TP2 MRR

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IV. Cookies Study

First, let's go back to our previous results: We saw that with linear regression without penalty, only 2 explanatory variables were significant. We can deduce that we can also use a Lasso regression to select the most important features to predict the fat value of a cookie. Futhermore, we can use a Ridge regression to see if the model is overfitting or not.

Imports

```
cookies_data <- read.csv("cookies.csv")
```

Features extraction

For each line (meaning, for each cookie), we will use the different spectral values to compute: the mean, the standard deviation, the slope, the minimum and the maximum.

```
# Computation (mean, standard deviation, minimum and maximum)

cookies_data$mean <- rowMeans(cookies_data[, 2:701])
cookies_data$stDev <- apply(cookies_data[, 2:701], 1, sd)
cookies_data$min <- apply(cookies_data[, 2:701], 1, min)
cookies_data$max <- apply(cookies_data[, 2:701], 1, max)


# Computation (slope)

# Function: compute_slope
# @param: spectrum_values of a cookie (here, column 2 to 701)
# @return: slope of the spectrum curve for a cookie
compute_slope <- function(spectrum_values) {
  pos <- 1:length(spectrum_values)
  lm_model <- lm(spectrum_values ~ pos)
  slope <- coef(lm_model)[2]
  return(slope)
}

cookies_data$slope <- apply(cookies_data[, 2:701], 1, compute_slope)
```

```r
# Display of the new columns
head(cookies_data[,702:706])
```

```
##        mean     stDev      min     max       slope
## 1 0.9851499 0.4111868 0.259270 1.73946 0.001914311
## 2 1.0355417 0.4123933 0.266864 1.66273 0.001898164
## 3 1.0010620 0.4025158 0.251654 1.60960 0.001860203
## 4 1.0280481 0.4040351 0.277777 1.63881 0.001861782
## 5 1.0655011 0.4158252 0.288328 1.70320 0.001910926
## 6 1.0840236 0.4262425 0.284625 1.74356 0.001967228
```

Regression model

Now, we have the different features of the spectra.

```r
# Only features and fat values are retrieved

cookies_features <- cookies_data[c("fat", "mean", "stDev", "slope", "min", "max")]
head(cookies_features)
```

```
##     fat      mean     stDev       slope      min     max
## 1 12.57 0.9851499 0.4111868 0.001914311 0.259270 1.73946
## 2 15.13 1.0355417 0.4123933 0.001898164 0.266864 1.66273
## 3 12.63 1.0010620 0.4025158 0.001860203 0.251654 1.60960
## 4 13.85 1.0280481 0.4040351 0.001861782 0.277777 1.63881
## 5 15.25 1.0655011 0.4158252 0.001910926 0.288328 1.70320
## 6 13.66 1.0840236 0.4262425 0.001967228 0.284625 1.74356
```

```r
X <- as.matrix(cookies_features[, -1]) # co-variables
y <- cookies_features$fat # target variable
```

```
## Le chargement a nécessité le package : Matrix
```

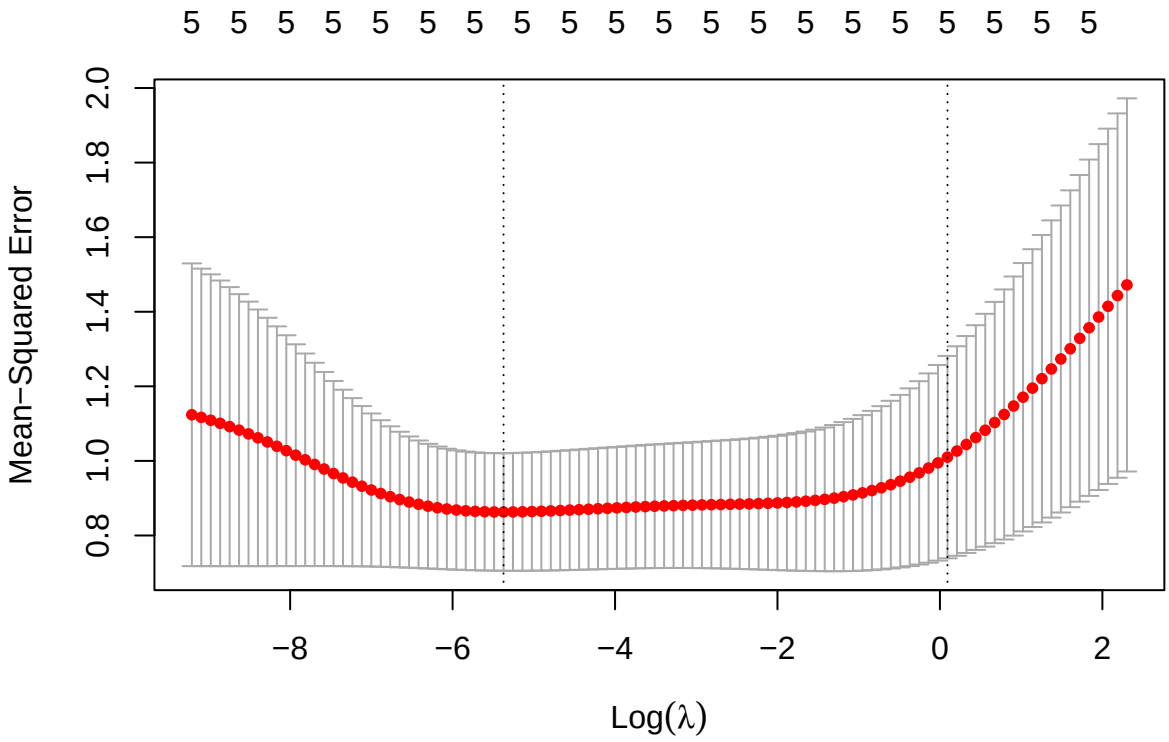
```
## Loaded glmnet 4.1-8
```

Ridge regression

We're going to do the ridge regression first, using a cross validation k-fold to choose the best value for λ.

```r
# Cross validation
lambdas_log <- 10^seq(-4, 1, length.out = 100)
cv_ridge <- cv.glmnet(X, y, alpha=0, lambda = (lambdas_log), standardize = TRUE)

plot(cv_ridge)
```



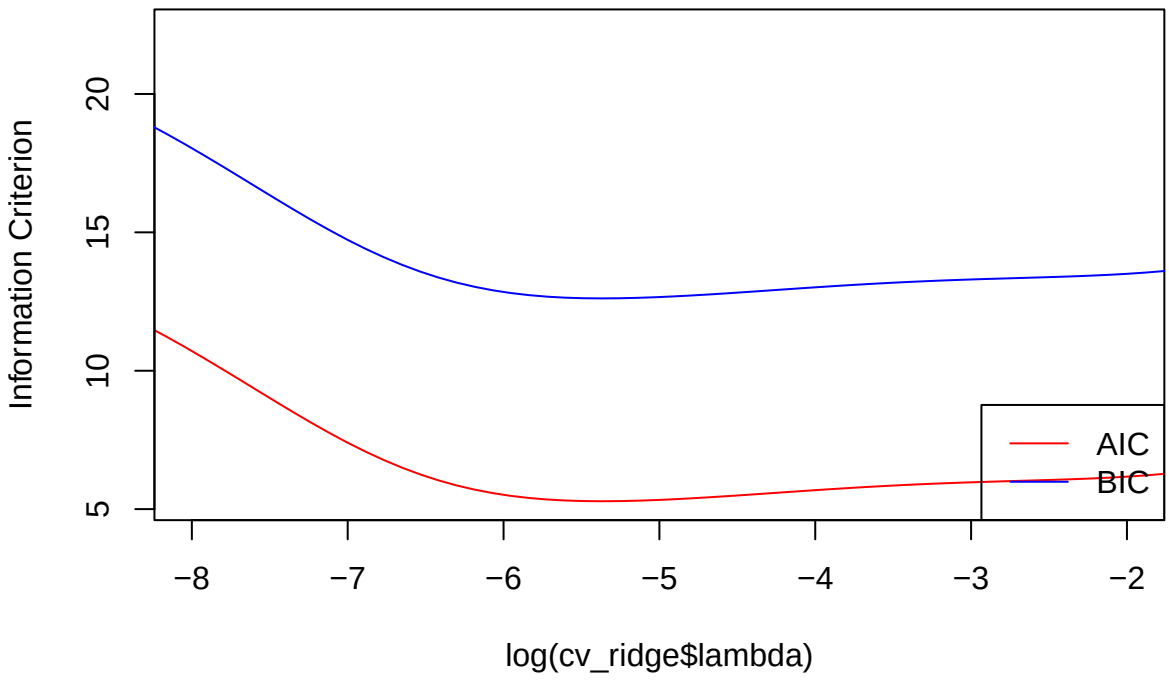

```
best_lambda <- cv_ridge$lambda.min # lambda that gives the lowest MSE
print(paste("The best value for lambda is", best_lambda))
```

```
## [1] "The best value for lambda is 0.00464158883361278"
```

Let's use AIC and BIC criteria to recheck this value.

```
# AIC and BIC
n <- nrow(X)
p <- ncol(X)
aic <- n * log(cv_ridge$cvm) + 2 * p
bic <- n * log(cv_ridge$cvm) + log(n) * p

plot(log(cv_ridge$lambda), aic, col = "red1", type = "l", xlim = c(-8, -2), ylab = "Information Criteri
lines(log(cv_ridge$lambda), bic, col = "blue1")
legend("bottomright", lwd = 1, col = c("red1", "blue1"), legend = c("AIC", "BIC"))
```




```
best_lambda_aic <- cv_ridge$lambda[which.min(aic)] # lambda that gives the lowest AIC
best_lambda_bic <- cv_ridge$lambda[which.min(bic)] # lambda that gives the lowest BIC
```

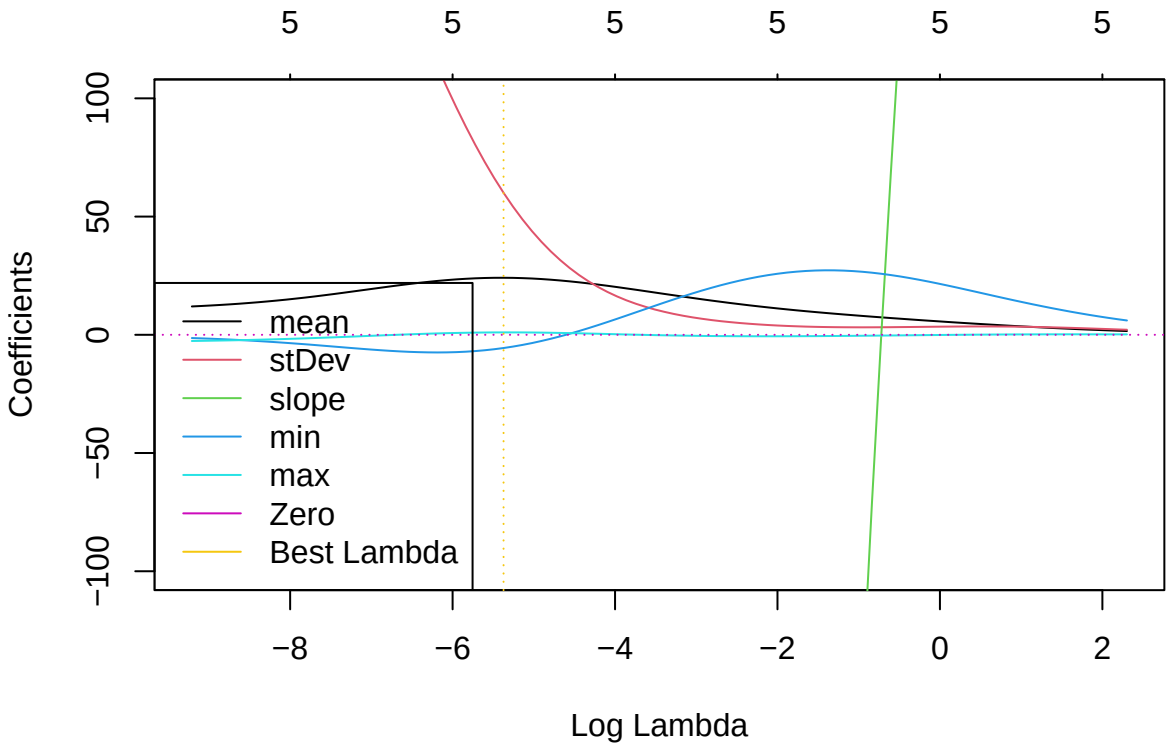
Now we can compare the different values for λ we found.

```
lambda_values <- c(best_lambda, best_lambda_aic, best_lambda_bic)
lambda_values
```

```
## [1] 0.004641589 0.004641589 0.004641589
```

