MTH 451/551 – Lab 4

1. Read **Example 12.5** in [TB]. Produce a figure similar to **Figure 12.1** by doing the following. For various n values, construct a monic polynomial with roots at each of the integers 1 through n. Then attempt to solve for the roots of these polynomials. Plot the estimated roots in the complex plane (they should be real numbers, in fact integers, but in general roots are complex numbers and we can more easily see any errors by plotting in the complex plane).

   In particular, type

   ```
   for n=21:30,plot(roots(poly(1:n)),'o');hold on;end;hold off
   ```

   (but be sure to understand what this is doing).

   Just by looking at the plot, and assuming that the relative errors in the coefficients of the polynomial are close to machine epsilon, can you estimate the relative condition number of the problem of finding say the 20^{th} root of the degree 30 Wilkinson polynomial? (There is no need to use the formula derived in the book for this example.) (Technical note: for $n = 20$ all of the estimated roots are real unless you artificially add random error to the polynomial coefficients, as done in [TB]. In this case `plot` means something different, since for real-valued vectors it plots the value of the element vs the index, whereas for complex-valued vectors it plots the imaginary part vs the real part.)

2. Read the **Experiment** of Lecture 16 in [TB].

3. (20 points) Do 16.2 in [TB].

   - To build a random orthogonal matrix you may use

     ```
     [U,X]=qr(randn(50));
     ```

   - To build a random Σ matrix you may use

     ```
     S=diag(flipud(sort(rand(50,1))));
     ```

   - The 2-norm is fine to use for this problem, i.e.,

     ```
     norm(U-U2)
     ```

   - To find out which columns of $U2$ are opposite columns of U use

     ```
     flip=sign(diag(U2'*U));
     ```

   - To multiply the appropriate columns of $U2$ by -1 to match U use

     ```
     U2=U2*diag(flip);
     ```

     Be sure to do the same for $V2$.

   - Rather than showing the values of the norms you are asked to compute, it may be more interesting to plot them on the same axes. For example

```
for i=1:numberofmatrices
  ...
  normU(i)=norm(U-U2);
  normV(i)=norm(V-V2);
  ...
end
semilogy(normU,'s')
hold on
semilogy(normV,'x')
...
legend('\delta U','\delta V', '\delta S', '\delta A');
```

You do not have to do it this way, but if you do, you could show a lot more than just five example matrices, maybe 100.

4. Read the MATLAB examples in Lecture 19.