

## 1.1

Def: A **metric** is a function  $d : X^2 \rightarrow \mathbb{R}$  such that: 1.  $d(x, y) \geq 0$ , and  $d(x, y) = 0$  if and only if  $x = y$  (Positive Definiteness) 2.  $d(x, y) = d(y, x)$  (Symmetry) 3.  $d(x, z) \leq d(x, y) + d(y, z)$  (Triangle Inequality)

Def: A **metric space** is a set  $X$  with a metric  $d$  on it, denoted  $(X, d)$ .

## 1.3

Def: Let  $(X, d)$  be metric space. An **open ball** of radius  $r$  centered at  $x$  is  $b_r(x) = \{y \in X \mid d(y, x) < r\}$ . (Note the strict inequality)

Let  $S \subseteq X$ . An element  $x_0 \in S$  is an **interior point** of  $S$  if there exists  $r > 0$  such that  $b_r(x_0) \subseteq S$ .

Let  $S^\circ$  denote the set of all interior points of  $S$ . Then  $S$  is **open** if  $S = S^\circ$ . (Note that it is sufficient to show  $S \subseteq S^\circ$ ).

A set  $R \subseteq X$  is **closed** if  $R^c$  (the complement) is open.