There are the same. We can also plot the Regularization Path.

```
plot(cv_ridge$glmnet.fit, xvar = "lambda", ylim = c(-100, 100))
abline(h = 0, col = 6, lty = 3)
abline(v = log(best_lambda), col = 7, lty = 3)
legend("bottomleft", legend = c(colnames(X), "Zero", "Best Lambda"), col = 1:7, lty = 1)
```



Now we have the best value for λ, we do another ridge regression with this parameter and there is its results :

```
## 6 x 1 sparse Matrix of class "dgCMatrix"
##                         s0
## (Intercept) -6.227597e-01
## mean         2.412271e+01
## stDev        6.019353e+01
## slope       -1.852240e+04
## min         -5.621932e+00
## max          1.015760e+00
```

```r
predictions <- predict(best_model_ridge, newx = X)

# RMSE
rmse <- sqrt(mean((predictions - y)^2))
print(paste("RMSE ridge model :", rmse))
```

```
## [1] "RMSE ridge model : 0.761277527254414"
```

```r
# R^2
r_squared <- 1 - sum((y - predictions)^2) / sum((y - mean(y))^2)
print(paste("R^2 ridge model :", r_squared))
```

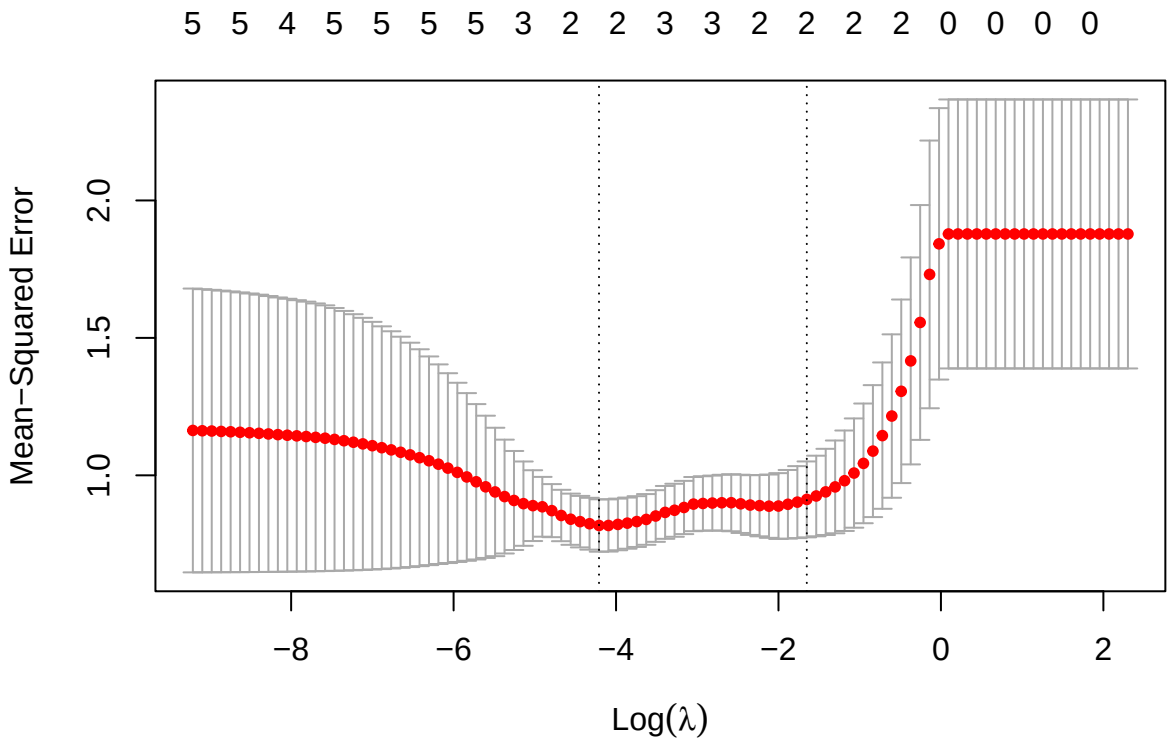
```
## [1] "R^2 ridge model : 0.671003019992278"
```

Conclusion of Ridge regression We can see that the coefficient of the slope is very important in absolute value relatively to the others (variables are scaled). It means that the slope is a very important feature to predict the fat value of a cookie. Futhermore, the coefficient of the mean, the standard deviation and the minimum are not null but negligible compared to the slope. It means that these features are not very important to predict the fat value of a cookie, but are more important than the max.

Lasso regression

```r
cv_lasso <- cv.glmnet(X, y, alpha=1, lambda = (lambdas_log), standardize = TRUE)

plot(cv_lasso)
```



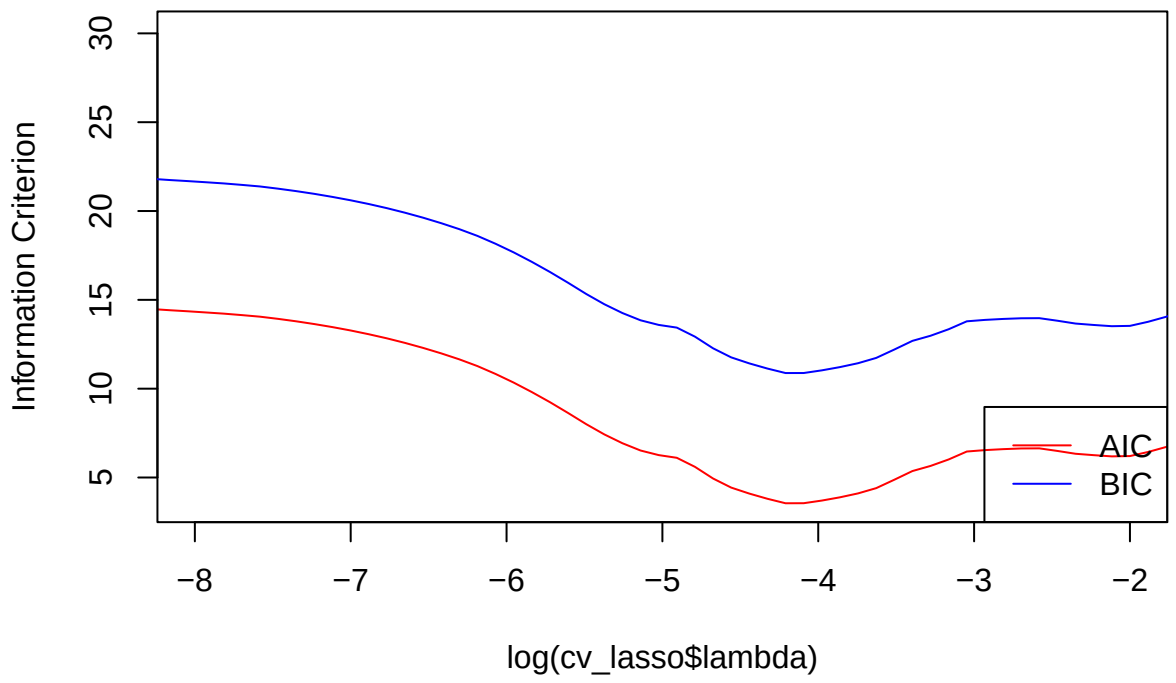
```r
best_lambda_lasso <- cv_lasso$lambda.min # lambda that gives the lowest MSE
print(paste("The best value for lambda is", best_lambda_lasso))
```

```
## [1] "The best value for lambda is 0.0148496826225447"
```

Let's use AIC and BIC criteria to recheck this value.

```r
# AIC and BIC
n <- nrow(X)
p <- ncol(X)
lasso_aic <- n * log(cv_lasso$cvm) + 2 * p
lasso_bic <- n * log(cv_lasso$cvm) + log(n) * p

plot(log(cv_lasso$lambda), lasso_aic, col = "red1", type = "l", xlim = c(-8, -2), ylab = "Information C
lines(log(cv_lasso$lambda), lasso_bic, col = "blue1")
legend("bottomright", lwd = 1, col = c("red1", "blue1"), legend = c("AIC", "BIC"))
```




```r
best_lambda_lasso_aic <- cv_lasso$lambda[which.min(lasso_aic)] # lambda that gives the lowest AIC
best_lambda_lasso_bic <- cv_lasso$lambda[which.min(lasso_bic)] # lambda that gives the lowest BIC
```

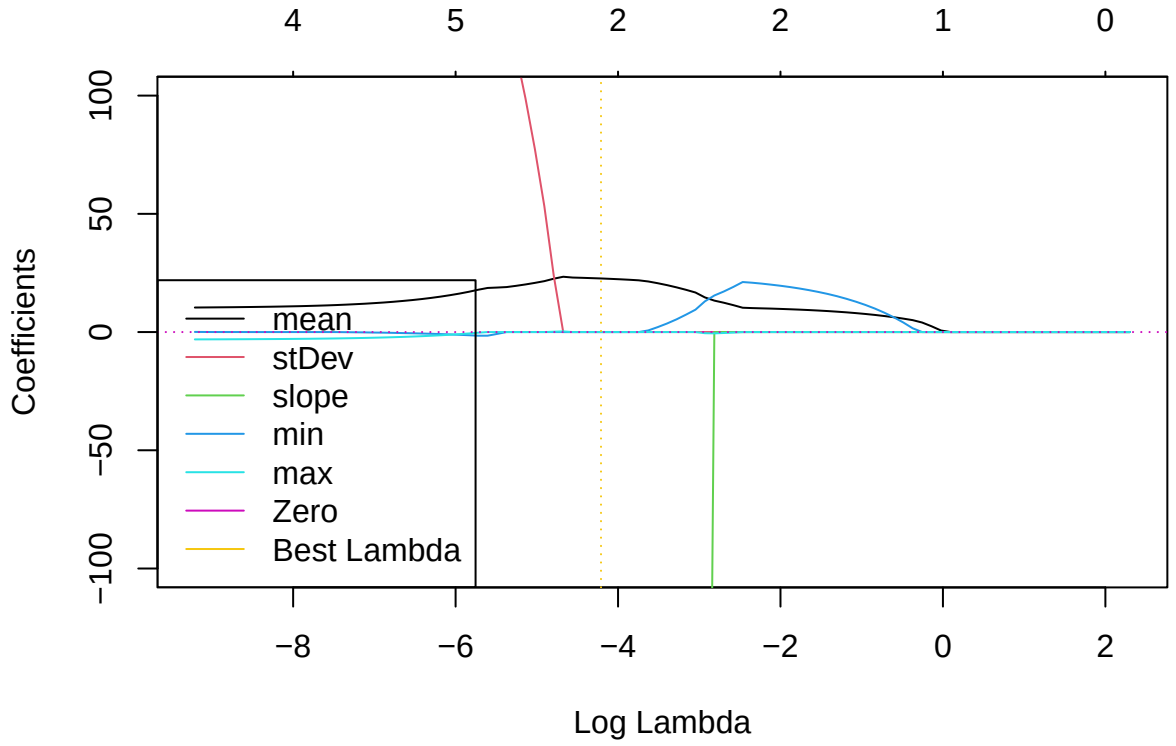
Now we can compare the different values for λ we found.

```
lambda_lasso_values <- c(best_lambda_lasso, best_lambda_lasso_aic, best_lambda_lasso_bic)
lambda_lasso_values
```

```
## [1] 0.01484968 0.01484968 0.01484968
```

There are the same. We can also plot the Regularization Path.

```
plot(cv_lasso$glmnet.fit, xvar = "lambda", ylim = c(-100, 100))
abline(h = 0, col = 6, lty = 3)
abline(v = log(best_lambda_lasso), col = 7, lty = 3)
legend("bottomleft", legend = c(colnames(X), "Zero", "Best Lambda"), col = 1:7, lty = 1)
```



Now we have the best value for λ, we do another lasso regression with this parameter and there is its results :

```
best_model_lasso <- glmnet(X, y, alpha=1, lambda = best_lambda_lasso)
coef(best_model_lasso)
```

```
## 6 x 1 sparse Matrix of class "dgCMatrix"
##                       s0
## (Intercept)    -2.387106
```

```
## mean           22.663461
## stDev           .
## slope       -3708.804403
## min             .
## max             .
```

```r
predictions <- predict(best_model_lasso, newx = X)

# RMSE
rmse <- sqrt(mean((predictions - y)^2))
print(paste("RMSE lasso model :", rmse))
```

```
## [1] "RMSE lasso model : 0.798650159017692"
```

```r
# R^2
r_squared <- 1 - sum((y - predictions)^2) / sum((y - mean(y))^2)
print(paste("R^2 lasso model :", r_squared))
```

```
## [1] "R^2 lasso model : 0.637907893912154"
```

Conclusion of Lasso regression

We can see that the coefficient of the slope is also very important in absolute value relatively to the others (variables are scaled). It means that the slope is a very important feature to predict the fat value of a cookie. In this case, we can also see that the mean is not null wheras all the others are. It means that the mean could be a feature to predict the fat value of a cookie relatively to the others, but is less important than the slope.

