \in \mathbb{R}, xy = 1$

(d) $\forall x \in \mathbb{Q}, \exists y \in \mathbb{Z}, xy \in \mathbb{N}_0 = \mathbb{N} \cup \{0\}$

(e) all of the above are false

18. The negation of the statement

$$\forall a > 0, \, \exists x \in \mathbb{R}, \, \frac{x+a}{a} > a$$

is which of the following?

(a) $\exists a > 0, \, \forall x \in \mathbb{R}, \, \frac{x+a}{a} > a$

(b) $\forall a < 0, \, \exists x \in \mathbb{R}, \, \frac{x+a}{a} < a$

(c) $\exists a < 0, \, \forall x \in \mathbb{R}, \, \frac{x+a}{a} > a$

(d) $\forall a > 0, \, \exists x \in \mathbb{R}, \, \frac{x+a}{a} \leq a$

(e) $\exists a > 0, \, \forall x \in \mathbb{R}, \, \frac{x+a}{a} \leq a$

19. Let P, Q, R be simple statements, then the argument

$(P \wedge Q) \vee R.$
$\sim R.$
Therefore $P \vee Q$.

is which of the following?

(a) a tautology

(b) invalid

(c) valid

(d) a contradiction

(e) none of the above

20. Let A be the set of even natural numbers, B the set of odd natural numbers, and P the set of prime numbers. Then the set $\{2\}$ can be written as

(a) $(P - A) - B$

(b) $(A \cap B) \cup P$

(c) $((A \cap P) \cup P) - B$

(d) $(P - B) \cup A$

(e) none of the above

21. Let U be the universe $\mathbb{N}$, and consider the sets $A = \{1, 2, 3, 4, 5, 6\}$ and $B = \{2, 4, 6, 8, 10\}$. Then $(\overline{A} \cap B) \cup (A - B)$ is the set

(a) $\{3, 5, 8\}$

(b) $\{1, 3, 5, 8, 10\}$

(c) $\{8, 10\}$

(d) $\{2, 4, 6\}$

(e) none of the above

22. Let $A = \{a, b, c, \{\emptyset\}\}$. Which of the following is **true**?

(a) $\{\emptyset\} \in A$ and $\{a, b, c\} \subseteq \mathcal{P}(A)$

(b) $\{a, b, c\} \in \mathcal{P}(A)$ and $\{\emptyset\} \in \mathcal{P}(A)$

(c) $a \in \mathcal{P}(A)$ and $\emptyset \subseteq \mathcal{P}(A)$

(d) $\{\emptyset\} \in \mathcal{P}(A)$ and $\{\{b\}\} \subseteq \mathcal{P}(A)$

(e) $\emptyset \subseteq A$ and $\{\{\emptyset\}\} \in \mathcal{P}(A)$

23. Let $A = \{n^2 - 2n : n \in \mathbb{Z}\}$ and $B = \{m^2 + 2m : m \in \mathbb{Z}\}$. Which of the following is **true**?

(a) $A \cup B = \mathbb{Z}$

(b) $A \subseteq B$ and $A \neq B$

(c) $B \subseteq A$ and $B \neq A$

(d) $A = B$

(e) $A \cap B = \emptyset$

24. Let A, B and C be subsets of a universe U. Then

$$\left(\overline{(A-B)\cup(A\cap B)}\cup A\right)-C$$

can be rewritten as

(a) $A\cup C$

(b) $\emptyset$

(c) $\overline{C}$

(d) A

(e) U

25. A car license plate consists of 7 characters. The first 3 characters can be any letters from A to F, but no letter can be repeated. The next 3 characters can be any digit from 1 to 9, but no digit can be repeated. The last character can be any of the letters X, Y, or Z. An example of this format is $BFA438X$. How many license plates are possible with these rules?

(a) 5,040

(b) 181,440

(c) 362,880

(d) 472,392

(e) 4,084,080

26. Consider an alphabet $A=\{a,b,c,d\}$. How many A-sequences of *length* n contain exactly one a?

(a) 4^n-3^n

(b) $4\times 3^{n-1}$

(c) 4^n-1

(d) $n\times 3^{n-1}$

(e) None of the above.

27. Let $A = \{a, b, c\}$ and $B = \{1, 2, 3\}$. Which of the following sets is a relation from $A \times \mathbb{R}$ to B?

(a) $\{((a, x), 1) : x \in \mathbb{Z}\}$

(b) $\{(a, -1, 1), (b, -2, 2), (c, -3, 3)\}$

(c) $\{((1, \sqrt{2}), b)\}$

(d) $\{(y, \sqrt{3}, 1) : y \in A\}$

(e) $\{((a, 7), 2), ((2, 7), a)\}$

28. Let $A = \{w, x, y, z\}$ and $R = \{(w, w), (w, x), (x, x), (x, w), (x, y), (y, y), (y, x), (z, z)\} \subset A \times A$. Which of the following is **true**?

(a) R is an equivalence relation

(b) R is reflexive and transitive

(c) R is reflexive but not symmetric

(d) R is reflexive and symmetric

(e) R is symmetric but not reflexive

29. Consider the relation R on $\mathbb{R}$ given by

$$R = \{(x, y) : x - y \in \mathbb{Z}\},$$

which we know is an equivalence relation. Which of the following is **false**?

