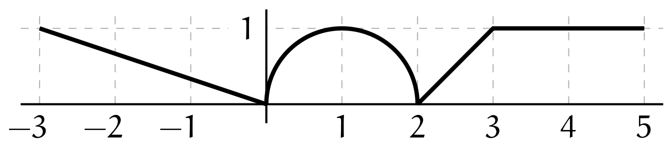
Problems (+practice) from Butler Fall 16 corresponding to Exam 3

1. Find $\displaystyle\int_0^{\pi/4} \big(2-\sec^2(\theta)\big)^{1/2}\sec^2(\theta)\,d\theta$. (Hint: recall that $\tan^2(\theta)+1=\sec^2(\theta)$.)

2. Find $\displaystyle\int \frac{dy}{y^{1/3}(y^{2/3}+1)}$.

3. Given $f(x)$ is shown below, and $F(x)=\displaystyle\int_{3-x}^{2x^2} f(t)\,dt$, determine the following.



   (a) Find $F(1)$.

   (b) Find $F(-1)$.

   (c) Find $F(0)$.

   (d) Find $F'\big(-\sqrt{2}\big)$.

4. Find $\displaystyle\int_1^{3\sqrt{3}} \frac{dy}{y^{2/3}(y^{2/3}+1)}$.

5. Rewrite the following as a *single* integral, i.e., of the form $\displaystyle\int_a^b f(u)\,du$:

$$\int_0^{\ln(3)} e^x f(e^x)\,dx + \int_3^6 \sin(2\pi x) f\big(\sin(\pi x)\big)\,dx + \int_6^{10} \tfrac{1}{2} f\big(8-\tfrac{1}{2}x\big)\,dx.$$

6. Simplify the following to a single integral:

$$\int_1^6 f(t)\,dt + \int_1^4 f(t)\,dt + 2\int_2^1 f(t)\,dt + \int_6^4 f(t)\,dt.$$

7. Find $\displaystyle\int_{-5}^{5} \big(3+t^7\cos(t)\big)\,dt$.

8. Find $\displaystyle\int \frac{1}{\big(\sin(\frac{1}{2}\theta)-\cos(\frac{1}{2}\theta)\big)^2}\,d\theta$.

9. Find $\displaystyle\int \frac{(x-1)^2+2}{x}\,dx$.

10. Approximate the area under the curve $y=x^2-3x+5$ from $x=0$ to $x=4$ using a Riemann sum with four equally spaced intervals and using the *right* endpoint of each interval.

11. Given that $v(t) = \dfrac{5t}{t^2+1}$ is a velocity of the particle, estimate the distance it has traveled from $t = 0$ to $t = 4$ by using a Riemann sum with four equally spaced intervals and using the *left* endpoints of each interval.

12. Find the area between $y = \sqrt{1-x^2}$ and $y = 2|x| - 2$ for $-1 \leqslant x \leqslant 1$.

13. Find $\displaystyle\int_0^8 h(x)\,dx$ where

$$h(x) = \begin{cases} 2 - |x-2| & \text{if } 0 \leqslant x \leqslant 3; \\ 3 - |x-5| & \text{if } 3 \leqslant x \leqslant 8. \end{cases}$$

14. For $x > 0$, find $f(x)$ given that

$$\int_x^2 tf(t)\,dt = e^{3x-6} + C\cos(\pi x) + 2x + 1.$$

(Note that you will have to solve for "C".)

15. Find $\dfrac{d}{dx}\left(\displaystyle\int_x^{x^2} \frac{\arctan t}{t^4+1}\,dt\right)$.

16. Evaluate $\displaystyle\int_1^3 \frac{dy}{y^{1/2}+y^{3/2}}$ by substituting $u = y^{1/2}$.

17. Find $\displaystyle\int x\big(\sin(x^2) + \cos(x^2)\big)^2 \sin(x^2)\,dx$.

18. Find the area bounded by $f(x) = 2\sin x$ and $g(x) = \sec x \tan x$ between $x = 0$ and the smallest $x > 0$ where the two curves intersect.

19. Find the average value of the function $f(x) = \dfrac{x}{x^2+1}$ for $0 \leqslant x \leqslant 2$.

20. Approximate the area under the curve $y = \frac{2x}{4x^2+1}$ from $x = 0$ to $x = 4$ using a Riemann sum with the interval split into four equally sized parts and using the central point in each part.