No penalization

```r
model_linear <- lm(y ~ X)

predictions_linear <- predict(model_linear, newdata = data.frame(X))

# RMSE
rmse_linear <- sqrt(mean((predictions_linear - y)^2))
print(paste("RMSE linear model :", rmse_linear))
```

```
## [1] "RMSE linear model : 0.706440945059225"
```

```r
# R^2
r_squared_linear <- 1 - sum((y - predictions_linear)^2) / sum((y - mean(y))^2)
print(paste("R^2 linear model :", r_squared_linear))
```

```
## [1] "R^2 linear model : 0.716692796197633"
```

Conclusion

We can see that for both Lasso and Ridge regressions, the RMSE is higher and the R^2 is lower than those of the linear model. It means that the linear model is better than the Lasso and Ridge models to predict the fat value of a cookie in our study. There could be several reasons for that. First, the number of features is not very high, so the penalization is not very useful and we actually lost to much information. Secondly, the model was not overfitting. Still, the values of the RMSE and R^2 are very close, so the difference is not very important, and those penalizations show us that the slope and mean are indead the most important features to predict the fat value of a cookie.

V2

```
cookies_data <- read.csv("cookies.csv")
head(cookies_data)
```

```
##     fat    X1100    X1102    X1104    X1106    X1108    X1110    X1112    X1114
## 1 12.57 0.259748 0.259482 0.259535 0.259270 0.259376 0.259270 0.259642 0.259801
## 2 15.13 0.267625 0.267320 0.267117 0.266864 0.266965 0.267016 0.267422 0.267878
## 3 12.63 0.251753 0.251654 0.251851 0.251900 0.252047 0.252342 0.252735 0.253177
## 4 13.85 0.278077 0.277877 0.277827 0.278077 0.277777 0.278027 0.278177 0.278427
## 5 15.25 0.288900 0.288484 0.288328 0.288328 0.288328 0.288796 0.289316 0.289783
## 6 13.66 0.285423 0.284891 0.284625 0.284625 0.284678 0.284731 0.284785 0.284944
##      X1116    X1118    X1120    X1122    X1124    X1126    X1128    X1130
## 1 0.260172 0.260863 0.261977 0.263198 0.264685 0.266702 0.268878 0.271427
## 2 0.268538 0.269452 0.270720 0.271989 0.273613 0.275947 0.278636 0.281833
## 3 0.254012 0.254798 0.256124 0.258138 0.260103 0.262707 0.265507 0.268602
## 4 0.278727 0.279328 0.280178 0.281628 0.283279 0.285229 0.287630 0.290681
## 5 0.290095 0.290823 0.291655 0.292746 0.294566 0.296541 0.298984 0.301895
## 6 0.285370 0.286115 0.287179 0.288509 0.290319 0.292553 0.295108 0.298247
##      X1132    X1134    X1136    X1138    X1140    X1142    X1144    X1146
## 1 0.274771 0.278805 0.283212 0.288149 0.293563 0.299668 0.306038 0.312780
## 2 0.285588 0.290104 0.295280 0.300710 0.307104 0.313954 0.321211 0.328771
## 3 0.272384 0.276854 0.281865 0.287514 0.293801 0.300286 0.307114 0.314285
## 4 0.293882 0.297983 0.302734 0.307886 0.313937 0.320389 0.327141 0.334243
## 5 0.305585 0.309691 0.314526 0.320035 0.326013 0.332718 0.339735 0.346961
## 6 0.302025 0.306654 0.311763 0.317563 0.324054 0.330918 0.338102 0.345711
##      X1148    X1150    X1152    X1154    X1156    X1158    X1160    X1162
## 1 0.319947 0.327432 0.335182 0.342880 0.350683 0.358434 0.366078 0.373404
## 2 0.336535 0.344451 0.352621 0.360842 0.369113 0.377131 0.384945 0.392455
## 3 0.321801 0.329612 0.337766 0.345969 0.354075 0.361983 0.369794 0.377359
## 4 0.341645 0.349297 0.357299 0.365352 0.373304 0.381206 0.388658 0.396110
## 5 0.354653 0.362554 0.370611 0.378564 0.386673 0.394625 0.402318 0.409595
## 6 0.353693 0.361940 0.370507 0.379128 0.387694 0.396155 0.404296 0.412225
##      X1164    X1166    X1168    X1170    X1172    X1174    X1176    X1178
## 1 0.380358 0.386888 0.392993 0.398673 0.403981 0.409024 0.414121 0.419588
## 2 0.399458 0.406055 0.412093 0.417827 0.423359 0.428636 0.434066 0.439546
## 3 0.384285 0.390818 0.396713 0.402509 0.408060 0.413513 0.418965 0.424516
## 4 0.403012 0.409514 0.415616 0.421367 0.426719 0.432020 0.437222 0.442723
## 5 0.416509 0.423110 0.429243 0.435065 0.440315 0.445461 0.450763 0.456325
```

```
## 6 0.419568 0.426485 0.432817 0.439043 0.444790 0.450430 0.456017 0.461870
##      X1180    X1182    X1184    X1186    X1188    X1190    X1192    X1194
## 1 0.425269 0.431161 0.437160 0.442734 0.448095 0.453192 0.457810 0.462216
## 2 0.445229 0.451014 0.456799 0.462432 0.467709 0.472631 0.477147 0.481258
## 3 0.430362 0.436207 0.442249 0.447849 0.453105 0.458116 0.462881 0.466958
## 4 0.448275 0.454076 0.459878 0.465579 0.470931 0.475782 0.480183 0.484185
## 5 0.462094 0.467864 0.473634 0.479091 0.484289 0.489071 0.493490 0.497440
## 6 0.467830 0.474109 0.480388 0.486454 0.492147 0.497469 0.502204 0.506355
##      X1196    X1198    X1200    X1202    X1204    X1206    X1208    X1210
## 1 0.466091 0.469807 0.473046 0.475647 0.477717 0.478885 0.479097 0.477823
## 2 0.484962 0.488413 0.491457 0.493994 0.495618 0.496430 0.496126 0.494451
## 3 0.470740 0.474179 0.477175 0.479533 0.481204 0.482088 0.481842 0.480172
## 4 0.487836 0.491137 0.493987 0.496288 0.497838 0.498689 0.498038 0.496488
## 5 0.501027 0.504353 0.507160 0.509239 0.510695 0.511370 0.510955 0.509343
## 6 0.510133 0.513325 0.515986 0.518274 0.519817 0.520722 0.520296 0.518487
##      X1212    X1214    X1216    X1218    X1220    X1222    X1224    X1226
## 1 0.475222 0.471241 0.465932 0.459350 0.451971 0.444114 0.436417 0.428613
## 2 0.491356 0.487093 0.481562 0.475067 0.467760 0.460097 0.452384 0.444925
## 3 0.477175 0.472853 0.467155 0.460523 0.453155 0.445442 0.437485 0.429920
## 4 0.493687 0.489686 0.484385 0.478233 0.471331 0.463979 0.456477 0.449275
## 5 0.506380 0.502430 0.497180 0.490891 0.483821 0.476544 0.469475 0.462510
## 6 0.515347 0.510878 0.505291 0.498426 0.491136 0.483368 0.475492 0.467883
##      X1228    X1230    X1232    X1234    X1236    X1238    X1240    X1242
## 1 0.421128 0.414068 0.407697 0.402017 0.396868 0.392196 0.388321 0.384817
## 2 0.437973 0.431224 0.425236 0.419705 0.414732 0.410267 0.406461 0.403010
## 3 0.422748 0.416067 0.409927 0.404376 0.399513 0.395239 0.391702 0.388411
## 4 0.442323 0.435871 0.430020 0.424818 0.419967 0.415966 0.412515 0.409364
## 5 0.455753 0.449411 0.443746 0.438548 0.433922 0.429763 0.426281 0.423058
## 6 0.460753 0.454048 0.448142 0.442608 0.437819 0.433243 0.429518 0.426219
##      X1244    X1246    X1248    X1250    X1252    X1254    X1256    X1258
## 1 0.381685 0.378766 0.376058 0.373722 0.371705 0.369741 0.368042 0.366503
## 2 0.399915 0.397073 0.394637 0.392252 0.390121 0.388244 0.386468 0.384894
## 3 0.385366 0.382615 0.380208 0.377899 0.375787 0.374018 0.372299 0.370776
## 4 0.406463 0.403862 0.401412 0.399211 0.397210 0.395410 0.393759 0.392359
## 5 0.420199 0.417548 0.415105 0.412922 0.410947 0.409335 0.407724 0.406269
## 6 0.423293 0.420579 0.418025 0.415790 0.413608 0.411799 0.410043 0.408553
##      X1260    X1262    X1264    X1266    X1268    X1270    X1272    X1274
## 1 0.365123 0.363742 0.362575 0.361354 0.360239 0.359177 0.358168 0.357107
## 2 0.383474 0.382154 0.380886 0.379668 0.378551 0.377486 0.376471 0.375304
## 3 0.369401 0.368124 0.366994 0.366061 0.364980 0.363997 0.363015 0.361934
## 4 0.391009 0.389708 0.388608 0.387508 0.386457 0.385507 0.384557 0.383507
## 5 0.405177 0.403930 0.402682 0.401591 0.400499 0.399511 0.398472 0.397328
## 6 0.407276 0.405999 0.404882 0.403764 0.402913 0.402221 0.401104 0.399986
##      X1276    X1278    X1280    X1282    X1284    X1286    X1288    X1290
## 1 0.355992 0.354771 0.353603 0.352488 0.351427 0.350471 0.349569 0.348719
## 2 0.374137 0.373122 0.372056 0.371092 0.370229 0.369418 0.368758 0.368098
## 3 0.360755 0.359625 0.358496 0.357317 0.356187 0.355057 0.354124 0.353240
## 4 0.382356 0.381156 0.380006 0.378955 0.377955 0.376905 0.376055 0.375204
## 5 0.396237 0.395197 0.394158 0.393222 0.392338 0.391455 0.390727 0.389947
## 6 0.398816 0.397538 0.396315 0.395091 0.393973 0.392909 0.391951 0.391047
##      X1292    X1294    X1296    X1298    X1300    X1302    X1304    X1306
## 1 0.347870 0.347286 0.346861 0.346596 0.346596 0.346808 0.347180 0.347711
## 2 0.367591 0.367337 0.367185 0.367286 0.367591 0.368047 0.368809 0.369773
## 3 0.352454 0.351815 0.351422 0.351275 0.351176 0.351373 0.351815 0.352454
```

```
## 4 0.374454 0.374004 0.373604 0.373504 0.373654 0.374004 0.374554 0.375254
## 5 0.389375 0.389064 0.389012 0.389220 0.389635 0.389895 0.390519 0.391403
## 6 0.390302 0.389716 0.389238 0.389131 0.389291 0.389610 0.390302 0.391100
##      X1308    X1310    X1312    X1314    X1316    X1318    X1320    X1322
## 1 0.348507 0.349569 0.350896 0.352435 0.354399 0.356735 0.359177 0.361937
## 2 0.370940 0.372513 0.374390 0.376623 0.379160 0.382053 0.385199 0.388548
## 3 0.353338 0.354615 0.356236 0.358103 0.360264 0.362671 0.365422 0.368468
## 4 0.376355 0.377655 0.379305 0.381356 0.383607 0.386207 0.389058 0.392209
## 5 0.392598 0.394313 0.396185 0.398472 0.400967 0.403826 0.406944 0.410375
## 6 0.392271 0.393707 0.395570 0.397698 0.400359 0.403232 0.406212 0.409564
##      X1324    X1326    X1328    X1330    X1332    X1334    X1336    X1338
## 1 0.365016 0.368414 0.372289 0.376430 0.380942 0.385507 0.390338 0.395328
## 2 0.392303 0.396413 0.400879 0.405700 0.410825 0.416153 0.421633 0.427114
## 3 0.371759 0.375541 0.379618 0.384039 0.388853 0.393962 0.399218 0.404671
## 4 0.395560 0.399411 0.403662 0.408364 0.413215 0.418416 0.423868 0.429669
## 5 0.414429 0.418692 0.423214 0.428100 0.433350 0.438860 0.444525 0.450347
## 6 0.413395 0.417705 0.422335 0.427337 0.432764 0.438511 0.444098 0.450004
##      X1340    X1342    X1344    X1346    X1348    X1350    X1352    X1354
## 1 0.400637 0.406158 0.411891 0.417677 0.423623 0.429834 0.435992 0.442097
## 2 0.432898 0.438886 0.445229 0.451471 0.457662 0.464106 0.470500 0.476995
## 3 0.410172 0.416165 0.422404 0.428741 0.435372 0.441955 0.448832 0.455611
## 4 0.435371 0.441373 0.447624 0.453876 0.460178 0.466630 0.473132 0.479883
## 5 0.456065 0.462250 0.468592 0.475089 0.481638 0.488240 0.494841 0.501598
## 6 0.456283 0.462881 0.469639 0.476503 0.483580 0.490870 0.498107 0.505078
##      X1356    X1358    X1360    X1362    X1364    X1366    X1368    X1370
## 1 0.448255 0.454519 0.460730 0.466782 0.472727 0.478726 0.484778 0.491095
## 2 0.483135 0.489529 0.495821 0.502063 0.508355 0.514495 0.520990 0.527638
## 3 0.462046 0.468333 0.474719 0.480958 0.487246 0.493681 0.499968 0.506649
## 4 0.486385 0.492887 0.499339 0.505741 0.512092 0.518294 0.524496 0.531148
## 5 0.508096 0.514593 0.520987 0.527432 0.533929 0.540427 0.546872 0.553525
## 6 0.512261 0.519285 0.526362 0.533226 0.540197 0.547167 0.554085 0.561162
##      X1372    X1374    X1376    X1378    X1380    X1382    X1384    X1386
## 1 0.497571 0.504366 0.511533 0.519336 0.528042 0.537438 0.547684 0.558301
## 2 0.534691 0.542151 0.550270 0.559048 0.568944 0.579904 0.591728 0.604363
## 3 0.513624 0.520796 0.528263 0.536368 0.545063 0.554838 0.565449 0.576845
## 4 0.537900 0.544951 0.552354 0.560506 0.569508 0.579511 0.590564 0.602017
## 5 0.560595 0.567976 0.575876 0.584765 0.594485 0.605297 0.617408 0.630091
## 6 0.568345 0.575529 0.583457 0.592131 0.601815 0.612404 0.623951 0.636455
##      X1388    X1390    X1392    X1394    X1396    X1398    X1400    X1402
## 1 0.569714 0.581712 0.594187 0.607511 0.621526 0.636178 0.652209 0.669356
## 2 0.617810 0.631714 0.646582 0.662516 0.679312 0.697276 0.716152 0.736450
## 3 0.588880 0.601603 0.614817 0.628915 0.644044 0.660058 0.677497 0.695770
## 4 0.614421 0.627624 0.641528 0.656332 0.672037 0.688741 0.707047 0.726202
## 5 0.643605 0.657899 0.672921 0.689399 0.706708 0.725212 0.744913 0.765496
## 6 0.649598 0.663486 0.678439 0.694349 0.711217 0.729202 0.748677 0.769323
##      X1404    X1406    X1408    X1410    X1412    X1414    X1416    X1418
## 1 0.687193 0.705507 0.724034 0.742720 0.761353 0.779774 0.797505 0.815288
## 2 0.757103 0.778213 0.799068 0.820026 0.840526 0.860672 0.879955 0.898730
## 3 0.714928 0.734528 0.754275 0.774022 0.793573 0.812927 0.831987 0.850654
## 4 0.746107 0.766163 0.786619 0.806974 0.827230 0.846885 0.866391 0.885496
## 5 0.786704 0.808223 0.829534 0.851158 0.872365 0.893209 0.913013 0.932713
## 6 0.790660 0.812211 0.834027 0.855791 0.877288 0.898146 0.918845 0.939119
##      X1420    X1422    X1424    X1426    X1428    X1430    X1432    X1434
## 1 0.832647 0.849582 0.865985 0.881858 0.897306 0.911586 0.924698 0.936111
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## 2 0.916846 0.934200 0.950438 0.965763 0.980225 0.993317 1.005390 1.016150
