

# Image filtering – Efficient computation of region statistics

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# Finding the mean intensity of all rectangular regions of a given size

```
def region_sums(image, reg_rows, reg_cols):  
    rs = np.zeros((image.shape[0]-reg_rows+1, image.shape[1]-reg_cols+1))  
    for i in range(rs.shape[0]):  
        for j in range(rs.shape[1]):  
            rs[i,j] = np.sum(image[i:i+reg_rows, j:j+reg_cols])  
    return rs
```

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```

**Problem:** lots of repeated computation

# Finding the mean intensity of all rectangular regions of a given size

**Clever idea:** the integral image trick

Let  $I$  be a 2D array representing a gray-level image

We define the integral  $S$  image as follows:

$$S[r,c] = \sum_{i=0}^{r-1} \sum_{j=0}^{c-1} I[i,j]$$

That is,  $S[r,c]$  contains the sum of the pixels in  $I$  that are above and to the left of  $I[r,c]$

Example:

$I =$	4	5	9	0	6	5	8	6
	6	6	6	1	5	7	1	1
	5	2	0	3	1	0	2	6
	4	8	5	1	6	7	5	6
	9	5	6	9	2	4	3	9
$S =$	0	0	0	0	0	0	0	0
	0	4	9	18	18	24	29	37
	0	10	21	36	37	48	60	69
	0	15	28	43	47	59	71	82
	0	19	40	60	65	83	102	118
	0	28	54	80	94	114	137	156

$S$  can easily be computed in using a single pass through the image

# Finding the mean intensity of all rectangular regions

S can easily be computed in using a single pass through the image

```
S = np.zeros((image.shape[0]+1, image.shape[1]+1), dtype=np.int32)
```

```
S[1:,1:] = I
```

0	0	0	0	0	0	0	0	0
0	4	5	9	0	6	5	8	6
0	6	6	6	1	5	7	1	1
0	5	2	0	3	1	0	2	6
0	4	8	5	1	6	7	5	6
0	9	5	6	9	2	4	3	9

```
S = np.cumsum(S,axis=0)
```

0	0	0	0	0	0	0	0	0
0	4	5	9	0	6	5	8	6
0	10	11	15	1	11	12	9	7
0	15	13	15	4	12	12	11	13
0	19	21	20	5	18	19	16	19
0	28	26	26	14	20	23	19	28

```
S = np.cumsum(S,axis=1)
```

0	0	0	0	0	0	0	0	0
0	4	9	18	18	24	29	37	43
0	10	21	36	37	48	60	69	76
0	15	28	43	47	59	71	82	95
0	19	40	60	65	83	102	118	137
0	28	54	80	94	114	137	156	184

# Finding the mean intensity of all rectangular regions of a given size

$$I = \begin{bmatrix} 4 & 5 & 9 & 0 & 6 & 5 & 8 & 6 \\ 6 & 6 & 6 & 1 & 5 & 7 & 1 & 1 \\ 5 & 2 & 0 & 3 & 1 & 0 & 2 & 6 \\ 4 & 8 & 5 & 1 & 6 & 7 & 5 & 6 \\ 9 & 5 & 6 & 9 & 2 & 4 & 3 & 9 \end{bmatrix} \quad S = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 4 & 9 & 18 & 18 & 24 & 29 & 37 & 43 \\ 0 & 10 & 21 & 36 & 37 & 48 & 60 & 69 & 76 \\ 0 & 15 & 28 & 43 & 47 & 59 & 71 & 82 & 95 \\ 0 & 19 & 40 & 60 & 65 & 83 & 102 & 118 & 137 \\ 0 & 28 & 54 & 80 & 94 & 114 & 137 & 156 & 184 \end{bmatrix}$$

Taking advantage of the following observation, the sum of any region can be found by accessing 4 elements of S

For example, the sum of the 2-by-3 region with upper-left corner [1,3] is

$$R[1,3] = a - b - c + d$$

The diagram illustrates the calculation of the sum of a 2x3 region R[1,3] using the sum table S. The region R[1,3] is highlighted in green in the first grid. The calculation is shown as the sum of four 2x2 regions (a, b, c, d) derived from S, with overlapping areas being subtracted or added accordingly. The regions are defined by their upper-left corners in the original image grid I:

- a** (blue): 2x2 region starting at (1,1) with values [6, 6, 1, 5].
- b** (yellow): 2x2 region starting at (1,3) with values [6, 6, 1, 5].
- c** (grey): 2x2 region starting at (3,1) with values [5, 2, 0, 3].
- d** (red): 2x2 region starting at (3,3) with values [3, 1, 0, 2].

