

# Image filtering – Efficient computation of region statistics

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# Finding the mean intensity of all rectangular regions of a given size

```
def region_sums(image, reg_rows, reg_cols):  
    rs = np.zeros((image.shape[0]-reg_rows+1, image.shape[1]-reg_cols+1))  
    for i in range(rs.shape[0]):  
        for j in range(rs.shape[1]):  
            rs[i,j] = np.sum(image[i:i+reg_rows, j:j+reg_cols])  
    return rs
```

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```

**Problem:** lots of repeated computation

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**Clever idea:** the integral image trick

Let  $I$  be a 2D array representing a gray-level image

We define the integral  $S$  image as follows:

$$S[r,c] = \sum_{i=0}^{r-1} \sum_{j=0}^{c-1} I[i,j]$$

That is,  $S[r,c]$  contains the sum of the pixels in  $I$  that are above and to the left of  $I[r,c]$

Example:

4	5	9	0	6	5	8	6
6	6	6	1	5	7	1	1
5	2	0	3	1	0	2	6
4	8	5	1	6	7	5	6
9	5	6	9	2	4	3	9

$I =$

$S =$

0	0	0	0	0	0	0	0	0
0	4	9	18	18	24	29	37	43
0	10	21	36	37	48	60	69	76
0	15	28	43	47	59	71	82	95
0	19	40	60	65	83	102	118	137
0	28	54	80	94	114	137	156	184

$S$  can easily be computed in using a single pass through the image

# Finding the mean intensity of all rectangular regions

S can easily be computed in using a single pass through the image

```
S = np.zeros((image.shape[0]+1,image.shape[1]+1),dtype=np.int32)
```

```
S[1:,1:] = I
```

0	0	0	0	0	0	0	0	0
0	4	5	9	0	6	5	8	6
0	6	6	6	1	5	7	1	1
0	5	2	0	3	1	0	2	6
0	4	8	5	1	6	7	5	6
0	9	5	6	9	2	4	3	9

```
S = np.cumsum(S, axis=0)
```

0	0	0	0	0	0	0	0	0
0	4	5	9	0	6	5	8	6
0	10	11	15	1	11	12	9	7
0	15	13	15	4	12	12	11	13
0	19	21	20	5	18	19	16	19
0	28	26	26	14	20	23	19	28

```
S = np.cumsum(S, axis=1)
```

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Taking advantage of the following observation, the sum of any region can be found by accessing 4 elements of  $S$

For example, the sum of the 2-by-3 region with upper-left corner [1,3] is

$$\begin{array}{ccccccccc}
 4 & 5 & 9 & 0 & 6 & 5 & 8 & 6 & \\
 6 & 6 & 6 & 1 & 5 & 7 & 1 & 1 & \\
 5 & 2 & 0 & 3 & 1 & 0 & 2 & 6 & \\
 4 & 8 & 5 & 1 & 6 & 7 & 5 & 6 & \\
 9 & 5 & 6 & 9 & 2 & 4 & 3 & 9 & \\
 \end{array} = 
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 9 & 5 & 6 & 9 & 2 & 4 & 3 & 9 & \\
 \end{array} + 
 \begin{array}{ccccccccc}
 4 & 5 & 9 & 0 & 6 & 5 & 8 & 6 & \\
 6 & 6 & 6 & 1 & 5 & 7 & 1 & 1 & \\
 5 & 2 & 0 & 3 & 1 & 0 & 2 & 6 & \\
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 \end{array}$$

$$R[1,3] = a - b - c + d$$

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$$R = A - B - C + D$$

$$= \begin{matrix} 36 & 37 & 48 & 60 & 69 & 76 \\ 43 & 47 & 59 & 71 & 82 & 95 \\ 60 & 65 & 83 & 102 & 118 & 137 \\ 80 & 94 & 114 & 137 & 156 & 184 \end{matrix} - \begin{matrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 18 & 18 & 24 & 29 & 37 & 43 \\ 36 & 37 & 48 & 60 & 69 & 76 \\ 43 & 47 & 59 & 71 & 82 & 95 \end{matrix} - \begin{matrix} 0 & 10 & 21 & 36 & 37 & 48 \\ 0 & 15 & 28 & 43 & 47 & 59 \\ 0 & 19 & 40 & 60 & 65 & 83 \\ 0 & 28 & 54 & 80 & 94 & 114 \end{matrix} + \begin{matrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 4 & 9 & 18 & 18 & 24 \\ 0 & 10 & 21 & 36 & 37 & 48 \\ 0 & 15 & 28 & 43 & 47 & 59 \end{matrix} = \begin{matrix} 36 & 27 & 27 & 24 & 32 & 28 \\ 25 & 18 & 16 & 17 & 16 & 17 \\ 24 & 19 & 16 & 18 & 21 & 26 \\ 37 & 34 & 29 & 29 & 27 & 34 \end{matrix}$$

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```
def integral_image(image):
    S = np.zeros((image.shape[0]+1,image.shape[1]+1))
    S [1:,1:] = np.cumsum(np.cumsum(image, axis=0), axis=1)
    return S
```

```
def region_sums(image,r, c):
    S = integral_image(image)
    return S[r:,c:] - S[r:,:-c] - S[:,-r,c:] + S[:-r,:-c]
```