## 3 0.868976 0.887102 0.904983 0.922618 0.939614 0.955530 0.969726 0.981761
## 4 0.904151 0.922006 0.939761 0.956866 0.973521 0.989025 1.002630 1.014580
## 5 0.951685 0.969878 0.986875 1.003400 1.018790 1.033080 1.046080 1.057410
## 6 0.959073 0.978388 0.997544 1.016220 1.034370 1.051130 1.066400 1.079170
##      X1436    X1438   X1440    X1442    X1444    X1446    X1448    X1450
## 1 0.945295 0.952408 0.95777 0.961698 0.964618 0.966741 0.968599 0.970245
## 2 1.025440 1.033400 1.04036 1.046190 1.051010 1.054920 1.058120 1.060710
## 3 0.991291 0.998021 1.00234 1.005090 1.007010 1.008480 1.009560 1.010650
## 4 1.024330 1.031590 1.03694 1.040790 1.043690 1.045790 1.047540 1.049090
## 5 1.067230 1.075240 1.08179 1.087140 1.091350 1.094630 1.097330 1.099510
## 6 1.089280 1.096300 1.10088 1.103910 1.105830 1.107160 1.108330 1.109390
##      X1452    X1454    X1456    X1458    X1460    X1462    X1464    X1466
## 1 0.971732 0.973112 0.974386 0.975288 0.976031 0.976403 0.976615 0.976562
## 2 1.062940 1.064660 1.066080 1.067250 1.068110 1.068320 1.068110 1.067560
## 3 1.011730 1.012860 1.013940 1.014920 1.015800 1.016440 1.016690 1.016590
## 4 1.050540 1.051890 1.052990 1.054040 1.054840 1.055340 1.055590 1.055490
## 5 1.101590 1.103150 1.104500 1.105490 1.106220 1.106480 1.106270 1.105750
## 6 1.110560 1.111630 1.112590 1.113380 1.114130 1.114710 1.114980 1.114870
##      X1468    X1470    X1472    X1474    X1476    X1478    X1480    X1482
## 1 0.976191 0.975288 0.973961 0.972262 0.970139 0.967909 0.965467 0.962813
## 2 1.066640 1.065070 1.063240 1.060860 1.058220 1.055170 1.051930 1.048630
## 3 1.016150 1.015260 1.014180 1.012510 1.010550 1.008290 1.005880 1.003230
## 4 1.055040 1.053990 1.052740 1.050990 1.048940 1.046490 1.043890 1.041090
## 5 1.104820 1.103410 1.101540 1.099250 1.096600 1.093590 1.090310 1.086880
## 6 1.114400 1.113540 1.112210 1.110460 1.108330 1.106150 1.103700 1.100990
##     X1484    X1486    X1488    X1490    X1492    X1494    X1496    X1498
## 1 0.96000 0.957027 0.953895 0.950550 0.947418 0.944233 0.940942 0.937704
## 2 1.04518 1.041680 1.037820 1.033860 1.029650 1.025290 1.020820 1.016250
## 3 1.00048 0.997481 0.994435 0.991291 0.988049 0.984709 0.981368 0.977881
## 4 1.03809 1.034890 1.031490 1.027890 1.024180 1.020230 1.016280 1.012280
## 5 1.08345 1.079760 1.076020 1.072120 1.067810 1.063600 1.059230 1.055230
## 6 1.09806 1.094650 1.091250 1.087580 1.083640 1.079600 1.075340 1.070870
##      X1500    X1502    X1504    X1506    X1508    X1510    X1512    X1514
## 1 0.934306 0.930856 0.927193 0.923530 0.920026 0.916469 0.912753 0.909091
## 2 1.011580 1.006870 1.002150 0.997478 0.993216 0.988953 0.984843 0.980834
## 3 0.974295 0.970758 0.967270 0.963783 0.960049 0.956267 0.952730 0.949095
## 4 1.008130 1.004180 1.000030 0.995927 0.992026 0.988275 0.984674 0.981173
## 5 1.051020 1.046860 1.042800 1.038800 1.034900 1.030850 1.026900 1.023570
## 6 1.066290 1.061500 1.056820 1.052250 1.047880 1.043730 1.040010 1.036120
##      X1516    X1518    X1520    X1522    X1524    X1526    X1528    X1530
## 1 0.905640 0.902030 0.898420 0.894917 0.891732 0.888812 0.886317 0.883769
## 2 0.977383 0.973933 0.970685 0.967539 0.964545 0.961906 0.959369 0.957086
## 3 0.945558 0.941923 0.938681 0.935586 0.932590 0.929642 0.926842 0.924386
## 4 0.977972 0.974921 0.971920 0.969069 0.966369 0.963718 0.961317 0.959017
## 5 1.019880 1.016300 1.012920 1.009430 1.006210 1.003040 0.999922 0.997063
## 6 1.032560 1.029260 1.026230 1.023250 1.020480 1.017980 1.015640 1.013400
##      X1532    X1534    X1536    X1538    X1540    X1542    X1544    X1546
## 1 0.881380 0.879097 0.876868 0.875063 0.873470 0.872302 0.871187 0.869966
## 2 0.954904 0.952772 0.950793 0.948814 0.946886 0.945009 0.943182 0.941406
## 3 0.921930 0.919572 0.917313 0.915348 0.913432 0.911664 0.909993 0.908372
## 4 0.956616 0.954365 0.952015 0.950064 0.948114 0.946313 0.944513 0.942962
## 5 0.994204 0.991501 0.988902 0.986407 0.984120 0.982197 0.980430 0.978715
## 6 1.011220 1.009040 1.007070 1.005210 1.003500 1.001910 1.000260 0.998821
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##      X1548    X1550    X1552    X1554    X1556    X1558    X1560    X1562
## 1 0.868852 0.867790 0.866675 0.865773 0.864923 0.864127 0.863437 0.862481
## 2 0.939782 0.938107 0.936484 0.934911 0.933287 0.931815 0.930344 0.928973
## 3 0.907095 0.905916 0.904688 0.903558 0.902576 0.901741 0.900808 0.899972
## 4 0.941462 0.939911 0.938411 0.936860 0.935660 0.934360 0.933159 0.932059
## 5 0.977103 0.975596 0.974192 0.972789 0.971489 0.970190 0.968735 0.967435
## 6 0.997544 0.996267 0.995203 0.994458 0.993660 0.993075 0.992170 0.991425
##      X1564    X1566    X1568    X1570    X1572    X1574    X1576    X1578
## 1 0.861420 0.860305 0.859031 0.857598 0.856164 0.854519 0.852714 0.850431
## 2 0.927350 0.925726 0.924051 0.922377 0.920296 0.918216 0.915831 0.913040
## 3 0.898941 0.897909 0.896681 0.895306 0.893783 0.892113 0.890295 0.888134
## 4 0.930659 0.929258 0.927808 0.926207 0.924457 0.922556 0.920406 0.918105
## 5 0.966136 0.964576 0.963069 0.961353 0.959586 0.957455 0.955324 0.952933
## 6 0.990574 0.989403 0.988179 0.986796 0.985412 0.983710 0.981688 0.979399
##      X1580    X1582    X1584    X1586    X1588    X1590    X1592    X1594
## 1 0.847936 0.845388 0.842734 0.839761 0.836682 0.833391 0.829568 0.825481
## 2 0.910300 0.907559 0.904464 0.901216 0.897817 0.894264 0.890560 0.886754
## 3 0.885973 0.883369 0.880569 0.877376 0.873839 0.870204 0.866422 0.862345
## 4 0.915655 0.913104 0.910203 0.907052 0.903651 0.900100 0.896199 0.892098
## 5 0.950334 0.947371 0.944356 0.941238 0.937755 0.934116 0.930322 0.926216
## 6 0.977005 0.974185 0.971152 0.967853 0.964128 0.960084 0.955827 0.951251
##      X1596    X1598    X1600    X1602    X1604    X1606    X1608    X1610
## 1 0.821340 0.817199 0.813112 0.808865 0.804512 0.799894 0.795275 0.790657
## 2 0.882695 0.878432 0.874119 0.869856 0.865391 0.860926 0.856460 0.851893
## 3 0.857875 0.853257 0.848590 0.843924 0.839208 0.834394 0.829580 0.824766
## 4 0.887747 0.883296 0.878544 0.873893 0.869242 0.864790 0.859939 0.855138
## 5 0.922109 0.917743 0.913325 0.908751 0.904280 0.899602 0.894976 0.890350
## 6 0.946462 0.941407 0.936299 0.931244 0.925976 0.920867 0.915759 0.910438
##      X1612    X1614    X1616    X1618    X1620    X1622    X1624    X1626
## 1 0.786197 0.781738 0.777385 0.773191 0.769051 0.764751 0.760451 0.756363
## 2 0.847377 0.842810 0.838395 0.834082 0.829972 0.825811 0.821599 0.817387
## 3 0.819854 0.815089 0.810570 0.806050 0.801580 0.797061 0.792591 0.788121
## 4 0.850336 0.845535 0.840884 0.836382 0.831931 0.827580 0.822979 0.818678
## 5 0.885568 0.880682 0.875952 0.871482 0.867011 0.862801 0.858435 0.854173
## 6 0.905223 0.900009 0.894901 0.889899 0.885057 0.880161 0.875319 0.870636
##      X1628    X1630    X1632    X1634    X1636    X1638    X1640    X1642
## 1 0.752488 0.748772 0.745162 0.741659 0.738261 0.734970 0.731785 0.728759
## 2 0.813378 0.809319 0.805462 0.801707 0.798358 0.794958 0.791710 0.788514
## 3 0.783798 0.779721 0.775791 0.772008 0.768521 0.765230 0.761889 0.758647
## 4 0.814326 0.810275 0.806224 0.802473 0.799072 0.795621 0.792270 0.788969
## 5 0.849754 0.845596 0.841594 0.837747 0.834005 0.830522 0.827091 0.823817
## 6 0.865901 0.861431 0.857014 0.853077 0.849192 0.845468 0.841956 0.838497
##      X1644    X1646    X1648    X1650    X1652    X1654    X1656    X1658
## 1 0.726051 0.723503 0.721115 0.718938 0.716868 0.715169 0.713683 0.712621
## 2 0.785469 0.782627 0.780090 0.777654 0.775472 0.773646 0.772022 0.770702
## 3 0.755749 0.753047 0.750493 0.748135 0.746023 0.744205 0.742584 0.741258
## 4 0.785968 0.783068 0.780317 0.777966 0.775716 0.773765 0.772015 0.770614
## 5 0.820594 0.817579 0.814772 0.812173 0.809990 0.808119 0.806352 0.804844
## 6 0.835092 0.831952 0.829079 0.826365 0.824024 0.821948 0.820086 0.818490
##      X1660    X1662    X1664    X1666    X1668    X1670    X1672    X1674
## 1 0.712090 0.711878 0.711931 0.712408 0.713417 0.715169 0.717823 0.721592
## 2 0.769840 0.769434 0.769383 0.769789 0.770855 0.772478 0.774914 0.778213
## 3 0.740472 0.740030 0.739931 0.740374 0.741553 0.743468 0.746317 0.750296
## 4 0.769514 0.768814 0.768664 0.768764 0.769964 0.771664 0.774215 0.777666
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## 5 0.803805 0.803025 0.802765 0.802973 0.803753 0.805208 0.807443 0.810770
## 6 0.817319 0.816574 0.816308 0.816574 0.817585 0.819341 0.822108 0.826046
##      X1676    X1678    X1680    X1682    X1684    X1686    X1688    X1690
## 1 0.726582 0.732581 0.739907 0.748241 0.757425 0.766821 0.776483 0.786197
## 2 0.782678 0.788158 0.794907 0.802823 0.811348 0.820635 0.830327 0.840222
## 3 0.755552 0.762184 0.770240 0.779426 0.788907 0.798387 0.807819 0.817005
## 4 0.782317 0.788369 0.795621 0.803923 0.812876 0.822028 0.831131 0.839933
## 5 0.815032 0.820594 0.827195 0.834992 0.843309 0.852301 0.861398 0.870494
## 6 0.831154 0.837805 0.845734 0.854833 0.864517 0.874148 0.883460 0.892453
##      X1692    X1694    X1696    X1698    X1700    X1702    X1704    X1706
## 1 0.795806 0.805468 0.815182 0.825215 0.835249 0.845122 0.854678 0.863702
## 2 0.850066 0.860063 0.870009 0.879802 0.889494 0.898984 0.907762 0.915831
## 3 0.825798 0.834590 0.843629 0.852962 0.862345 0.871678 0.880667 0.889116
## 4 0.848486 0.857088 0.865791 0.874693 0.883696 0.892448 0.900800 0.908603
## 5 0.879486 0.888479 0.897471 0.906464 0.915092 0.923721 0.931725 0.939106
## 6 0.900913 0.909427 0.918101 0.927146 0.936245 0.945185 0.953911 0.961840
##      X1708    X1710    X1712    X1714    X1716    X1718    X1720    X1722
## 1 0.872037 0.879840 0.887485 0.894917 0.902349 0.909568 0.916363 0.921778
## 2 0.923341 0.930597 0.937448 0.944197 0.950743 0.956984 0.962921 0.967742
## 3 0.897172 0.904688 0.912155 0.919720 0.927039 0.934063 0.940597 0.945656
## 4 0.915855 0.922606 0.929408 0.936160 0.942912 0.949464 0.955516 0.960467
## 5 0.945916 0.952361 0.958703 0.964836 0.971074 0.977051 0.982717 0.987395
## 6 0.969343 0.976473 0.983710 0.990840 0.997917 1.004830 1.011110 1.016060
##      X1724    X1726    X1728    X1730    X1732    X1734    X1736    X1738
## 1 0.925335 0.926450 0.924804 0.920663 0.914824 0.907870 0.900756 0.894598
## 2 0.970990 0.972055 0.970736 0.967133 0.962109 0.956426 0.950641 0.945770
## 3 0.948849 0.949586 0.947818 0.943495 0.937649 0.931018 0.924337 0.918590
## 4 0.963668 0.964668 0.963468 0.959967 0.954965 0.949164 0.943462 0.938561
## 5 0.990566 0.991813 0.990930 0.988019 0.983705 0.978663 0.973673 0.969410
## 6 1.019200 1.020000 1.018400 1.014520 1.009250 1.003240 0.997332 0.992383
##      X1740    X1742    X1744    X1746    X1748    X1750    X1752    X1754
## 1 0.889714 0.886370 0.884884 0.885149 0.886795 0.889661 0.893377 0.897252
## 2 0.942167 0.940036 0.939680 0.940695 0.942979 0.946023 0.949677 0.953432
## 3 0.914414 0.911860 0.910976 0.911565 0.913432 0.916428 0.920211 0.923944
## 4 0.934810 0.932659 0.932109 0.932959 0.934910 0.937961 0.941412 0.945063
## 5 0.966447 0.964732 0.964680 0.965824 0.967955 0.971074 0.974504 0.978091
## 6 0.988711 0.986689 0.986264 0.987328 0.989616 0.992809 0.996587 1.000360
##      X1756    X1758    X1760    X1762    X1764    X1766    X1768    X1770
## 1 0.900597 0.902986 0.904047 0.903782 0.902349 0.899588 0.895872 0.891785
## 2 0.956730 0.959166 0.960283 0.960181 0.959014 0.956933 0.954041 0.950692
## 3 0.927285 0.929642 0.930527 0.929986 0.928218 0.925320 0.921635 0.917657
## 4 0.948214 0.950764 0.951715 0.951615 0.950264 0.947913 0.944813 0.941262
## 5 0.981313 0.983705 0.985004 0.985056 0.984120 0.982197 0.979702 0.976739
## 6 1.003770 1.006060 1.007120 1.006750 1.005260 1.002870 0.999513 0.995895
##      X1772    X1774    X1776    X1778    X1780    X1782    X1784    X1786
## 1 0.887379 0.883132 0.879097 0.875275 0.871506 0.868055 0.864658 0.861207
## 2 0.947241 0.943740 0.940391 0.936991 0.933591 0.930344 0.927147 0.923798
## 3 0.913678 0.909600 0.905769 0.901986 0.898302 0.894815 0.891474 0.888232
## 4 0.937661 0.934060 0.930659 0.927208 0.923857 0.920606 0.917305 0.914104
## 5 0.973569 0.970502 0.967331 0.964160 0.961042 0.957975 0.955064 0.951997
## 6 0.992064 0.988445 0.984774 0.981209 0.977803 0.974238 0.970886 0.967533
##      X1788    X1790    X1792    X1794    X1796    X1798    X1800    X1802
## 1 0.857757 0.854412 0.851174 0.848042 0.844910 0.841725 0.838381 0.835142
## 2 0.920448 0.916998 0.913649 0.910350 0.906900 0.903398 0.900049 0.896852
```

