To Sell or Not to Sell: Distributions of Best Selling Time

Carmen Gutierrez, Noah Lowery, Brittany Russell, Dylan Scofield

Dr. Sixian Jin: Math 490

California State University SAN MARCOS

Brownian Motion

For investors, a question that is often thrown around is what is the best time to sell. This is referred as ***First Exit Time*** (or time to exit). The goal of this is to maximize the profits, or minimize loss, of an investment. This is a difficult question however due to the random nature of financial entities such as stocks. This leads to the use of mathematical models that can simulate randomness. A common model used by mathematicians in finance is a ***Brownian Motion***.
Brownian motion has a couple properties that makes it suited for this kind of question. For the sake of brevity, we will only mention the properties that most relate with our question. First $B_0 = 0$. This states that our model initially starts at zero. In our case, this does not imply an entity worth \$0, but rather a change of 0% from the initial price. It also has ***Stationary Increments*** which means that from each time period in the graph, the value of the next is not determined by what came before; the disjoint time periods are independent. Finally, it is a continuous function that is differentiable nowhere.

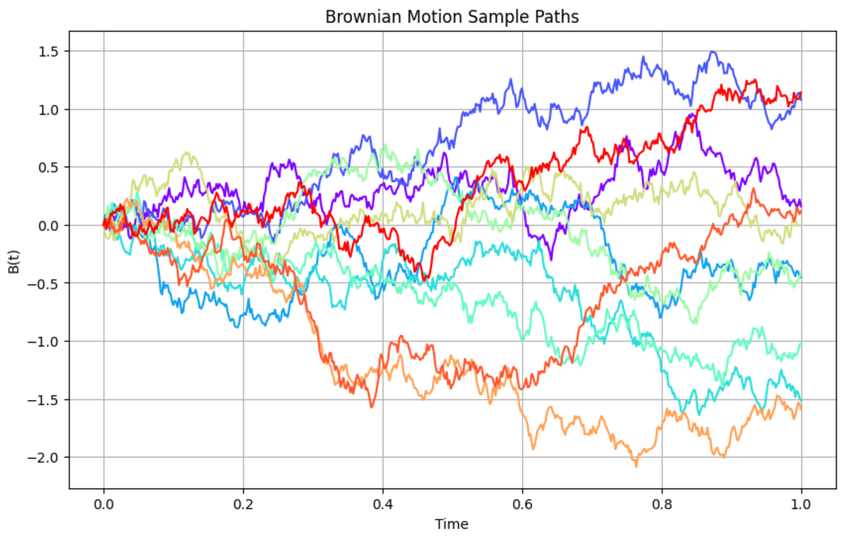


Figure 1: 10 Sample Paths of Brownian Motion

From this graph we can see how Brownian motion imitates the behavior of stock price.

Goals and Overview

Our goal is to understand the behavior of asset prices modeled by **Geometric Brownian Motion (GBM)** and analyze when an investor is most likely to sell based on predefined gain/loss thresholds.

- We simulate thousands of GBM paths and study the first time each path hits either a $+10\%$ profit or a -5% loss.
- These first-exit times are visualized and summarized through histograms and cumulative distributions.
- We seek to answer: *When are gains or losses most likely to trigger a sell decision? How do these times distribute across simulations?*
- Our analysis gives insight into the timing of investor decisions under a stochastic model and may help in optimizing sell strategies based on statistical behavior.

