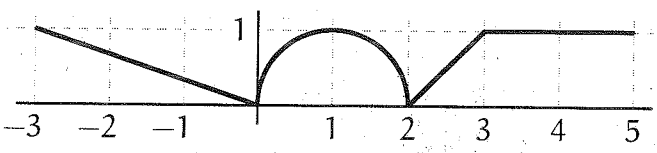
Problems (+practice) from Butler Fall 16 corresponding to Exam 3

1. Find $\int_0^{\pi/4} \left(2-\sec^2(\theta)\right)^{1/2} \sec^2(\theta)\,d\theta$. (Hint: recall that $\tan^2(\theta)+1=\sec^2(\theta)$.) $\pi/4$

2. Find $\int \frac{dy}{y^{1/3}(y^{2/3}+1)}$. $= \frac{3}{2}\ln(y^{2/3}+1)+C$

3. Given $f(x)$ is shown below, and $F(x)=\int_{3-x}^{2x^2} f(t)\,dt$, determine the following.



(a) Find $F(1)$. 0

(b) Find $F(-1)$. $3/2$

(c) Find $F(0)$. $\frac{\pi+1}{2}$

(d) Find $F'(-\sqrt{2})$. $-4\sqrt{2}+1$

4. Find $\int_1^{3\sqrt{3}} \frac{dy}{y^{2/3}(y^{2/3}+1)}$. $= \frac{\pi}{4}$

5. Rewrite the following as a *single* integral, i.e., of the form $\int_a^b f(u)\,du$:

$$\int_0^{\ln(3)} e^x f(e^x)\,dx + \int_3^6 \sin(2\pi x) f(\sin(\pi x))\,dx + \int_6^{10} \tfrac{1}{2} f(8-\tfrac{1}{2}x)\,dx. \quad = \int_1^5 f(u)\,du$$

6. Simplify the following to a single integral:

$$\int_1^6 f(t)\,dt + \int_1^4 f(t)\,dt + 2\int_2^1 f(t)\,dt + \int_6^4 f(t)\,dt. \quad = 2\int_2^4 f(t)\,dt$$

7. Find $\int_{-5}^{5} \left(3+t^7\cos(t)\right)dt$. $= 30.$

8. Find $\int \frac{1}{\left(\sin(\frac{1}{2}\theta)-\cos(\frac{1}{2}\theta)\right)^2}\,d\theta$. $= \tan\theta - \sec\theta + C$

9. Find $\int \frac{(x-1)^2+2}{x}\,dx$. $\frac{1}{2}x^2 - 2x + 3\ln|x| + C$

10. Approximate the area under the curve $y=x^2-3x+5$ from $x=0$ to $x=4$ using a Riemann sum with four equally spaced intervals and using the *right* endpoint of each interval.

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11. Given that $v(t) = \dfrac{5t}{t^2+1}$ is a velocity of the particle, estimate the distance it has traveled from $t = 0$ to $t = 4$ by using a Riemann sum with four equally spaced intervals and using the *left* endpoints of each interval. 6

12. Find the area between $y = \sqrt{1-x^2}$ and $y = 2|x| - 2$ for $-1 \leqslant x \leqslant 1$. $\frac{\pi}{2} + 2$

13. Find $\displaystyle\int_0^8 h(x)\,dx$ where $= 10$

$$h(x) = \begin{cases} 2 - |x-2| & \text{if } 0 \leqslant x \leqslant 3; \\ 3 - |x-5| & \text{if } 3 \leqslant x \leqslant 8. \end{cases}$$

14. For $x > 0$, find $f(x)$ given that

$$\int_x^2 tf(t)\,dt = e^{3x-6} + C\cos(\pi x) + 2x + 1.$$

$f(x) = \dfrac{-3e^{3x-6} - 6\pi \sin \pi x + 2}{x}$

(Note that you will have to solve for "C".)

15. Find $\dfrac{d}{dx}\left(\displaystyle\int_x^{x^2} \frac{\arctan t}{t^4+1}\,dt\right)$. $= \dfrac{2x\arctan(x^2)}{x^8+1} - \dfrac{\arctan x}{x^4+1}$

16. Evaluate $\displaystyle\int_1^3 \frac{dy}{y^{1/2} + y^{3/2}}$ by substituting $u = y^{1/2}$. $= \pi/6$

17. Find $\displaystyle\int x\left(\sin(x^2) + \cos(x^2)\right)^2 \sin(x^2)\,dx$. $= -\frac{1}{2}\cos x^2 + \frac{1}{3}\sin x^2 + C$

18. Find the area bounded by $f(x) = 2\sin x$ and $g(x) = \sec x \tan x$ between $x = 0$ and the smallest $x > 0$ where the two curves intersect. $3 - 2\sqrt{2}$

19. Find the average value of the function $f(x) = \dfrac{x}{x^2+1}$ for $0 \leqslant x \leqslant 2$. $\dfrac{\ln 5}{4}$

20. Approximate the area under the curve $y = \frac{2x}{4x^2+1}$ from $x = 0$ to $x = 4$ using a Riemann sum with the interval split into four equally sized parts and using the central point in each part. $\dfrac{368}{325} \approx 1.13231$

21. Find $\displaystyle\int 2te^{\sin(t^2)}\cos(t^2)\,dt$. $= e^{\sin t^2} + C$

22. Given that $\displaystyle\int_0^{2x} f(t)\,dt = e^{x^2} - Ce^x$, find $f(t)$ and C. $C = 1$ $\quad f(t) = \frac{1}{2}te^{t^2/4} - \frac{1}{2}e^{t/2}$