(a) $[\pi] \cap \mathbb{Z} = \emptyset$

(b) $[\pi] \cup [2] = \mathbb{R}$

(c) $[n] = \mathbb{Z}$ for all integers n

(d) $[q] \subseteq \mathbb{Q}$ for all rationals q

(e) One of the above is false

30. Which of the following functions has an inverse function?

(a) $f : \mathbb{R} \to \mathbb{R}$ given by $f(x) = x^2$

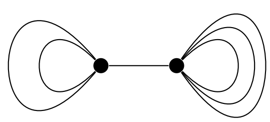
(b) $f : \mathbb{R} \to \{x \in \mathbb{R} : x \geq 0\}$ given by $f(x) = x^2$

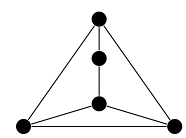
(c) $f : \{x \in \mathbb{R} : x \geq 0\} \to \{x \in \mathbb{R} : x \geq 0\}$ given by $f(x) = 1 - x$

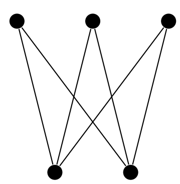
(d) $f : \mathbb{R} \to \mathbb{R}$ given by $f(x) = x^4$

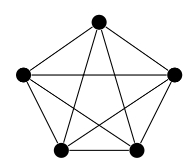
(e) $f : \{x \in \mathbb{R} : x \geq 0\} \to \{x \in \mathbb{R} : x \geq 0\}$ given by $f(x) = \sqrt{x}$

31. The graph $K_{2,3}$ is

(a) 

(b) 

(c) 

(d) 

(e) None of the above

32. Which of the following descriptions of a graph is possible?

(a) A graph with 5 vertices with degrees 1, 1, 2, 2 and 3

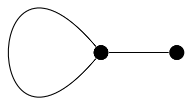
(b) A simple graph with 6 vertices with degrees 1, 2, 2, 4, 6 and 6

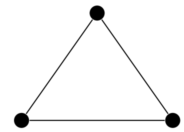
(c) A simple graph with 5 vertices with degrees 2, 2, 2, 5 and 5

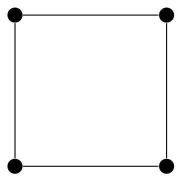
(d) A graph with 5 vertices, 5 edges and total degree 10

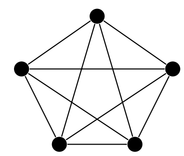
(e) None of the above

33. Which of the following graphs is bipartite?

(a) 

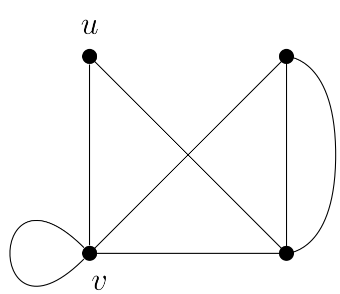
(b) 

(c) 

(d) 

(e) None of the above

34. Consider the graph



How many paths are there from u to v?

(a) 2

(b) 4

(c) 3

(d) 1

(e) None of the above

35. Which one of the following facts about a graph G which is a tree is **false**

(a) G has no closed circuits

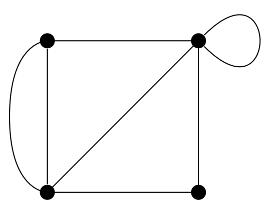
(b) G has no parallel edges

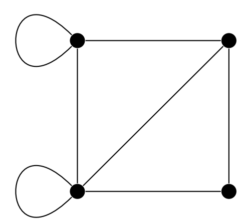
(c) The number of vertices of G exceeds the number of edges of G by one

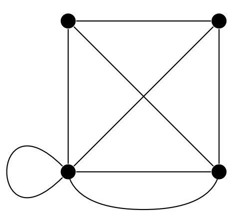
(d) The number of edges of G exceeds the number of vertices of G by one

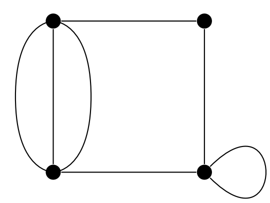
(e) G has no loops.

36. Which of the following graphs has an Eulerian circuit?

(a) 

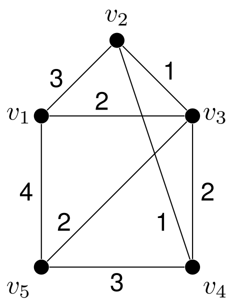
(b) 

(c) 

(d) 

(e) None of the above

Questions 37 and 38 refer to the following weighted graph and table.



Edge	Weight	Will adding edge make a circuit?	Action taken	Cumulative Weight of subgraph
(v_2, v_3)	1	N	A	1
(v_2, v_4)	1	N	A	2
(v_1, v_3)	2	N	A	4
(v_3, v_4)	2	Y	NA	4
(v_3, v_5)	2	Y	NA	4
(v_1, v_2)	3	N	A	7
(v_4, v_5)	3	Y	NA	7
(v_1, v_5)	4	Y	NA	7

37. Reading the table from top to bottom, which is the first incorrect line of the table shown above when implementing Kruskal's algorithm?

(a) Line 3

(b) Line 4

(c) Line 5

(d) Line 6

(e) All lines are correct.

38. The weight of the minimum spanning tree is

(a) 7

(b) 8

(c) 9

(d) 6

(e) None of the above

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