Number Bases & Flow Charts

Introduction

This unit covers how to convert to different number bases and how to add and subtract in each base. This unit will also introduce you to flow charts.

Number Bases

When working with number bases the base will be listed in subscript after the number. The base for a number system tells us how many possibilities there are for a single digit. When a number's base is not given, the base is assumed to be 10.

Examples:

8549_{10}	Decimal
$AF42_{16}$	Hexadecimal
743_{8}	Octal
10101011_{2}	Binary

Base 10 (Decimal)

In base 10 there are ten possibilities for each digit (0, 1, 2, 3, 4, 5, 6, 7, 8 and 9).

The places values will start from the right and be 10^0, 10^1, 10^2 and so on.

___	___	___	___	___
10^4	10^3	10^2	10^1	10^0
10,000s place	1,000s place	100s place	10s place	1s place

Adding

When adding in base 10 carry a 1 for every 10.

Example 1: 434 + 89

Carry		1	1	
Number 1		4	3	4
Number 2	+		8	9
Result		5	2	3

Example 2: 333 + 496

Carry		1		
Number 1		3	3	3
Number 2	+	4	9	6
Result		8	2	9

Subtracting

When subtracting in base 10 borrow 10 as needed.

Example 1: 492 - 89

After Borrow			8	12
Number 1		4	~~9~~	~~2~~
Number 2	-		8	9
Result		4	0	3

Example 2: 128 - 36

After Borrow		0	12	
Number 1		~~1~~	~~2~~	8
Number 2	-		3	6
Result			9	2

Base 2 (Binary)

In base 2 there are two possibilities for each digit (0 and 1). Each digit in binary is called a **bit**. A group of 8 bits is called a **byte**.

The below chart shows the first 5 place values for a base 2:

2^4	2^3	2^2	2^1	2^0
16s place	8s place	4s place	2s place	1s place

Finding the Decimal Value from Binary

Multiply each place value by the value it stores and then add all the results together.

Example 1: 1010_2

Digit	1	0	1	0
	2^3	2^2	2^1	2^0
Place Value	8s	4s	2s	1s
Value	8	0	2	0

Decimal Value: 10

Example 2: 1101_2

Digit	1	1	0	1
	2^3	2^2	2^1	2^0
Place Value	8s	4s	2s	1s
Value	8	4	0	1

Decimal Value: 13

Finding the Binary Value from Decimal

1. Find the first place value that is larger than your number.
2. Move right one place
3. Check to see if the place value is smaller than your number
    a. If it is not write a 0
    b. If it is write a 1 and reduce your number by the place value
4. Repeat Steps 2-3 until you have done the 1s place

Example 1: 22_{10}

Digit		1	0	1	1	0
	2^5	2^4	2^3	2^2	2^1	2^0
Place Value	32s	16s	8s	4s	2s	1s
Number	22	6	6	2	0	0

Example 2: 25_{10}

Digit		1	1	0	0	1
	2^5	2^4	2^3	2^2	2^1	2^0
Place Value	32s	16s	8s	4s	2s	1s
Number	25	9	1	1	1	0

Adding

When adding in base 2 carry a 1 for every 2.

Example 1: $1011_2 + 1010_2$

Carry		1		1		
Number 1			1	0	1	1
Number 2	+		1	0	1	0
Result		1	0	1	0	1

Example 2: $1101_2 + 1011_2$

Carry			1	1	1	
Number 1			1	1	0	1
Number 2	+		1	0	1	1
Result		1	1	0	0	0

Subtracting

When subtracting in base 2 borrow 2 as needed.

Example 1: $1101_2 - 1010_2$

After Borrow			0	2	
Number 1		1	~~1~~	~~0~~	1
Number 2	-	1	0	1	0
Result		0	0	1	1

Example 2: $101_2 - 11_2$

After Borrow		0	2	
Number 1		~~1~~	~~0~~	1
Number 2	-		1	1
Result			1	0

Base 8 (Octal)

In base 8 there are eight possibilities for each digit (0, 1, 2, 3, 4, 5, 6 and 7).

