# Hw #2 - DSP

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1. $A = 20$

   $\omega_0 = 2\pi f \quad f = 25\,Hz$

   $\phi = 5s$

2. Euler's formula:

$$e^{j\theta} = \cos_2\theta + j\sin\theta$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} \cdots$$

$$\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots$$

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots$$

$$\cos(\theta) + j\sin(\theta) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots = e^{j\theta}$$

3. a) $\cos(\theta_1 + \theta_2) = \dfrac{e^{j(\theta_1+\theta_2)} + e^{-j(\theta_1+\theta_2)}}{2} = \dfrac{e^{j\theta_1}e^{j\theta_2} + e^{-j\theta_1}e^{-j\theta_2}}{2}$

$$= \frac{1}{4}\left(e^{j\theta_1}e^{-j\theta_2} + e^{-j\theta_1}e^{-j\theta_2}\right) + \frac{1}{4}\left(e^{j\theta_1}e^{j\theta_2} + e^{-j\theta_1}e^{-j\theta_2}\right)$$

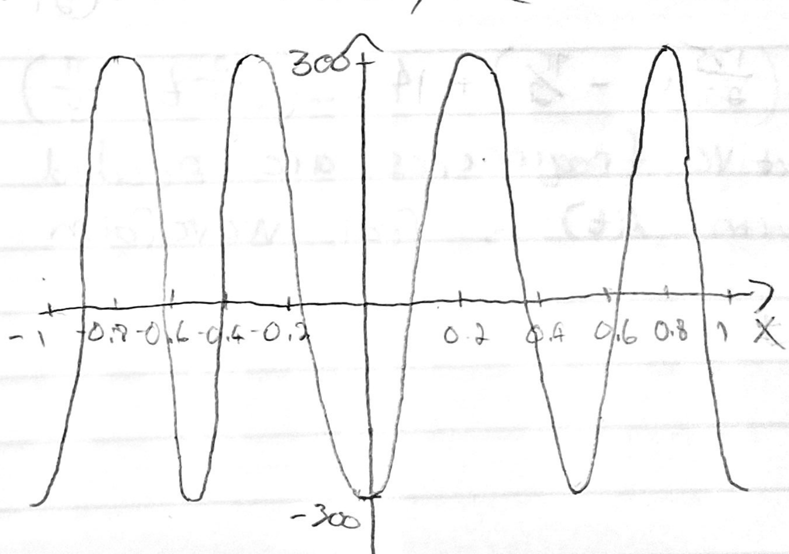
$$= \cos\theta_1 \cos\theta_2 + \sin\theta_1 \sin\theta_2$$

b) $\cos(\theta_1 - \theta_2) = \dfrac{e^{j(\theta_1-\theta_2)} + e^{-j(\theta_1-\theta_2)}}{2} = \dfrac{e^{j\theta_1}e^{-j\theta_2} + e^{-j\theta_1}e^{j\theta_2}}{2}$

$$\frac{1}{4}\left(e^{j\theta_1}e^{-j\theta_2} + e^{-j\theta_1}e^{j\theta_2}\right) + \frac{1}{4}\left(e^{+j\theta_1}e^{-j\theta_2} + e^{-j\theta_1}e^{j\theta_2}\right)$$

$$= \cos\theta_1 \cos\theta_2 - \sin\theta_1 \sin\theta_2$$

4. $f(x) = Re\left(300\, e^{j2\pi \cdot 2(x-0.75)}\right) = Re\left(300\, e^{j4\pi(x-0.75)}\right)$



7. $x(t) = 10\cos(800\pi t + \pi/4) + 7\cos(1200\pi t - \pi/3) - 3\cos(1600\pi t)$

$= \frac{10}{2}\left(e^{j(800\pi t + \pi/4)} + e^{-j(800\pi t + \pi/4)}\right) + 7\cos(1200\pi t - \pi/3) - 3\cos(1600\pi t)$

$= 5\left(e^{j800\pi t} e^{j\pi/4} + e^{-j800\pi t} e^{j\pi/4}\right) + 7\cos(1200\pi t - \pi/3) - 3\cos(1600\pi t$