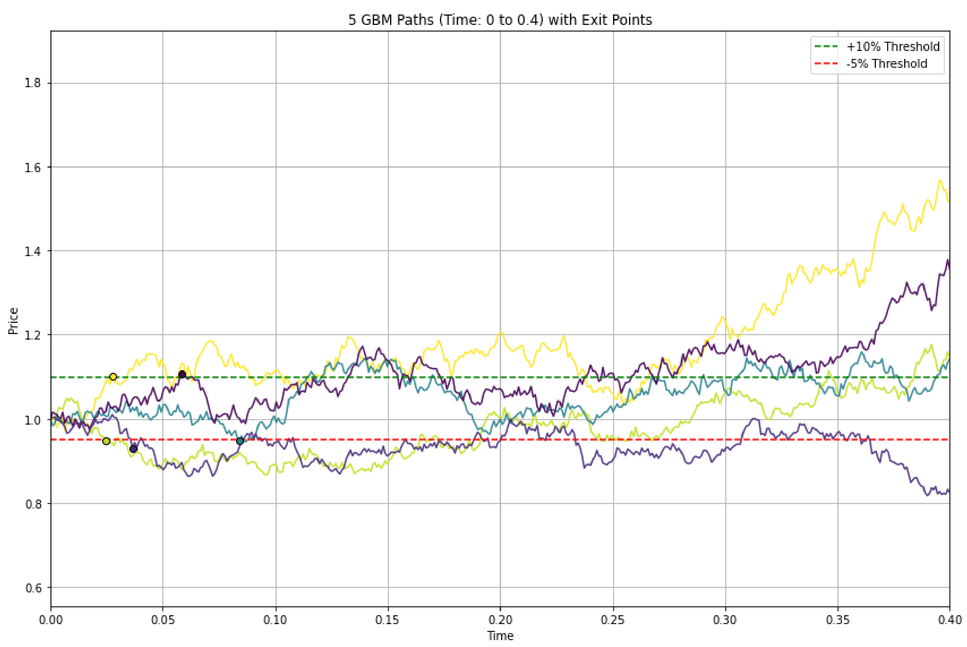
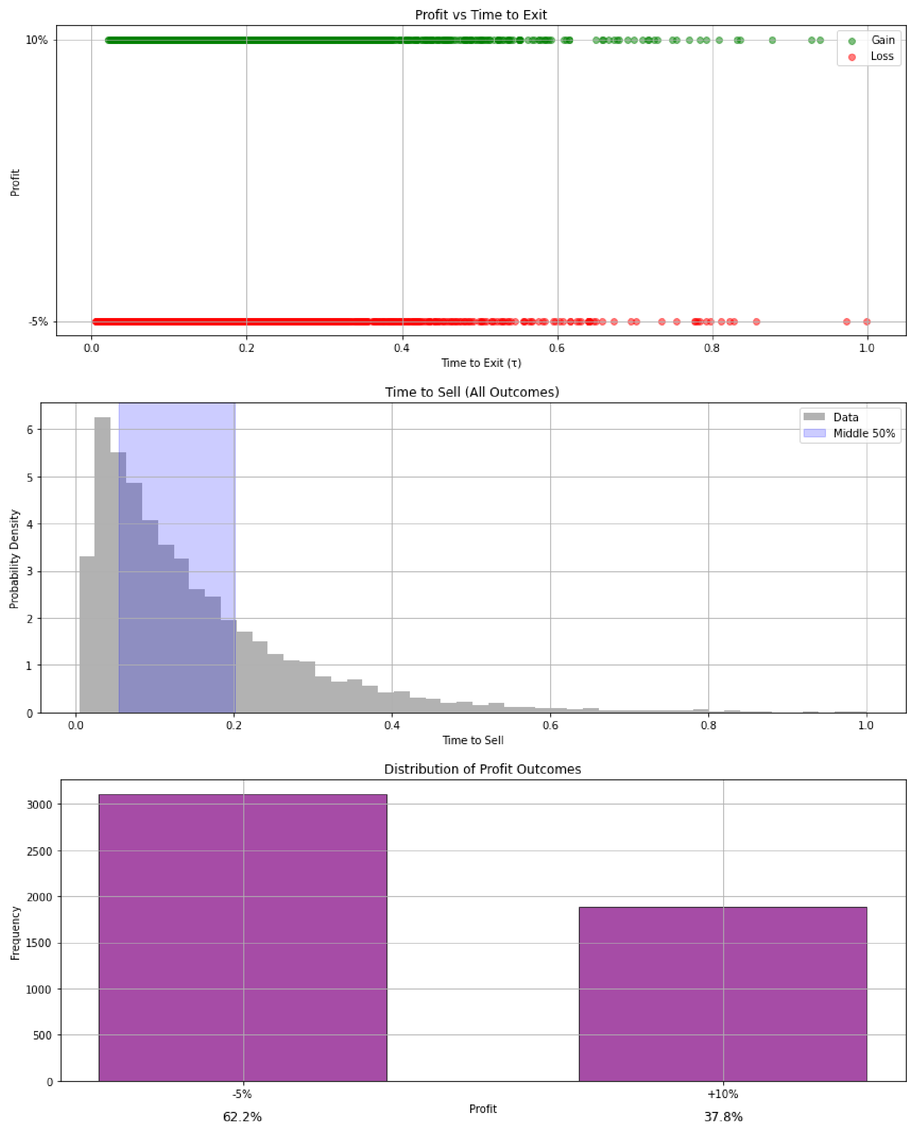