The below chart shows the first 4 place values for a base 8:

___	___	___	___
8^3	8^2	8^1	8^0
512s place	64s place	8s place	1s place

Finding the Decimal Value from Octal

Multiply each place value by the value it stores and then add all the results together.

Example 1: 56_8

Digit	5	6
	8^1	8^0
Place Value	8s	1s
Value	40	6

Decimal Value: 46

Example 2: 321_8

Digit	3	2	1
	8^2	8^1	8^0
Place Value	64s	8s	1s
Value	192	16	1

Decimal Value: 209

Finding the Octal Value from Decimal

1. Find the first place value that is larger than your number.
2. Move right one place
3. Check to see if the place value is smaller than your number
    a. If it is not write a 0
    b. If it is write down how many times the place value goes into the number. Next reduce your number by the number you wrote down times the place value.
4. Repeat Steps 2-3 until you have done the 1s place

Example 1: 22_{10}

Digit	0	2	6
	8^2	8^1	8^0
Place Value	64s	8s	1s
Number	22	6	0

Example 2: 75_{10}

Digit	0	1	1	3
	8^3	8^2	8^1	8^0
Place Value	512s	64s	8s	1s
Number	75	11	3	0

Adding

When adding in base 8 carry a 1 for every 8.

Example 1: $123_8 + 125_8$

Carry			1	
Number 1		1	2	3
Number 2	+	1	2	5
Result		2	5	0

Example 2: $333_8 + 257_8$

Carry		1	1	
Number 1		3	3	3
Number 2	+	2	5	7
Result		6	1	2

Subtracting

When subtracting in base 8 borrow 8 as needed.

Example 1: $123_8 – 52_8$

After Borrow		0	10	
Number 1		~~1~~	~~2~~	3
Number 2	-		5	2
Result			5	1

Example 2: $754_2 - 55_2$

After Borrow 2		6	12	
After Borrow 1		~~7~~	~~4~~	12
Number 1		7	~~5~~	4
Number 2	-		5	5
Result		6	7	7

Base 16 (Hexadecimal)

In base 16 there are sixteen possibilities for each digit (0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E and F).

A	10
B	11
C	12
D	13
E	14
F	15

The below chart shows the first 3 place values for a base 16:

___	___	___
16^2	16^1	16^0
256s place	16s place	1s place

Finding the Decimal Value from Hexadecimal

Multiply each place value by the value it stores and then add all the results together in decimal.

Example 1: $A3_{16}$

Digit	A	3
	16^1	16^0
Place Value	16s	1s
Value	160	3

Decimal Value: 163

Example 2: $9C_{16}$

Digit	9	C
	16^1	16^0
Place Value	16s	1s
Value	144	12

Decimal Value: 156

Finding the Hexadecimal Value from Decimal

1. Find the first place value that is larger than your number.
2. Move right one place
3. Check to see if the place value is smaller than your number
    a. If it is not write a 0
    b. If it is write down how many times the place value goes into the number. Next reduce your number by the number you wrote down times the place value.
4. Repeat Steps 2-3 until you have done the 1s place

Example 1: 52_{10}

Digit	0	3	4
	16^2	16^1	16^0
Place Value	256s	16s	1s
Number	52	4	0

Example 2: 200_{10}

Digit	0	C	8
	16^2	16^1	16^0
Place Value	256s	16s	1s
Number	200	8	0

Adding

When adding in base 16 carry a 1 for every 16.

Example 1: $A3F_{16} + AA9_{16}$

Carry		1		1	
Number 1			A	3	F
Number 2	+		A	A	9
Result		1	4	E	8

Example 2: $B3_{16} + 5F_{16}$

Carry		1	1	
Number 1			B	3
Number 2	+		5	F
Result		1	1	2

Subtracting

When subtracting in base 16 borrow 16 as needed.

