Homework 3

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Exercises

5.2 A discrete random variable Y has discrete distribution given in the following table:

y_i	$f(y_i)$
0	.1
1	.2
2	.3
5	.4

(a) **Calculate P(0 < Y < 2).**

$$P(0 < Y < 2) = P(Y = 1) = f(1) = .2$$

(b) **Calculate E(Y).**

$$E(Y) = \sum_i y_i f(y_i)$$
$$= 0 * .1 + 1 * .2 + 2 * .3 + 5 * .4 = 2.8$$

(c) **Calculate Var(Y).**

$$Var(Y) = E((Y - E(Y))^2) = E(Y^2) - E(Y)^2$$

$$E(Y^2) = \sum_i y_i^2 f(y_i)$$
$$= 0^2 * .1 + 1^2 * .2 + 2^2 * .3 + 5^2 * .4 = 11.4$$

$$\therefore Var(Y) = 11.4 - (2.8)^2 = 3.56$$

(d) **Let W = 3Y − 1. Calculate E(W).**

$$E(W) = E(3Y - 1) = 3E(Y) + E(-1) = 3 * 2.8 - 1 = 7.4$$

(e) **Calculate Var(W).**

$$E(W^2) = E((3Y - 1)^2) = E(9Y^2 - 6Y + 1) = 9E(Y^2) - 6E(Y) + 1 = 9 * 11.4 - 6 * 2.8 + 1 = 86.8$$

$$Var(W) = E(W^2) - E(W)^2 = 86.8^2 - 7.4^2 = 7479.48$$

5.4 Let Y be *binomial* $(n = 4, \pi = .3)$.

(a) **Calculate the mean and variance by filling in the following table:**

"'r n <- 4 p <- 0.3 ys <- c(0,1,2,3,4) fs <- dbinom(ys, n, p) y.fs <- ys * fs y.y.fs <- ys * ys * fs sums <- c(sum(fs), sum(y.fs), sum(y.y.fs)) "'

y_i	$f(y_i)$	$y_i \times f(y_i)$	$y_i^2 \times f(y_i)$
0	0.2401	0	0
1	0.4116	0.4116	0.4116
2	0.2646	0.5292	1.0584
3	0.0756	0.2268	0.6804
4	0.0081	0.0324	0.1296
Sum	1	1.2	2.28

i. $\mathbf{E}(\mathbf{Y}) = 1.2$

ii. $\mathbf{Var}(\mathbf{Y}) = E(Y^2) - E(Y)^2 = 2.28 - 1.2^2 = 0.84$

(b) **Calculate the mean and variance of Y using Equations 5.7 and 5.8, respectively. Do you get the same as you got in part (a)?**

$$E(Y|n, \pi) = n\pi \tag{5.7}$$

$$Var(Y|n, \pi) = n\pi(1 - \pi) \tag{5.8}$$

So $E(Y|n = 4, \pi = 0.3) = 4 * 0.3 = 1.2$,

and $Var(Y|n = 4, \pi = 0.3) = 4 * 0.3 * (1 - 0.3) = 0.84$.

These are indeed the same values we computed in part (a).