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## 3 0.884941 0.881650 0.878359 0.875067 0.871727 0.868387 0.865046 0.861804
## 4 0.910903 0.907652 0.904401 0.901050 0.897750 0.894549 0.891198 0.887997
## 5 0.948723 0.945292 0.941809 0.938379 0.934948 0.931517 0.928139 0.924812
## 6 0.964181 0.960776 0.957423 0.953911 0.950453 0.946834 0.943322 0.939811
##      X1804    X1806    X1808    X1810    X1812    X1814    X1816    X1818
## 1 0.832063 0.829144 0.826596 0.824207 0.821977 0.819694 0.817306 0.814811
## 2 0.893858 0.890865 0.888124 0.885486 0.883050 0.880614 0.878077 0.875438
## 3 0.858710 0.855910 0.853208 0.850752 0.848590 0.846478 0.844169 0.841713
## 4 0.884796 0.881895 0.879194 0.876694 0.874193 0.871992 0.869642 0.867291
## 5 0.921745 0.918731 0.915976 0.913377 0.910934 0.908439 0.906048 0.903449
## 6 0.936511 0.933319 0.930552 0.928157 0.925710 0.923528 0.921187 0.918739
##      X1820    X1822    X1824    X1826    X1828    X1830    X1832    X1834
## 1 0.812262 0.809767 0.806954 0.803928 0.800690 0.797345 0.793948 0.790710
## 2 0.872647 0.869958 0.867167 0.864122 0.861027 0.857881 0.854684 0.851639
## 3 0.839159 0.836359 0.833412 0.830268 0.826878 0.823489 0.819854 0.816513
## 4 0.864690 0.861940 0.858989 0.855838 0.852537 0.849186 0.845785 0.842534
## 5 0.900954 0.898043 0.895236 0.892065 0.888999 0.885724 0.882553 0.879330
## 6 0.915972 0.913152 0.909959 0.906501 0.902989 0.899264 0.895805 0.892400
##      X1836    X1838    X1840    X1842    X1844    X1846    X1848    X1850
## 1 0.787578 0.784658 0.781951 0.779349 0.777120 0.775262 0.773935 0.773245
## 2 0.848645 0.845854 0.843165 0.840882 0.839004 0.837634 0.836873 0.836771
## 3 0.813222 0.809980 0.807033 0.804478 0.802415 0.800794 0.799615 0.799124
## 4 0.839433 0.836533 0.833832 0.831381 0.829331 0.827830 0.826830 0.826580
## 5 0.876368 0.873561 0.871066 0.868779 0.866959 0.865660 0.864880 0.864880
## 6 0.888994 0.885855 0.883035 0.880480 0.878405 0.876862 0.875904 0.875745
##      X1852    X1854    X1856    X1858    X1860    X1862    X1864    X1866
## 1 0.773510 0.774306 0.775740 0.777969 0.780889 0.784764 0.789860 0.796284
## 2 0.837583 0.839106 0.841490 0.844637 0.848798 0.854024 0.860520 0.868639
## 3 0.799468 0.800549 0.802464 0.805215 0.808850 0.813271 0.819068 0.826141
## 4 0.827130 0.828330 0.830431 0.833182 0.836883 0.841634 0.847786 0.855338
## 5 0.865660 0.867323 0.869662 0.873145 0.877511 0.882917 0.889518 0.897835
## 6 0.876436 0.877980 0.880214 0.883354 0.887398 0.892666 0.899264 0.907245
##      X1868    X1870    X1872    X1874    X1876    X1878    X1880    X1882
## 1 0.804193 0.813430 0.824313 0.837000 0.851493 0.867843 0.886264 0.906967
## 2 0.878534 0.890154 0.903652 0.919078 0.936534 0.955919 0.977231 1.001180
## 3 0.834640 0.844710 0.856548 0.870253 0.885874 0.903755 0.923502 0.945656
## 4 0.864540 0.875293 0.887897 0.902251 0.918605 0.936710 0.956766 0.979572
## 5 0.908075 0.919874 0.933545 0.949242 0.966967 0.986667 1.008240 1.032410
## 6 0.917090 0.928849 0.942099 0.957477 0.975036 0.994139 1.015580 1.039310
##      X1884    X1886    X1888   X1890   X1892   X1894   X1896   X1898   X1900
## 1 0.929741 0.954160 0.979854 1.00730 1.03623 1.06665 1.09834 1.13046 1.16199
## 2 1.027210 1.055020 1.084250 1.11490 1.14677 1.17975 1.21355 1.24724 1.27997
## 3 0.969923 0.995859 1.023370 1.05191 1.08207 1.11331 1.14573 1.17791 1.20979
## 4 1.004280 1.030790 1.058890 1.08825 1.11906 1.15107 1.18393 1.21679 1.24895
## 5 1.058870 1.086830 1.116200 1.14697 1.17894 1.21210 1.24568 1.27962 1.31258
## 6 1.065440 1.093380 1.122540 1.15303 1.18490 1.21762 1.25104 1.28403 1.31633
##     X1902   X1904   X1906   X1908   X1910   X1912   X1914   X1916   X1918
## 1 1.19267 1.22176 1.24873 1.27400 1.29757 1.31960 1.33977 1.35809 1.37454
## 2 1.31148 1.34056 1.36745 1.39227 1.41490 1.43545 1.45418 1.47087 1.48523
## 3 1.24000 1.26859 1.29482 1.31933 1.34227 1.36320 1.38240 1.39950 1.41502
## 4 1.27976 1.30826 1.33477 1.35928 1.38193 1.40259 1.42135 1.43780 1.45245
## 5 1.34397 1.37287 1.39938 1.42360 1.44538 1.46633 1.48473 1.50105 1.51561
## 6 1.34719 1.37550 1.40189 1.42504 1.44744 1.46740 1.48538 1.50161 1.51571
##     X1920   X1922   X1924   X1926   X1928   X1930   X1932   X1934   X1936
```

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## 1 1.38861 1.40050 1.41075 1.41903 1.42535 1.42965 1.43209 1.43289 1.43225
## 2 1.49807 1.50893 1.51761 1.52471 1.53004 1.53349 1.53547 1.53598 1.53511
## 3 1.42794 1.43899 1.44857 1.45604 1.46125 1.46513 1.46709 1.46798 1.46689
## 4 1.46541 1.47621 1.48501 1.49236 1.49757 1.50132 1.50347 1.50412 1.50342
## 5 1.52756 1.53817 1.54653 1.55303 1.55807 1.56135 1.56312 1.56353 1.56249
## 6 1.52784 1.53811 1.54631 1.55285 1.55754 1.56062 1.56217 1.56291 1.56174
##     X1938   X1940   X1942   X1944   X1946   X1948   X1950   X1952   X1954
## 1 1.43044 1.42752 1.42370 1.41898 1.41340 1.40725 1.40072 1.39371 1.38660
## 2 1.53339 1.53075 1.52730 1.52293 1.51811 1.51284 1.50690 1.50071 1.49431
## 3 1.46493 1.46213 1.45830 1.45388 1.44872 1.44278 1.43619 1.42941 1.42244
## 4 1.50167 1.49892 1.49547 1.49066 1.48576 1.48006 1.47396 1.46751 1.46091
## 5 1.56057 1.55781 1.55444 1.55002 1.54529 1.53998 1.53422 1.52782 1.52138
## 6 1.55956 1.55652 1.55264 1.54854 1.54333 1.53806 1.53221 1.52614 1.51992
##     X1956   X1958   X1960   X1962   X1964   X1966   X1968   X1970   X1972
## 1 1.37938 1.37200 1.36419 1.35634 1.34837 1.34030 1.33245 1.32433 1.31642
## 2 1.48772 1.48092 1.47402 1.46676 1.45981 1.45260 1.44494 1.43692 1.42901
## 3 1.41532 1.40814 1.40092 1.39355 1.38624 1.37877 1.37111 1.36315 1.35534
## 4 1.45405 1.44695 1.43975 1.43245 1.42545 1.41799 1.41039 1.40249 1.39449
## 5 1.51467 1.50812 1.50116 1.49398 1.48686 1.47953 1.47215 1.46456 1.45708
## 6 1.51326 1.50677 1.49996 1.49304 1.48581 1.47868 1.47160 1.46383 1.45601
##     X1974   X1976   X1978   X1980   X1982   X1984   X1986   X1988   X1990
## 1 1.30808 1.29996 1.29189 1.28419 1.27628 1.26848 1.26084 1.25330 1.24597
## 2 1.42094 1.41297 1.40516 1.39714 1.38912 1.38105 1.37319 1.36527 1.35776
## 3 1.34758 1.33962 1.33166 1.32370 1.31599 1.30833 1.30101 1.29364 1.28642
## 4 1.38664 1.37913 1.37148 1.36378 1.35608 1.34847 1.34102 1.33342 1.32612
## 5 1.44928 1.44133 1.43364 1.42579 1.41789 1.40999 1.40229 1.39450 1.38753
## 6 1.44840 1.44042 1.43265 1.42499 1.41748 1.40988 1.40248 1.39508 1.38795
##     X1992   X1994   X1996   X1998   X2000   X2002   X2004   X2006   X2008
## 1 1.23896 1.23249 1.22644 1.22113 1.21640 1.21232 1.20876 1.20568 1.20287
## 2 1.35056 1.34350 1.33670 1.33061 1.32493 1.31970 1.31498 1.31082 1.30717
## 3 1.27964 1.27355 1.26790 1.26294 1.25857 1.25473 1.25198 1.24953 1.24702
## 4 1.31937 1.31322 1.30731 1.30176 1.29686 1.29276 1.28941 1.28636 1.28341
## 5 1.38020 1.37308 1.36643 1.36055 1.35510 1.35026 1.34605 1.34226 1.33888
## 6 1.38135 1.37539 1.37023 1.36528 1.36092 1.35746 1.35454 1.35198 1.34980
##     X2010   X2012   X2014   X2016   X2018   X2020   X2022   X2024   X2026
## 1 1.20064 1.19867 1.19719 1.19623 1.19581 1.19612 1.19719 1.19904 1.20154
## 2 1.30433 1.30179 1.29991 1.29854 1.29768 1.29763 1.29834 1.29971 1.30138
## 3 1.24501 1.24309 1.24147 1.24034 1.23985 1.24010 1.24137 1.24344 1.24648
## 4 1.28126 1.27936 1.27781 1.27660 1.27585 1.27610 1.27710 1.27896 1.28171
## 5 1.33638 1.33410 1.33243 1.33139 1.33082 1.33082 1.33155 1.33311 1.33519
## 6 1.34810 1.34624 1.34469 1.34384 1.34336 1.34352 1.34448 1.34629 1.34943
##     X2028   X2030   X2032   X2034   X2036   X2038   X2040   X2042   X2044
## 1 1.20488 1.20897 1.21359 1.21847 1.22405 1.22989 1.23631 1.24311 1.25006
## 2 1.30372 1.30656 1.30966 1.31306 1.31681 1.32112 1.32569 1.33046 1.33533
## 3 1.25012 1.25449 1.25940 1.26471 1.27080 1.27748 1.28475 1.29217 1.29978
## 4 1.28496 1.28866 1.29316 1.29806 1.30356 1.30956 1.31587 1.32247 1.32927
## 5 1.33799 1.34096 1.34428 1.34782 1.35234 1.35717 1.36227 1.36762 1.37287
## 6 1.35326 1.35757 1.36225 1.36768 1.37369 1.38056 1.38785 1.39524 1.40243
##     X2046   X2048   X2050   X2052   X2054   X2056   X2058   X2060   X2062
## 1 1.25675 1.26328 1.26949 1.27549 1.28117 1.28637 1.29147 1.29646 1.30102
## 2 1.33995 1.34447 1.34873 1.35294 1.35725 1.36060 1.36395 1.36720 1.37040
## 3 1.30710 1.31427 1.32061 1.32665 1.33250 1.33800 1.34311 1.34802 1.35327
## 4 1.33582 1.34177 1.34757 1.35328 1.35863 1.36368 1.36853 1.37288 1.37738
## 5 1.37802 1.38337 1.38831 1.39283 1.39704 1.40104 1.40479 1.40843 1.41175
```

