1 Equations from Kepler's Laws

1.1 Determining Tangential Velocity Given r and Position Given θ

The first step in modeling the orbits of Earth and Mars was to find their tangential velocity at a given radius from the centre of their elliptical orbit. This was determined using Kepler's law of areas, and one of its resulting equations (equations (1) and (2)), both found in Classical Mechanics by Taylor sections 3.5 and 8.6, and resulted in equation(4) - the equation for L (equation (3)) was taken from the Georgia State University website in which the eccentricities and semi-major axis values were provided.

$$\frac{dA}{dt} = \frac{1}{2} r v_\theta \tag{1}$$

$$\frac{dA}{dt} = \frac{L}{2\mu} \tag{2}$$

$$L = \mu\sqrt{GMa(1-e^2)} \tag{3}$$

$$v_\theta = \frac{\sqrt{GMa(1-e^2)}}{r} \tag{4}$$

Equation (7) was formed through reference to Classical Mechanics by Taylor secton 8.6. Equation (8) was taken from the Georigia State website. Equation (9) was based on common knowledge.

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \tag{5}$$

$$\frac{b}{a} = \sqrt{1-e^2} \tag{6}$$

$$\frac{x^2}{a^2} + \frac{y^2}{a^2(1-e^2)} = 1 \tag{7}$$

$$r(\theta) = \frac{a(1-e^2)}{1+ecos(\theta)} \tag{8}$$

$$d = \sqrt{(x_2-x_1)^2 - (y_2-y_1)^2}, x = rcos(\theta), y = rsin(\theta) \tag{9}$$

1.2 Approximating Position at Time t

In order to find an exact equation for position at time t, one would have to integrate a variation of Kepler's third law. This was initially attempted, but then abandoned in favor of an approximation using the equations listed in section 1.1. First, functions were defined for equations (4) and (8). Then, a funtion called DeltaT was used to approximate the time elapsed between two tenths of a degree. Next, a distance function was written which takes a inputed time and adds predetermined Δt's in a while-loop until it reaches the given time. As these chuncks of time are summed, the function also sums the degrees elapse; the sum of the degrees is the actual output of the function.

The results of the distance funtion and the radius function are then fed into a function which converts the polar coordinates to cartesian coordinates, and then into a function which determines the distance between the planets using equation (9).