23. Find the average value for $f(x) = |x^2 - 4|$ for the interval $-3 \leqslant x \leqslant 5$. $\rightarrow 5$

24. Find the following.

$$\int \sin(x)\cos(x)\sqrt{3 - \cos(x)}\,dx.$$

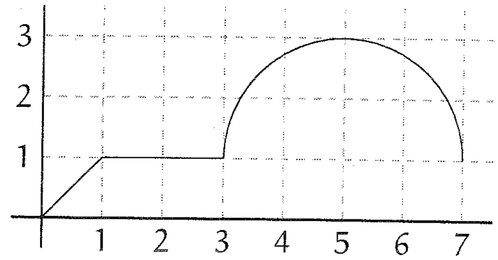
$= 2(3-\cos x)^{3/2} - \frac{2}{5}(3-\cos x)^{5/2} + C$

25. Find the area between the curves $y = x^2$ and $x = y^3$. $5/12$

26. Find $\dfrac{d}{dx}\left(\displaystyle\int_{x^2}^{e^x} \sin(t^2)\,dt\right)$. $= e^x \sin(e^{2x}) - 2x\sin(x^4)$.

27. Let $f(x)$ be the function defined piecewise on the interval $[0, 7]$ by the following:

$$f(x) = \begin{cases} x & \text{if } 0 \leqslant x \leqslant 1, \\ 1 & \text{if } 1 \leqslant x \leqslant 3, \\ 1 + \sqrt{4 - (x-5)^2} & \text{if } 3 \leqslant x \leqslant 7. \end{cases}$$



Given that $F(x) = \int_2^x f(t)\,dt$ find the following.

(a) $F(0) =$ $-\frac{3}{2}$

(b) $F(2) =$ 0

(c) $F(7) =$ $5 + 2\pi$

28. Approximate the area under the curve $y = \frac{1}{4}x^2 - x$ from $x = 1$ to $x = 5$ using a Riemann sum with the interval split into four equally sized parts and using the central point in each part.

$-\frac{7}{4}$

29. Find $\int e^{x+e^x}\,dx$. $e^{e^x} + C$

30. Find the average value of $f(x) = \dfrac{x}{x^2+3}$ for $-2 \leqslant x \leqslant 5$. $= \frac{1}{14} \ln 4$

31. Consider the following table which gives information about the function $f(x)$ at some points:

$x =$	-1	-0.5	0	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5	5.5	6	6.5	7
$f(x) =$	2.7	1.6	1.3	1.1	0.8	0.7	0.6	0.8	1.2	1.5	1.6	1.8	2.0	2.1	1.8	1.3	0.5

Use this information to approximate the area under the curve $y = xf(2x)$ from $x = 0$ to $x = 3$ using Riemann sums with six equally sized parts and using the *right* endpoint in each part.

8.2

32. Find $\int_0^1 \dfrac{\sqrt{y}}{y^3+1}\,dy$. (Hint: you do **not** need to know that $y^3 + 1 = (y+1)(y^2 - y + 1)$.)

$= \frac{2}{3} \ln 2$

33. Find $\int_{1/2}^1 r(4x)\,dx$ given the following information $= (-1)$

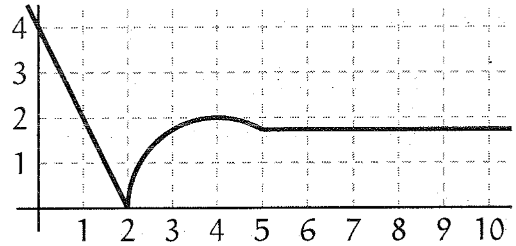
$$\int_1^3 r(2x)\,dx = 5, \qquad \int_1^2 r(3x)\,dx = 4, \qquad \int_4^3 r(x)\,dx = 2 \quad \text{and} \quad \int_0^1 r(5x)\,dx = 6.$$

34. Find $\int \dfrac{e^x}{e^{2x} + 2e^x + 2}\,dx$. (Hint: $2 = 1 + 1$) $= \arctan(e^x + 1) + C$

35. Find $\int_0^8 \dfrac{dx}{1 + \sqrt[3]{x}}$. $= 3 \ln 3$

36. Let $H(x) = \int_{x^2+1}^{x+3} h(t)\,dt$ where $h(t)$ is the function defined piecewise by

$$h(t) = \begin{cases} 4-2t & \text{if } t \leqslant 2, \\ \sqrt{4-(t-4)^2} & \text{if } 2 \leqslant t \leqslant 5, \\ \sqrt{3} & \text{if } t \geqslant 5. \end{cases}$$



Find the following values.

(a) $H(1) =$ π

(b) $H'(1) =$ 2

(c) $H(2) =$ 0

(d) $H(3) =$ $-4\sqrt{3}$

37. Find $\frac{d}{dt}\left(\int_{t^3}^{e^t} \cos(x^2)\,dx\right)$. $= \cos\left(e^{2x}\right)\cdot e^{x} - 3x^2 \cdot \cos\left(x^6\right)$.

38. Find $\int_{1}^{\sqrt{3}} \arctan(t)\,dt + \int_{\pi/4}^{\pi/3} \tan(t)\,dt.$ $= \ln 2$

(Hint: think of integrals as areas and how they connect to each other; $\tan(\frac{\pi}{4}) = 1$, $\tan(\frac{\pi}{3}) = \sqrt{3}$.)

39. Find the area between the curves $y = 2x^2 - x\sin^2 x + 1$ and $y = x^2 + x\cos^2 x + 3$. $\frac{9}{2}$ square units.