Figure 2: 5 Samples with a **Time To Exit (TTE)** of -5% and $+10\%$ over-layed on the graphs.

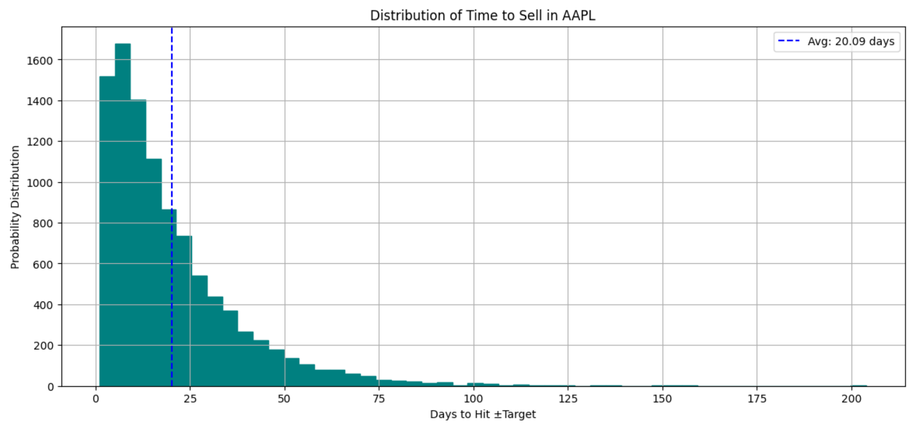
Distributions



Explanation of Graphs

- **Graph 1:** Simulated profit paths from *5,000 GBM simulations*, ending at either a $+10\%$ gain (green) or a -5% loss (red).
- **Graph 2:** Distribution of first exit times (gain or loss), with the middle 50% of outcomes highlighted.
- **Graph 3:** Cumulative distribution of profit outcomes over all paths.
- The distributions suggest that the most likely exit, whether due to a gain or a loss, occurs between **0.05–0.20 time units** (18–74 days).
- All graphs share the same time axis (in trading days); the model runs over one year using a **time grid of 1,095 steps** (8 trading hours per step).

Observation & Conclusion



- Taking into account (AAPL) stock historical volatility and drift, we applied our **GBM** model with an exit strategy of +10% profit / 5% loss.
- In the simulation the average **TTE** is *20 days*, and the *probability of hitting +10% before 5%:* $\sim$38.6%. Given the thresholds, the model agrees with our theoretical results. This suggests that investors are more inclined to incur a loss prior to realizing a gain, despite the average **TTE** being comparable for both outcomes.
- **Altering the thresholds alters the distribution.** Investing conservatively (e.g., +5% / 2%) or aggressively (+20% / 10%) results in different exit probabilities, timelines, and risk characteristics.