Example 1: $A5_{16} – 5C_{16}$

After Borrow		9	21
Number 1		~~A~~	~~5~~
Number 2	-	5	C
Result		4	9

Example 2: $987_{16} – 3CD_{16}$

After Borrow 2			23	
After Borrow 1		8	~~7~~	23
Number 1		~~9~~	~~8~~	7
Number 2	-	3	C	D
Result		5	B	A

Conversion Tricks

This section covers tricks for converting from one base to another quickly.

Binary to Octal

Starting from the right, make groups of 3 digits. Next, find the decimal value of each group, and then write those values under the groups that generated them.

Example 1: 1001110011_2 to Octal

Binary	1	001	110	011
Octal	1	1	6	3

Example 2: 100111_2 to Octal

Binary	100	111
Octal	4	7

Binary to Hexadecimal

Starting from the right, make groups of 4 digits. Next, find the hexadecimal value of each group, and then write those values under the groups that generated them.

Example 1: 1001111011_2 to Hexadecimal

Binary	10	0111	1011
Hexadecimal	2	7	B

Example 2: 0101111_2 to Hexadecimal

Binary	010	1111
Hexadecimal	2	F

Octal to Binary

Use each octal digital to create 3 binary digits.

Example 1: 7531_8 to Binary

Octal	7	5	3	1
Binary	111	101	011	001

Example 2: 62_8 to Binary

Binary	6	2
Octal	101	010

Hexadecimal to Binary

Use each hexadecimal digital to create 4 binary digits.

Example 1: $D56F_{16}$ to Binary

Hexadecimal	D	5	6	F
Binary	1101	0101	0110	1111

Example 2: 84_{16} to Binary

Binary	8	4
Octal	1000	0100

Bitwise Operations

Bitwise operations are operations used on the binary values of numbers.

Here are the Bitwise operations and an example of each:

- ~ (inverse) –gives the inverse of a single number
  - Example: ~23

Binary	1	0	1	1	1
Place Value	16s	8s	4s	2s	1s
Binary Result	0	1	0	0	0

Result: 8

- << (Shift Left) –shifts the bits left the given number of digits
  - Example: 23 << 2

Binary			1	0	1	1	1
Place Value	64s	32s	16s	8s	4s	2s	1s
Binary Result	1	0	1	1	1	0	0

Result: 92

- >> (Shift Right) –shifts the bits right the given number of digits
  - Example: 23 >> 2

Binary	1	0	1	1	1
Place Value	16s	8s	4s	2s	1s
Binary Result			1	0	1

Result: 5

- >>> (Shift Right) –shifts the bits right and shifts a 0 into the sign digit.
  - With positive values it works >>
  - With negative values it will give you are large positive value.

- & (Bitwise AND) –ANDs the bits of the two number
  - Example: 25 & 5

Binary of 25	1	1	0	0	1
Binary of 5	0	0	1	0	1
Place Value	16s	8s	4s	2s	1s
Binary Result	0	0	0	0	1

Result: 1

- | (Bitwise OR) –ORs the bits of the two numbers
  - Example: 25 | 5

Binary of 25	1	1	0	0	1
Binary of 5	0	0	1	0	1
Place Value	16s	8s	4s	2s	1s
Binary Result	1	1	1	0	1

Result: 29

- ^ (Bitwise XOR) –XORs the bits of the two numbers
  - Example: 25 ^ 5

Binary of 25	1	1	0	0	1
Binary of 5	0	0	1	0	1
Place Value	16s	8s	4s	2s	1s
Binary Result	1	1	1	0	0

Result: 28

Storing Various Base Values into an Integer

Storing a Binary Value into an Integer

Format:

int name = 0bbinaryNumber;

Example:

```
int b = 0b011;
System.out.println(b);
```

Example Output:

```
3
```

Storing an Octal Value into an Integer

Format:

int name = 0octalNumber;

Example:

```
int o = 011;
System.out.println(o);
```

Example Output:

```
9
```

Storing a Hexadecimal Value into an Integer

Format:

int name = 0xHexadecimalNumber;