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## 6 1.40961 1.41642 1.42291 1.42861 1.43430 1.43967 1.44489 1.44989 1.45457
##     X2064   X2066   X2068   X2070   X2072   X2074   X2076   X2078   X2080
## 1 1.30543 1.30946 1.31302 1.31636 1.31939 1.32231 1.32464 1.32656 1.32815
## 2 1.37349 1.37623 1.37857 1.38095 1.38313 1.38542 1.38765 1.38953 1.39146
## 3 1.35819 1.36246 1.36673 1.37027 1.37327 1.37602 1.37818 1.37980 1.38098
## 4 1.38168 1.38544 1.38884 1.39189 1.39464 1.39684 1.39874 1.40029 1.40179
## 5 1.41539 1.41825 1.42090 1.42355 1.42594 1.42823 1.43005 1.43192 1.43353
## 6 1.45894 1.46346 1.46702 1.47027 1.47288 1.47490 1.47655 1.47772 1.47894
##     X2082   X2084   X2086   X2088   X2090   X2092   X2094   X2096   X2098
## 1 1.32942 1.33011 1.33027 1.33017 1.32974 1.32889 1.32762 1.32624 1.32480
## 2 1.39288 1.39379 1.39465 1.39501 1.39445 1.39359 1.39267 1.39206 1.39146
## 3 1.38157 1.38137 1.38113 1.38019 1.37887 1.37715 1.37513 1.37278 1.37061
## 4 1.40269 1.40319 1.40324 1.40284 1.40169 1.40014 1.39849 1.39659 1.39484
## 5 1.43473 1.43535 1.43613 1.43597 1.43540 1.43468 1.43348 1.43260 1.43135
## 6 1.47921 1.47889 1.47815 1.47687 1.47538 1.47314 1.47160 1.46931 1.46708
##     X2100   X2102   X2104   X2106   X2108   X2110   X2112   X2114   X2116
## 1 1.32380 1.32273 1.32141 1.31997 1.31849 1.31652 1.31466 1.31222 1.30946
## 2 1.39115 1.39095 1.39069 1.39014 1.38917 1.38785 1.38648 1.38415 1.38126
## 3 1.36850 1.36624 1.36403 1.36197 1.35976 1.35755 1.35548 1.35288 1.34984
## 4 1.39329 1.39169 1.39039 1.38859 1.38679 1.38483 1.38258 1.38008 1.37683
## 5 1.43088 1.43093 1.42995 1.42937 1.42823 1.42657 1.42470 1.42225 1.41945
## 6 1.46527 1.46325 1.46165 1.45957 1.45745 1.45505 1.45276 1.44989 1.44654
##     X2118   X2120   X2122   X2124   X2126   X2128   X2130   X2132   X2134
## 1 1.30606 1.30192 1.29715 1.29237 1.28791 1.28430 1.28138 1.27819 1.27517
## 2 1.37801 1.37349 1.36796 1.36248 1.35751 1.35355 1.35056 1.34756 1.34437
## 3 1.34625 1.34188 1.33711 1.33191 1.32714 1.32302 1.31943 1.31604 1.31270
## 4 1.37308 1.36863 1.36348 1.35803 1.35318 1.34943 1.34627 1.34302 1.33952
## 5 1.41534 1.41066 1.40526 1.39964 1.39486 1.39049 1.38717 1.38420 1.38088
## 6 1.44265 1.43744 1.43175 1.42547 1.41967 1.41525 1.41190 1.40870 1.40514
##     X2136   X2138   X2140   X2142   X2144   X2146   X2148   X2150   X2152
## 1 1.27214 1.26933 1.26662 1.26460 1.26285 1.26078 1.25802 1.25441 1.25022
## 2 1.34081 1.33792 1.33559 1.33376 1.33214 1.33071 1.32853 1.32544 1.32143
## 3 1.30956 1.30676 1.30425 1.30263 1.30111 1.29949 1.29737 1.29418 1.28996
## 4 1.33627 1.33332 1.33077 1.32897 1.32752 1.32582 1.32362 1.32047 1.31632
## 5 1.37745 1.37407 1.37142 1.36908 1.36762 1.36601 1.36393 1.36128 1.35764
## 6 1.40173 1.39870 1.39625 1.39492 1.39381 1.39285 1.39099 1.38875 1.38481
##     X2154   X2156   X2158   X2160   X2162   X2164   X2166   X2168   X2170
## 1 1.24602 1.24204 1.23843 1.23520 1.23185 1.22819 1.22431 1.22049 1.21640
## 2 1.31737 1.31392 1.31087 1.30813 1.30570 1.30270 1.29930 1.29590 1.29195
## 3 1.28524 1.28111 1.27733 1.27365 1.27016 1.26633 1.26220 1.25857 1.25464
## 4 1.31186 1.30791 1.30451 1.30156 1.29841 1.29476 1.29101 1.28711 1.28301
## 5 1.35364 1.34990 1.34714 1.34460 1.34189 1.33903 1.33560 1.33238 1.32838
## 6 1.38088 1.37688 1.37375 1.37061 1.36752 1.36401 1.36034 1.35656 1.35235
##     X2172   X2174   X2176   X2178   X2180   X2182   X2184   X2186   X2188
## 1 1.21226 1.20812 1.20430 1.20106 1.19926 1.19830 1.19750 1.19623 1.19469
## 2 1.28804 1.28368 1.27941 1.27551 1.27297 1.27165 1.27089 1.26972 1.26855
## 3 1.25031 1.24599 1.24221 1.23897 1.23681 1.23553 1.23494 1.23415 1.23243
## 4 1.27911 1.27470 1.27050 1.26725 1.26535 1.26445 1.26390 1.26300 1.26165
## 5 1.32365 1.31913 1.31460 1.31081 1.30847 1.30733 1.30655 1.30603 1.30473
## 6 1.34794 1.34357 1.33932 1.33613 1.33437 1.33352 1.33384 1.33341 1.33219
##     X2190   X2192   X2194   X2196   X2198   X2200   X2202   X2204   X2206
## 1 1.19230 1.18917 1.18577 1.18195 1.17813 1.17431 1.17032 1.16730 1.16507
## 2 1.26678 1.26414 1.26109 1.25764 1.25389 1.24983 1.24592 1.24252 1.23968
## 3 1.22993 1.22703 1.22335 1.21892 1.21480 1.21028 1.20610 1.20266 1.19982
```

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## 4 1.25930 1.25640 1.25285 1.24870 1.24470 1.24024 1.23614 1.23274 1.22999
## 5 1.30249 1.30021 1.29662 1.29334 1.28934 1.28539 1.28139 1.27775 1.27510
## 6 1.33033 1.32719 1.32383 1.31942 1.31537 1.31069 1.30627 1.30271 1.30031
##     X2208   X2210   X2212   X2214   X2216   X2218   X2220   X2222   X2224
## 1 1.16358 1.16252 1.16157 1.16077 1.15971 1.15881 1.15827 1.15817 1.15849
## 2 1.23801 1.23684 1.23562 1.23481 1.23384 1.23268 1.23161 1.23105 1.23085
## 3 1.19824 1.19731 1.19657 1.19613 1.19539 1.19461 1.19412 1.19407 1.19407
## 4 1.22859 1.22759 1.22649 1.22584 1.22474 1.22369 1.22294 1.22244 1.22219
## 5 1.27344 1.27245 1.27146 1.27063 1.26943 1.26839 1.26709 1.26621 1.26600
## 6 1.29877 1.29851 1.29781 1.29749 1.29670 1.29584 1.29515 1.29489 1.29473
##     X2226   X2228   X2230   X2232   X2234   X2236   X2238   X2240   X2242
## 1 1.15960 1.16157 1.16454 1.16889 1.17494 1.18222 1.19071 1.20048 1.21163
## 2 1.23141 1.23293 1.23532 1.23882 1.24379 1.24998 1.25688 1.26470 1.27424
## 3 1.19490 1.19682 1.20006 1.20458 1.21067 1.21799 1.22688 1.23685 1.24776
## 4 1.22319 1.22529 1.22824 1.23239 1.23809 1.24510 1.25320 1.26215 1.27280
## 5 1.26647 1.26767 1.27006 1.27354 1.27853 1.28451 1.29147 1.29958 1.30904
## 6 1.29579 1.29755 1.30069 1.30526 1.31144 1.31915 1.32788 1.33788 1.34900
##     X2244   X2246   X2248   X2250   X2252   X2254   X2256   X2258   X2260
## 1 1.22341 1.23589 1.24857 1.26184 1.27549 1.28950 1.30357 1.31721 1.33112
## 2 1.28489 1.29590 1.30788 1.32046 1.33381 1.34711 1.36040 1.37349 1.38648
## 3 1.25945 1.27104 1.28298 1.29521 1.30764 1.31987 1.33191 1.34409 1.35612
## 4 1.28356 1.29491 1.30626 1.31827 1.33072 1.34237 1.35488 1.36738 1.37938
## 5 1.31933 1.33004 1.34137 1.35359 1.36663 1.37968 1.39231 1.40463 1.41716
## 6 1.36087 1.37327 1.38540 1.39742 1.40988 1.42233 1.43387 1.44627 1.45777
##     X2262   X2264   X2266   X2268   X2270   X2272   X2274   X2276   X2278
## 1 1.34455 1.35772 1.37109 1.38479 1.39753 1.40948 1.42030 1.42991 1.43862
## 2 1.39957 1.41170 1.42287 1.43408 1.44489 1.45544 1.46488 1.47346 1.48087
## 3 1.36821 1.38044 1.39213 1.40333 1.41443 1.42450 1.43349 1.44111 1.44793
## 4 1.39139 1.40249 1.41344 1.42455 1.43600 1.44580 1.45475 1.46266 1.47001
## 5 1.42932 1.44112 1.45230 1.46363 1.47501 1.48520 1.49430 1.50199 1.50921
## 6 1.46952 1.48160 1.49272 1.50310 1.51390 1.52353 1.53173 1.53923 1.54594
##     X2280   X2282   X2284   X2286   X2288   X2290   X2292   X2294   X2296
## 1 1.44664 1.45423 1.46107 1.46867 1.47652 1.48613 1.49680 1.50859 1.52074
## 2 1.48736 1.49330 1.49868 1.50436 1.50999 1.51695 1.52527 1.53379 1.54272
## 3 1.45447 1.45962 1.46488 1.47048 1.47628 1.48384 1.49190 1.50054 1.51086
## 4 1.47656 1.48106 1.48571 1.49151 1.49607 1.50312 1.51072 1.51817 1.52697
## 5 1.51503 1.51987 1.52403 1.52829 1.53292 1.53972 1.54612 1.55376 1.56208
## 6 1.55030 1.55519 1.55956 1.56440 1.57063 1.57781 1.58717 1.59532 1.60532
##     X2298   X2300   X2302   X2304   X2306   X2308   X2310   X2312   X2314
## 1 1.53253 1.54304 1.55238 1.55817 1.56040 1.55907 1.55488 1.54888 1.54219
## 2 1.55150 1.55947 1.56596 1.57018 1.57236 1.57114 1.56759 1.56246 1.55739
## 3 1.52009 1.52874 1.53616 1.54058 1.54210 1.54087 1.53793 1.53365 1.52845
## 4 1.53533 1.54318 1.55003 1.55433 1.55588 1.55488 1.55198 1.54798 1.54353
## 5 1.57128 1.58027 1.58786 1.59274 1.59498 1.59394 1.59181 1.58729 1.58365
## 6 1.61436 1.62288 1.62863 1.63262 1.63490 1.63437 1.63139 1.62910 1.62394
##     X2316   X2318   X2320   X2322   X2324   X2326   X2328   X2330   X2332
## 1 1.53539 1.52940 1.52451 1.52069 1.51814 1.51671 1.51735 1.51957 1.52271
## 2 1.55262 1.54775 1.54379 1.54059 1.53846 1.53780 1.53811 1.53963 1.54237
## 3 1.52294 1.51828 1.51449 1.51086 1.50953 1.50835 1.50796 1.50943 1.51115
## 4 1.53918 1.53538 1.53083 1.52777 1.52612 1.52462 1.52557 1.52682 1.52983
## 5 1.57861 1.57455 1.57122 1.56800 1.56566 1.56447 1.56478 1.56592 1.56868
## 6 1.62080 1.61692 1.61362 1.60995 1.60851 1.60676 1.60750 1.60883 1.61128
##     X2334   X2336   X2338   X2340   X2342   X2344   X2346   X2348   X2350
## 1 1.52743 1.53311 1.53853 1.54389 1.54750 1.54930 1.54893 1.54633 1.54245
```

```
## 2 1.54491 1.54952 1.55272 1.55708 1.55982 1.56109 1.56119 1.55876 1.55592
## 3 1.51543 1.51936 1.52294 1.52633 1.52805 1.52864 1.52736 1.52383 1.51995
## 4 1.53243 1.53678 1.54053 1.54378 1.54683 1.54783 1.54748 1.54473 1.54068
## 5 1.57138 1.57543 1.57871 1.58230 1.58412 1.58557 1.58547 1.58380 1.58053
## 6 1.61346 1.61745 1.62064 1.62389 1.62474 1.62655 1.62575 1.62352 1.61985
##     X2352   X2354   X2356   X2358   X2360   X2362   X2364   X2366   X2368
## 1 1.53800 1.53332 1.52998 1.52748 1.52616 1.52626 1.52653 1.52759 1.52871
## 2 1.55277 1.54937 1.54689 1.54491 1.54394 1.54450 1.54577 1.54597 1.54673
## 3 1.51562 1.51125 1.50772 1.50565 1.50477 1.50457 1.50546 1.50605 1.50693
## 4 1.53693 1.53348 1.52963 1.52757 1.52682 1.52632 1.52622 1.52712 1.52777
## 5 1.57684 1.57382 1.57117 1.56920 1.56863 1.56857 1.56935 1.56920 1.56946
## 6 1.61692 1.61335 1.61123 1.61048 1.60990 1.60958 1.60979 1.60974 1.60936
##     X2370   X2372   X2374   X2376   X2378   X2380   X2382   X2384   X2386
## 1 1.52918 1.53056 1.53237 1.53401 1.53683 1.53895 1.54134 1.54336 1.54527
## 2 1.54790 1.54805 1.54891 1.54927 1.54998 1.55150 1.55221 1.55450 1.55505
## 3 1.50752 1.50796 1.50909 1.50953 1.51140 1.51140 1.51238 1.51356 1.51361
## 4 1.52857 1.52903 1.52968 1.53038 1.53198 1.53323 1.53458 1.53603 1.53703
## 5 1.57044 1.56987 1.57055 1.57096 1.57258 1.57320 1.57517 1.57663 1.57855
## 6 1.61053 1.61037 1.61165 1.61144 1.61234 1.61282 1.61447 1.61596 1.61660
##     X2388   X2390   X2392   X2394   X2396   X2398   X2400   X2402   X2404
## 1 1.54739 1.54936 1.55201 1.55525 1.55833 1.56183 1.56571 1.56894 1.57399
## 2 1.55663 1.55932 1.56175 1.56459 1.56728 1.56957 1.57282 1.57550 1.57865
## 3 1.51459 1.51528 1.51675 1.51754 1.52083 1.52339 1.52727 1.53007 1.53385
## 4 1.53838 1.53938 1.54118 1.54308 1.54548 1.54858 1.55013 1.55248 1.55658
## 5 1.58022 1.58198 1.58422 1.58697 1.59009 1.59347 1.59727 1.59986 1.60480
## 6 1.61862 1.62054 1.62442 1.62644 1.62969 1.63294 1.63607 1.63868 1.64411
##     X2406   X2408   X2410   X2412   X2414   X2416   X2418   X2420   X2422
## 1 1.57861 1.58232 1.58646 1.59103 1.59464 1.60106 1.60589 1.61115 1.61635
## 2 1.58225 1.58459 1.58773 1.58941 1.59098 1.59398 1.59773 1.60042 1.60407
## 3 1.53773 1.54023 1.54485 1.54765 1.55011 1.55379 1.55630 1.55856 1.56150
## 4 1.55983 1.56333 1.56694 1.57094 1.57309 1.57809 1.58094 1.58429 1.58844
## 5 1.60849 1.61255 1.61556 1.61910 1.61982 1.62533 1.62788 1.63116 1.63402
## 6 1.64810 1.65214 1.65571 1.65896 1.65949 1.66529 1.66635 1.66875 1.67130
##     X2424   X2426   X2428   X2430   X2432   X2434   X2436   X2438   X2440
## 1 1.62092 1.62516 1.62898 1.63355 1.63780 1.64194 1.64671 1.65170 1.65516
## 2 1.60504 1.60905 1.61184 1.61468 1.61838 1.62021 1.62397 1.62787 1.62818
## 3 1.56288 1.56676 1.56892 1.57103 1.57329 1.57570 1.57752 1.57791 1.57919
## 4 1.58999 1.59284 1.59439 1.59584 1.59844 1.60004 1.60150 1.60310 1.60470
## 5 1.63734 1.63963 1.64238 1.64483 1.64571 1.64945 1.65148 1.65746 1.65943
## 6 1.67375 1.67822 1.67992 1.68370 1.68588 1.68822 1.69125 1.69551 1.69663
##     X2442   X2444   X2446   X2448   X2450   X2452   X2454   X2456   X2458
## 1 1.65855 1.66200 1.66726 1.67262 1.67777 1.68074 1.68531 1.68924 1.69306
## 2 1.63102 1.63391 1.63543 1.63670 1.63868 1.64066 1.64335 1.64482 1.64731
## 3 1.58140 1.58194 1.58366 1.58430 1.58582 1.58641 1.58842 1.59054 1.59098
## 4 1.60665 1.60750 1.60965 1.61215 1.61330 1.61450 1.61680 1.61995 1.62090
## 5 1.66328 1.66780 1.67160 1.67404 1.67617 1.67851 1.67929 1.68298 1.68371
## 6 1.70025 1.70136 1.70381 1.70892 1.71147 1.71477 1.71376 1.71967 1.71999
##     X2460   X2462   X2464   X2466   X2468   X2470   X2472   X2474   X2476
## 1 1.69746 1.70283 1.70686 1.71052 1.71392 1.71615 1.72056 1.72512 1.72772
## 2 1.64858 1.65091 1.65213 1.65238 1.65380 1.65522 1.65761 1.65867 1.65959
## 3 1.59417 1.59628 1.60066 1.60277 1.60483 1.60783 1.60773 1.60945 1.60960
## 4 1.62325 1.62340 1.62715 1.62870 1.63105 1.63420 1.63545 1.63605 1.63791
## 5 1.68714 1.68745 1.69046 1.69088 1.69072 1.69348 1.69234 1.69395 1.69488
## 6 1.72318 1.72547 1.73111 1.73201 1.73287 1.73281 1.73531 1.73702 1.73467
```

```
##     X2478   X2480   X2482   X2484   X2486   X2488   X2490   X2492   X2494
## 1 1.72836 1.72985 1.73128 1.73255 1.73378 1.73563 1.73516 1.73717 1.73871
## 2 1.65822 1.66050 1.66055 1.66182 1.66253 1.66273 1.66076 1.65999 1.65898
## 3 1.60734 1.60930 1.60704 1.60832 1.60635 1.60360 1.60414 1.60095 1.59776
## 4 1.63660 1.63746 1.63726 1.63881 1.63630 1.63635 1.63470 1.63395 1.63245
## 5 1.69499 1.69743 1.69977 1.70029 1.70133 1.70320 1.70211 1.70247 1.69930
## 6 1.73739 1.73622 1.73808 1.73558 1.73946 1.73702 1.73973 1.73867 1.73883
##     X2496   X2498
## 1 1.73946 1.73829
## 2 1.65847 1.65878
## 3 1.59781 1.59717
## 4 1.63255 1.63095
## 5 1.69837 1.69961
## 6 1.74356 1.74196
```

```
dim(cookies_data)
```

```
## [1]  32 701
```

