

Homework 2

Noah Johnson

February 17, 2018

Exercises

4.2 There are two events A and B . $P(A) = .5$ and $P(B) = .3$. The events A and B are independent.

(a) **Find $P(\tilde{A})$**

$$P(\tilde{A}) = 1 - P(A) = 1 - 0.5 = 0.5$$

(b) **Find $P(A \cap B)$**

$$\begin{aligned} P(A \cap B) &= P(B|A) * P(A) \\ &= P(B) * P(A) \quad (\text{since } A \text{ and } B \text{ are independent}) \\ &= .3 * .5 = .15 \end{aligned}$$

(c) **Find $P(A \cup B)$**

$$\begin{aligned} P(A \cup B) &= P(A) + P(B) - P(A \cap B) \\ &= .5 + .3 - .15 = .65 \end{aligned}$$

4.4 There are two events A and B . $P(A) = .7$ and $P(B) = .8$. $P(\tilde{A} \cap \tilde{B}) = .1$.

(a) **Are A and B independent events? Explain why or why not.**

$$\begin{aligned} P(A \cup B) &= 1 - P(\overline{A \cup B}) \\ &= 1 - P(\tilde{A} \cap \tilde{B}) \quad (\text{using De Morgan's Law}) \\ &= 1 - 0.1 = .9 \end{aligned}$$

$$\begin{aligned} P(A \cap B) &= P(A) + P(B) - P(A \cup B) \\ &= .7 + .8 - .9 = .6 \end{aligned}$$

$$\begin{aligned} P(B|A) &= \frac{P(A \cap B)}{P(A)} = \frac{.6}{.7} = \frac{6}{7} \\ &\neq \frac{4}{5} = P(B) \quad \therefore A \text{ and } B \text{ are not independent events} \end{aligned}$$

(b) **Find $P(A \cup B)$**

Refer to the first equation above. $P(A \cup B) = 0.9$.

4.6 Two fair dice, one red and one green, are rolled. Let the event A be “the sum of the faces showing is equal to seven.” Let the event B be “the faces showing on the two dice are equal.”

(a) **List out the sample space of the experiment.**

$$\Omega = \{2, 3, \dots, 12\}$$

- (b) List the outcomes in A , and find $P(A)$.

$$A = \{(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)\}$$

$$P(A) = \frac{6}{36} = \frac{1}{6}$$

- (c) List the outcomes in B , and find $P(B)$.

$$B = \{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)\}$$

$$P(B) = \frac{6}{36} = \frac{1}{6}$$

- (d) List the outcomes in $A \cap B$, and find $P(A \cap B)$.

$$A \cap B = \emptyset$$

$$P(A \cap B) = 0$$

- (e) Are the events A and B independent? Explain why or why not.

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{0}{1/6} = 0$$

$$\neq \frac{1}{6} = P(B) \therefore A \text{ and } B \text{ are not independent events}$$

- (f) How would you describe the relationship between event A and event B ?

As mutually exclusive, since one event occurring precludes the other.

4.8 Two dice are rolled. The red die has been loaded. Its probabilities are $P(1) = P(2) = P(3) = P(4) = \frac{1}{5}$ and $P(5) = P(6) = \frac{1}{10}$. The green die is fair. Let the event A be “the sum of the faces showing is an even number.” Let the event B be “the sum of the faces showing is divisible by 3.”

- (a) List the outcomes in A , and find $P(A)$.

- (b) List the outcomes in B , and find $P(B)$.

- (c) List the outcomes in $A \cap B$, and find $P(A \cap B)$.

- (d) Are the events A and B independent? Explain why or why not.

4.10 Suppose there is a medical screening procedure for a specific cancer that has *sensitivity* = .90, and *specificity* = .95. Suppose the underlying rate of the cancer in the population is .001. Let B be the event “the person has that specific cancer,” and let A be the event “the screening procedure gives a positive result.”

- (a) What is the probability that a person has the disease given the results of the screening is positive?

- (b) Does this show that screening is effective in detecting this cancer?