Example:

```
int h = 0xA1;
System.out.println(h);
```

Example Output:

```
161
```

Getting Various Base Values from an int

Format:

```
String name = Integer.toString(value, base);
```

Example:

```
String b = Integer.toString(50,2);
String o = Integer.toString(50,8);
String h = Integer.toString(50,16);
System.out.println(b);
System.out.println(o);
System.out.println(h);
```

Example Output:

```
110010
62
32
```

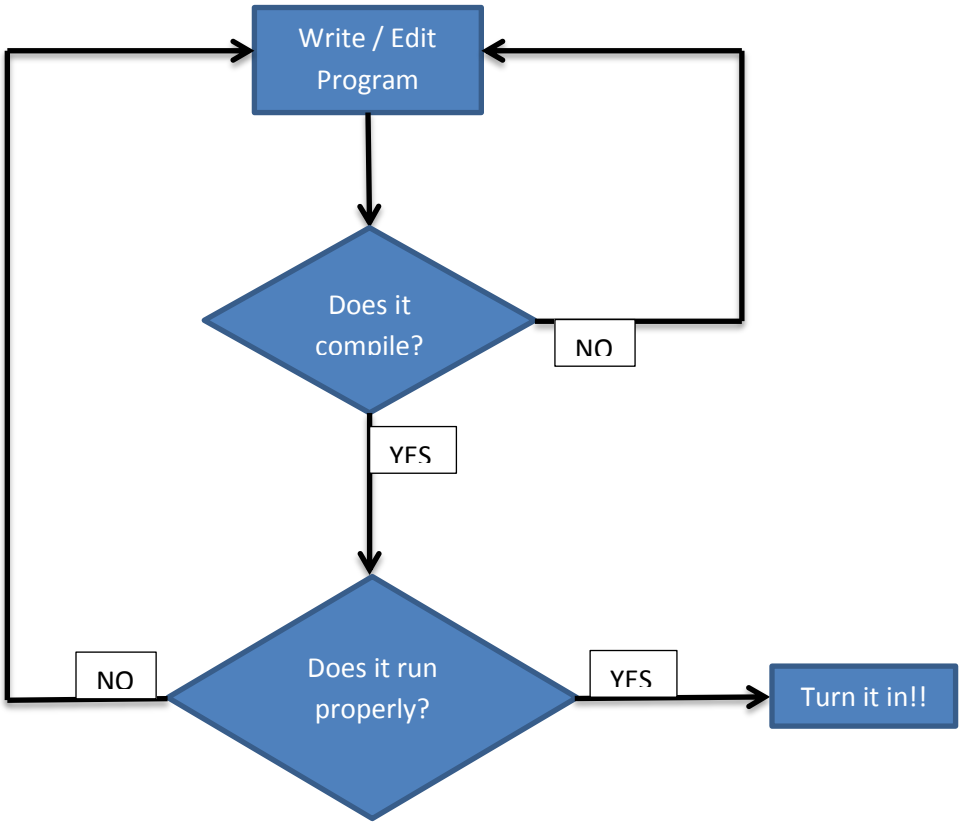
Flow Charts

A **flow chart** is a graphical representation of an **algorithm**. An algorithm is a sequence of steps to follow in order to complete a task. The follow charts we will be working with will consist of actions, yes/no decisions and paths.

Flow Chart Parts

- Actions will be represented with rectangles.
- Yes/No decisions will be represented with diamonds.
- The path of the program is show with arrows
    - Decisions will generate two paths. One for yes and one for no.

Example: Flow chart for building a program.



Blue Pelican Sections

Lesson 14
Lesson 27
Lesson 28

Terms

Algorithm	A sequence of steps to follow in order to complete a task.
Binary	A number system using only 0s and 1s.
Bitwise Operations	Operations that get preformed on the binary values of numbers.
Bit	A single digit of binary.
Byte	A group of 8 bits.
Decimal	A number system using only digits 0 through 9.
Flow Chart	A graphical representation of an algorithm.
Hexadecimal	A number system using only digits 0 through F.
Octal	A number system using only digits 0 through 7.
Number Base	A system of representing a number using a given range of digits.

Assignments

Number Base Practice
Bitwise Practice