$= 5e^{j800\pi t} e^{j\pi/4} + 5e^{-j800\pi t} e^{-j\pi/4} + \cdots$

$\omega_0 = 800\pi$

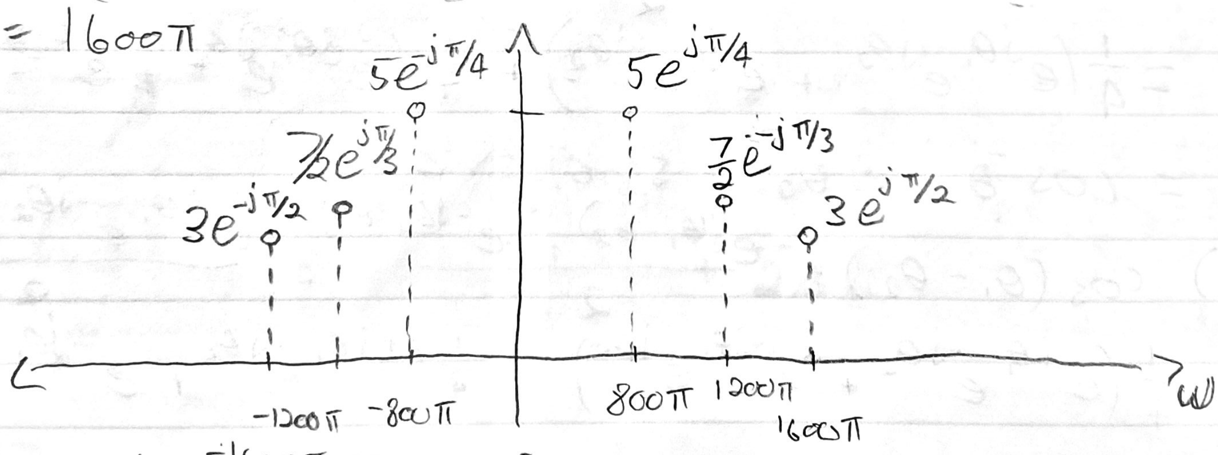
$7\cos(1200\pi t - \pi/3) = 7/2\left[e^{j(1200\pi t - \pi/3)} + e^{-j(1200\pi t - \pi/3)}\right]$

$= \frac{7}{2}\left[e^{j1200\pi t} e^{-j\pi/3} + e^{-j1200\pi/t} e^{j\pi/3}\right]$

$\omega_0 = 1200\pi$

$-3\cos(1600\pi t) = 3\cos(1600\pi t + \pi/2) = 3e^{j1600\pi} e^{j\pi/2} + 3e^{-j1600\pi} e^{-j\pi/2}$

$W_0 = 1600\pi$




8 a) $4e^{j\left(\frac{175}{2\pi}\right)t} e^{-j\pi/2} + 4e^{-j\,175/2\pi t} e^{j\pi/2} = 8\cos\left(\frac{175}{2\pi}t - \pi/2\right)$

$7e^{j\frac{50}{2\pi}t} e^{-j\pi/3} + 7e^{j\frac{50}{2\pi}t} e^{j\pi/3} = 14\cos\left(\frac{50}{2\pi}t - \pi/3\right)$

$X(t) = 8\cos\left(\frac{175}{2\pi}t - \pi/2\right) + 14\cos\left(\frac{50}{2\pi}t - \frac{\pi}{3}\right) + 11$ ?

b) the negative frequencies are needed because the waveform $X(t)$ is real waveform

9. $x(t) = 9\sin^3(27\pi t)$ $\quad \omega_0 = 27\pi \quad T_0 = \frac{2\cancel{\pi}}{27\cancel{\pi}} = \frac{2}{27}$

$$X[k] = \frac{1}{T_0}\int_0^{T_0} x(t)e^{-jk\omega_0 t}\,dt$$

$$= \frac{2}{27}\int_0^{2/27} 9\sin^3(27\pi t)e^{-jk\omega_0 t}\,dt$$

$$= \frac{18}{27}\int_0^{2/27} \sin^3(27\pi t)e^{-jk27\pi t}\,dt$$

$$= \frac{2}{3}\int_0^{2/27} \sin^3(27\pi t)e^{-jk27\pi t}\,dt$$

10. a) 1 $\quad x(t) = 2 + 3\cos\left(\frac{1.2}{2\pi}t + \frac{\pi}{2}\right)$

b) 5 $\quad x(t) = 3\cos\left(\frac{1.5}{2\pi}t + \pi\right)$

c) 3 $\quad x(t) = 2 + 3\cos\left(\frac{1.2}{2\pi} - \frac{\pi}{4}\right)$

d) 4 $\quad x(t) = 3\cos\left(\frac{1}{\pi}t + \pi\right) + 3\cos\left(\frac{1.2}{2\pi}t - \frac{\pi}{4}\right)$

e) 2 $\quad x(t) = 3\cos\left(\frac{1.5}{2\pi}t + \pi\right) + 3\cos\left(\frac{0.6}{2\pi}t - \frac{\pi}{4}\right)$