MATH221 – Mathematics for Computer Science – Autumn 2018

Assignment Two Answers

Question 1:

a)

Given $2 \cdot 1066 + 1492 \equiv n \ (mod\ 1776) \Rightarrow 3624 \equiv n \ (mod\ 1776)$

$1776 \mid (1776 - n), n \in \mathbb{N}$

$\therefore$ n is any of $\{\ldots, 72, 1848, 3480, \ldots\}$ ■

n is not unique because it's residue class has more than one element.

b)

Given $n \equiv 3 \ (mod\ 4)$

$n = 3 + 4r_1, r_1 \in \mathbb{N}$

Given $m \equiv 5 \ (mod\ 8)$

$m = 5 + 8r_2, r_2 \in \mathbb{N}$

Proof by contradiction:

$3 + 4r_1 = 5 + 8r_2$

$\Rightarrow 4r_1 = 8r_2 + 5 - 3$

$\Rightarrow 4r_1 = 8r_2 + 2$

$\Rightarrow 2r_1 = 4r_2 + 1$

$2r_1$ is an even number, $r_1 \in \mathbb{N}$

$4r_2 + 1$ is an odd number, $r_2 \in \mathbb{N}$

$\therefore 2r_1 \neq 4r_2 + 1$ ■

Question 2:

a) $\overline{B}$

b) $A - B$

c) $(A \cup B) - (A \cap B)$

d) $(A \cap B) \cup \overline{(A \cup B)}$

Question 3:

$$(-4+3x)^{12} = \sum_{k=0}^{12} \binom{12}{k} (3x)^{12-k}(-4)^k$$

To find the coefficient of x^5, let $k = 7$.

$\binom{12}{5} \cdot (-4)^7 \cdot (3x)^{12-7} = 792 \cdot -16384 \cdot 243x^5 = -3{,}153{,}199{,}104x^5$

$\therefore\ the\ coefficient\ of\ x^5\ is - 3{,}153{,}199{,}104$

Question 4:

a)

Given $a, b, c, d \in \mathbb{Z}\ with\ b, d \neq 0 : \frac{a}{b} R \frac{c}{d} \Leftrightarrow ad = bc\ \}$

Let $x = \frac{a}{b}, y = \frac{c}{d}, R = \{(x, y) \ni x, y\ belongs\ to\ \mathbb{Q} : x = y\}$.

Reflexive

If $x = x \Rightarrow (x, x) \in R$.

Symmetric

If $x = y \Leftrightarrow y = x$, then $(x, y) \in R \wedge (y, x) \in R$.

Transitive

Suppose $(x, y) \in R \wedge (y, z) \in R$, then $(x, z) \in R$.

If $x = y \Rightarrow y = z \Rightarrow x = z$.

Since $x = z \Leftrightarrow x = x$, then $(x, x) \in R$

$\therefore (x, z) \in R$.

Equivalence Classes

$[-1] = \{-1\}$

$\left[\frac{4}{5}\right] = \left\{\frac{4}{5}\right\}$

b)

Given $x, y \in \mathbb{R}, R = \{(x, y) \colon |x - y| \leq 1\}$

Let $x = 2, y = 3, z = 4, (2, 3) \in R \wedge (3, 4) \in R$.

$(2, 4) \notin R$

$\therefore R$ is not an equivalence relation as it is not transitive.