We see that there are 700 co-variables. We can assume that some of them are less important than the others. To see this, let's do a Ridge regression and look at the coefficient of each co-variables.

```
library(glmnet)
y <- cookies_data[, 1]
X <- cookies_data[, -1]

cv_ridge_model <- cv.glmnet(as.matrix(X), y, alpha=0)

print(cv_ridge_model)
```

```
##
## Call:  cv.glmnet(x = as.matrix(X), y = y, alpha = 0)
##
## Measure: Mean-Squared Error
##
##     Lambda Index Measure     SE Nonzero
## min  11.42   100  0.4742 0.1077     700
## 1se  22.95    85  0.5794 0.1250     700
```

```
best_lambda <- cv_ridge_model$lambda.min
print(paste("Best lambda :", best_lambda))
```

```
## [1] "Best lambda : 11.4243191334971"
```

```
final_ridge_model <- glmnet(as.matrix(X), y, alpha=0, lambda=best_lambda)

abs_coef <- abs(coef(final_ridge_model))
```

```
min(abs_coef)
```

```
## [1] 2.232542e-05
```

```r
paste("Number of value higher than 10^-1 : ", sum(abs_coef > 10^-1))
```

```
## [1] "Number of value higher than 10^-1 :  179"
```

```r
paste("Number of value higher than 10^-2 : ", sum(abs_coef > 10^-2))
```

```
## [1] "Number of value higher than 10^-2 :  630"
```

```r
paste("Number of value higher than 10^-3 : ", sum(abs_coef > 10^-3))
```

```
## [1] "Number of value higher than 10^-3 :  694"
```

```r
paste("Number of value higher than 10^-4 : ", sum(abs_coef > 10^-4))
```

```
## [1] "Number of value higher than 10^-4 :  700"
```

