

# MATH221 Mathematics for Computer Science

## Tutorial Sheet Week 7

Autumn 2017

1. Let  $X = \{a, b, c, d, e, f\}$ . Determine whether the following statements are true or false. Do *not* give reasons.

- |                                    |   |                                   |
|------------------------------------|---|-----------------------------------|
| (i) $X \in \mathcal{P}(X)$         | (ii) $\{\emptyset\} \in \mathcal{P}(X)$ | (iii) $a \in \mathcal{P}(X)$      |
| (iv) $\{a\} \in X$                 | (v) $a \in X$                           | (vi) $X \subseteq \mathcal{P}(X)$ |
| (vii) $a \subseteq \mathcal{P}(X)$ | (viii) $\{X\} \subseteq \mathcal{P}(X)$ |                                   |

2. Which of the following sets are equal? (In some but not all cases, you can list the elements of the sets explicitly, and this may help in answering the question.)

- (i)  $A = \{0, 1, 2\}$
- (ii)  $B = \{x \in \mathbb{R} : -1 \leq x < 3\}$
- (iii)  $C = \{x \in \mathbb{R} : -1 < x < 3\}$
- (iv)  $D = \{x \in \mathbb{Z} : -1 < x < 3\}$
- (v)  $E = \{x \in \mathbb{N} : -1 < x < 3\}$

3. Let  $U = \mathbb{R}$  and let  $A = \{1\}$ ,  $B = (0, 1) = \{x \in \mathbb{R} : 0 < x < 1\}$  and  $C = [0, 1] = \{x \in \mathbb{R} : 0 \leq x \leq 1\}$ . Find the sets below.

$A \cup B$	$A \cap B$	$B \cap C$	$A \cup C$	$A \cap C$
$\overline{A}$	$\overline{C}$	$C - A$	$C - B$	$A - C$

4. Prove or disprove the statement  $\{0, 1\} = \left\{n \in \mathbb{Z} : \exists k \in \mathbb{Z}, n = \frac{1 - (-1)^k}{2}\right\}$ .

5. Let  $U = \mathbb{N}$  and let  $A = \{x \in \mathbb{N} : x \text{ is odd}\}$ ,  $B = \{x \in \mathbb{N} : x \text{ is even}\}$ , and  $P = \{x \in \mathbb{N} : x \text{ is a prime number}\}$ . Find the sets below. Are  $A$  and  $B$  disjoint? Is  $P \subseteq A$ ?

$A \cup B$	$A \cap B$	$B \cap P$	$A \cup P$	$A \cap P$
$\overline{A}$	$\overline{P}$	$P - A$	$B - P$	$A - B$

6. Let  $U$  be the universal set and let  $A$ ,  $B$  and  $C$  be subsets of  $U$ . By using the properties of  $\cup$ ,  $\cap$  and  $\overline{\phantom{x}}$  (complement), and any results from lectures, simplify the following.

- |                                    |  |  |                                     |
|------------------------------------|--|--|-------------------------------------|
| (i) $(C \cap U) \cup \overline{C}$ | (ii) $\overline{(A \cap U)} \cup \overline{A}$ | (iii) $\overline{[(C \cup \emptyset) \cup C]}$ | (iv) $(A \cap B) \cap \overline{A}$ |
|------------------------------------|--|--|-------------------------------------|

7. Let  $U$  be a non-empty universal set, and let  $A$ ,  $B$  and  $C$  be subsets of  $U$ . Prove or disprove each of the following statements:

- (i)  $\overline{A} - \overline{B} = B - A$ .
- (ii)  $A - (B - C) = (A - B) - C$ . You may find the relation  $A - B = A \cap \overline{B}$ , the Distributive Laws and DeMorgan's Laws helpful.