21. Find $\displaystyle\int 2te^{\sin(t^2)}\cos(t^2)\,dt$.

22. Given that $\displaystyle\int_0^{2x} f(t)\,dt = e^{x^2} - Ce^x$, find $f(t)$ and C.

23. Find the average value for $f(x) = |x^2 - 4|$ for the interval $-3 \leqslant x \leqslant 5$.

24. Find the following.

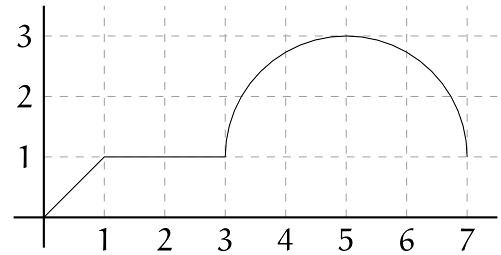
$$\int \sin(x)\cos(x)\sqrt{3-\cos(x)}\,dx.$$

25. Find the area between the curves $y = x^2$ and $x = y^3$.

26. Find $\dfrac{d}{dx}\left(\displaystyle\int_{x^2}^{e^x} \sin(t^2)\,dt\right)$.

27. Let $f(x)$ be the function defined piecewise on the interval $[0, 7]$ by the following:

$$f(x) = \begin{cases} x & \text{if } 0 \leqslant x \leqslant 1, \\ 1 & \text{if } 1 \leqslant x \leqslant 3, \\ 1 + \sqrt{4 - (x-5)^2} & \text{if } 3 \leqslant x \leqslant 7. \end{cases}$$



Given that $F(x) = \int_2^x f(t)\,dt$ find the following.

(a) $F(0) =$

(b) $F(2) =$

(c) $F(7) =$

28. Approximate the area under the curve $y = \frac{1}{4}x^2 - x$ from $x = 1$ to $x = 5$ using a Riemann sum with the interval split into four equally sized parts and using the central point in each part.

29. Find $\int e^{x+e^x}\,dx$.

30. Find the average value of $f(x) = \dfrac{x}{x^2+3}$ for $-2 \leqslant x \leqslant 5$.

31. Consider the following table which gives information about the function $f(x)$ at some points:

$x =$	-1	-0.5	0	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5	5.5	6	6.5	7
$f(x) =$	2.7	1.6	1.3	1.1	0.8	0.7	0.6	0.8	1.2	1.5	1.6	1.8	2.0	2.1	1.8	1.3	0.5

Use this information to approximate the area under the curve $y = xf(2x)$ from $x = 0$ to $x = 3$ using Riemann sums with six equally sized parts and using the *right* endpoint in each part.

32. Find $\int_0^1 \dfrac{\sqrt{y}}{y^3+1}\,dy$. (Hint: you do **not** need to know that $y^3 + 1 = (y+1)(y^2 - y + 1)$.)

33. Find $\int_{1/2}^1 r(4x)\,dx$ given the following information

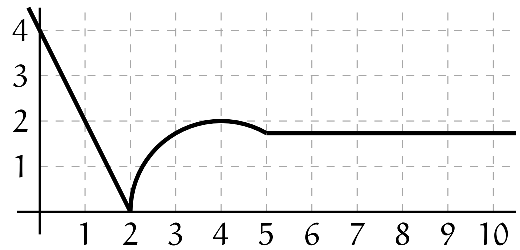
$$\int_1^3 r(2x)\,dx = 5, \qquad \int_1^2 r(3x)\,dx = 4, \qquad \int_4^3 r(x)\,dx = 2 \quad \text{and} \quad \int_0^1 r(5x)\,dx = 6.$$

34. Find $\int \dfrac{e^x}{e^{2x} + 2e^x + 2}\,dx$. (Hint: $2 = 1 + 1$)

35. Find $\int_0^8 \dfrac{dx}{1 + \sqrt[3]{x}}$.

36. Let $H(x) = \int_{x^2+1}^{x+3} h(t)\,dt$ where $h(t)$ is the function defined piecewise by

$$h(t) = \begin{cases} 4-2t & \text{if } t \leqslant 2, \\ \sqrt{4-(t-4)^2} & \text{if } 2 \leqslant t \leqslant 5, \\ \sqrt{3} & \text{if } t \geqslant 5. \end{cases}$$



Find the following values.

(a) $H(1) =$

(b) $H'(1) =$

(c) $H(2) =$

(d) $H(3) =$

37. Find $\dfrac{d}{dt}\left(\int_{t^3}^{e^t} \cos(x^2)\,dx\right)$.

38. Find $\int_1^{\sqrt{3}} \arctan(t)\,dt + \int_{\pi/4}^{\pi/3} \tan(t)\,dt$.
(Hint: think of integrals as areas and how they connect to each other; $\tan(\frac{\pi}{4}) = 1$, $\tan(\frac{\pi}{3}) = \sqrt{3}$.)

39. Find the area between the curves $y = 2x^2 - x\sin^2 x + 1$ and $y = x^2 + x\cos^2 x + 3$.