# Finding the mean intensity of all rectangular regions of a given size

$$I = \begin{bmatrix} 4 & 5 & 9 & 0 & 6 & 5 & 8 & 6 \\ 6 & 6 & 6 & 1 & 5 & 7 & 1 & 1 \\ 5 & 2 & 0 & 3 & 1 & 0 & 2 & 6 \\ 4 & 8 & 5 & 1 & 6 & 7 & 5 & 6 \\ 9 & 5 & 6 & 9 & 2 & 4 & 3 & 9 \end{bmatrix} \quad S = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 4 & 9 & 18 & 24 & 29 & 37 & 43 & 0 \\ 0 & 10 & 21 & 36 & 48 & 60 & 69 & 76 & 0 \\ 0 & 15 & 28 & 43 & 59 & 71 & 82 & 95 & 0 \\ 0 & 19 & 40 & 60 & 83 & 102 & 118 & 137 & 0 \\ 0 & 28 & 54 & 80 & 94 & 114 & 137 & 156 & 184 \end{bmatrix}$$

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# Finding the mean intensity of all rectangular regions of a given size

 $I =$ 

4	5	9	0	6	5	8	6
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5	2	0	3	1	0	2	6
4	8	5	1	6	7	5	6
9	5	6	9	2	4	3	9

 $S =$ 

0	0	0	0	0	0	0	0	0
0	4	9	18	18	24	29	37	43
0	10	21	36	37	48	60	69	76
0	15	28	43	47	59	71	82	95
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$$R = A - B - C + D$$

$$= \begin{array}{|c|c|c|c|c|c|} \hline 36 & 37 & 48 & 60 & 69 & 76 \\ \hline 43 & 47 & 59 & 71 & 82 & 95 \\ \hline 60 & 65 & 83 & 102 & 118 & 137 \\ \hline 80 & 94 & 114 & 137 & 156 & 184 \\ \hline \end{array} - \begin{array}{|c|c|c|c|c|c|} \hline 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 18 & 18 & 24 & 29 & 37 & 43 \\ \hline 36 & 37 & 48 & 60 & 69 & 76 \\ \hline 43 & 47 & 59 & 71 & 82 & 95 \\ \hline \end{array} - \begin{array}{|c|c|c|c|c|c|} \hline 0 & 10 & 21 & 36 & 37 & 48 \\ \hline 0 & 15 & 28 & 43 & 47 & 59 \\ \hline 0 & 19 & 40 & 60 & 65 & 83 \\ \hline 0 & 28 & 54 & 80 & 94 & 114 \\ \hline \end{array} + \begin{array}{|c|c|c|c|c|c|} \hline 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 4 & 9 & 18 & 18 & 24 \\ \hline 0 & 10 & 21 & 36 & 37 & 48 \\ \hline 0 & 15 & 28 & 43 & 47 & 59 \\ \hline \end{array} = \begin{array}{|c|c|c|c|c|c|} \hline 36 & 27 & 27 & 24 & 32 & 28 \\ \hline 25 & 18 & 16 & 17 & 16 & 17 \\ \hline 24 & 19 & 16 & 18 & 21 & 26 \\ \hline 37 & 34 & 29 & 29 & 27 & 34 \\ \hline \end{array}$$

# Finding the mean intensity of all rectangular regions of a given size

```
def integral_image(image):  
    S = np.zeros((image.shape[0]+1,image.shape[1]+1))  
    S [1:,1:] = np.cumsum(np.cumsum(image,axis=0),axis=1)  
    return S
```

```
def region_sums(image,r, c):  
    S = integral_image(image)  
    return S[r:,c:] - S[r,:-c] - S[:-r,c:] + S[:-r,:-c]
```

