Homework 2

Noah Johnson

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Exercises

4.2 There are two events A and B. $P(A) = .5$ and $P(B) = .3$. The events A and B are independent.

(a) **Find $\mathbf{P(\tilde{A})}$**

$$P(\tilde{A}) = 1 - P(A) = 1 - 0.5 = 0.5$$

(b) **Find $\mathbf{P(A \cap B)}$**

$$\begin{aligned} P(A \cap B) &= P(B|A) * P(A) \\ = P(B) * P(A) &\quad \text{(since A and B are independent)} \\ &= .3 * .5 = .15 \end{aligned}$$

(c) **Find $\mathbf{P(A \cup B)}$**

$$\begin{aligned} P(A \cup B) &= P(A) + P(B) - P(A \cap B) \\ &= .5 + .3 - .15 = .65 \end{aligned}$$

4.4 There are two events A and B. $P(A) = .7$ and $P(B) = .8$. $P(\tilde{A} \cap \tilde{B}) = .1$.

(a) **Are A and B independent events? Explain why or why not.**

$$\begin{aligned} P(A \cup B) &= 1 - P(\overline{A \cup B}) \\ = 1 - P(\tilde{A} \cap \tilde{B}) &\quad \text{(using De Morgan's Law)} \\ &= 1 - 0.1 = .9 \end{aligned}$$

$$\begin{aligned} P(A \cap B) &= P(A) + P(B) - P(A \cup B) \\ &= .7 + .8 - .9 = .6 \end{aligned}$$

$$\begin{aligned} P(B|A) &= \frac{P(A \cap B)}{P(A)} = \frac{.6}{.7} = \frac{6}{7} \\ \neq \frac{4}{5} = P(B) &\ \therefore \text{A and B are not independent events} \end{aligned}$$

(b) **Find $\mathbf{P(A \cup B)}$**

Refer to the first equation above. $P(A \cup B) = 0.9$.

4.6 Two fair dice, one red and one green, are rolled. Let the event A be "the sum of the faces showing is equal to seven." Let the event B be "the faces showing on the two dice are equal."

(a) **List out the sample space of the experiment.**

$\Omega = \{2, 3, \ldots, 12\}$

(b) **List the outcomes in A, and find P(A).**

$A = \{(1,6),(2,5),(3,4),(4,3),(5,2),(6,1)\}$

$P(A) = \frac{6}{36} = \frac{1}{6}$

(c) **List the outcomes in B, and find P(B).**

$B = \{(1,1),(2,2),(3,3),(4,4),(5,5),(6,6)\}$

$P(B) = \frac{6}{36} = \frac{1}{6}$

(d) **List the outcomes in $\mathbf{A \cap B}$, and find $\mathbf{P(A \cap B)}$.**

$A \cap B = \varnothing$

$P(A \cap B) = 0$

(e) **Are the events A and B independent? Explain why or why not.**

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{0}{1/6} = 0$$
$$\neq \frac{1}{6} = P(B) \ \therefore \text{A and B are not independent events}$$

(f) **How would you describe the relationship between event A and event B?**

As mutually exclusive, since one event occurring precludes the other.

4.8 Two dice are rolled. The red die has been loaded. Its probabilities are $P(1) = P(2) = P(3) = P(4) = \frac{1}{5}$ and $P(5) = P(6) = \frac{1}{10}$. The green die is fair. Let the event A be "the sum of the faces showing is an even number." Let the event B be "the sum of the faces showing is divisible by 3."

(a) **List the outcomes in A, and find P(A).**

$$A = \{(1,1),(1,1),(1,3),(1,3),(1,5),(1,5),(2,2),(2,2),(2,4),(2,4),(2,6),(2,6),(3,3),(3,3),(3,5),(3,5),$$
$$(4,2),(4,2),(4,4),(4,4),(5,1),(5,1),(5,3),(5,3),(5,5),(5,5),(6,2),(6,2),(6,4),(6,4),(6,6),(6,6)\}$$

$P(A) = \frac{6}{36} = \frac{1}{6}$

(b) **List the outcomes in B, and find P(B).**

$B = \{(1,1),(2,2),(3,3),(4,4),(5,5),(6,6)\}$

$P(B) = \frac{6}{36} = \frac{1}{6}$

(c) **List the outcomes in $\mathbf{A \cap B}$, and find $\mathbf{P(A \cap B)}$.**

(d) **Are the events A and B independent? Explain why or why not.**

4.10 Suppose there is a medical screening procedure for a specific cancer that has *sensitivity* = .90, and *specificity* = .95. Suppose the underlying rate of the cancer in the population is .001. Let B be the event "the person has that specific cancer," and let A be the event "the screening procedure gives a positive result."

(a) **What is the probability that a person has the disease given the results of the screening is positive?**

(b) **Does this show that screening is effective in detecting this cancer?**