We can see that the majority of the coefficients are lower than 10^{-1}. Then, we could think that a lot of our co-variables are useless to predict the target variable, aka the fat.

Let's do a Lasso regression to see which co-variables are actually useful to predict the fat :

```r
cv_lasso_model <- cv.glmnet(as.matrix(X), y, alpha=1)

best_lambda <- cv_lasso_model$lambda.min

best_model_lasso <- glmnet(as.matrix(X), y, lambda = best_lambda, alpha = 1)
print(best_model_lasso)
```

```
##
## Call:  glmnet(x = as.matrix(X), y = y, alpha = 1, lambda = best_lambda)
##
##   Df  %Dev  Lambda
## 1 31 98.09 0.01142
```

We can see that our model is pretty accurate (deviance of 98.09%) with only 31 co-variables used among the 700 existant.

Actually, we've shown that even less co-variables are useless than what we thought.

Now let's try to split our dataset into train and test dataset:

Ridge :

```r
# We split into 2 dataframe randomly
indice_train <- sample(1:nrow(cookies_data), 0.8 * nrow(cookies_data))
train_data <- cookies_data[indice_train, ]
test_data <- cookies_data[-indice_train, ]

# We define X & y for both dataframe
y_train <- train_data[, 1]
X_train <- train_data[, -1]
```

```r
y_test <- test_data[, 1]
X_test <- test_data[, -1]

# We train the model with the best value for lambda
cv_ridge_model <- cv.glmnet(as.matrix(X_train), y_train, alpha = 0)
```

```
## Warning: Option grouped=FALSE enforced in cv.glmnet, since < 3 observations per
## fold
```

```r
best_lambda_ridge <- cv_ridge_model$lambda.min
ridge_model <- glmnet(as.matrix(X_train), y_train, lambda = best_lambda_ridge, alpha = 0)

predictions_ridge <- predict(ridge_model, s = best_lambda_ridge, newx = as.matrix(X_test))

error_ridge <- sqrt(mean((predictions_ridge - y_test)^2))
print(paste("RMSE Ridge :", round(error_ridge, 2)))
```

```
## [1] "RMSE Ridge : 0.66"
```

Lasso :

```r
# We split into 2 dataframe randomly
indice_train <- sample(1:nrow(cookies_data), 0.8 * nrow(cookies_data))
train_data <- cookies_data[indice_train, ]
test_data <- cookies_data[-indice_train, ]

# We define X & y for both dataframe
y_train <- train_data[, 1]
X_train <- train_data[, -1]
y_test <- test_data[, 1]
X_test <- test_data[, -1]

# We train the model with the best value for lambda
cv_lasso_model <- cv.glmnet(as.matrix(X_train), y_train, alpha = 1)
```

```
## Warning: Option grouped=FALSE enforced in cv.glmnet, since < 3 observations per
## fold
```

```r
best_lambda <- cv_lasso_model$lambda.min
lasso_model <- glmnet(as.matrix(X_train), y_train, lambda = best_lambda, alpha = 1)

# We make prediction on the X_test
predictions_lasso <- predict(lasso_model, s = best_lambda, newx = as.matrix(X_test))

# We compute the RMSE
error_lasso  <- sqrt(mean((predictions_lasso - y_test)^2))
print(paste("RMSE :", round(error_lasso, 2)))
```

```
## [1] "RMSE : 0.19"
```

We see that the RMSE for the Lasso regression is way better than for the Ridge one. (de rien pour l'analyse)