Exercise 6 - Naïve Bayes

Friday, September 18, 2020 2:32 PM

The Naive Bayes Classifier

Naive Bayes Classifier

Assume target function $f : X \to V$, where each instance x described by attributes $\langle a_1, a_2 \ldots a_n \rangle$.
Most probable value of $f(x)$ is:

$$v_{MAP} = \underset{v_j \in V}{\text{argmax}} P(v_j | a_1, a_2 \ldots a_n)$$

$$v_{MAP} = \underset{v_j \in V}{\text{argmax}} \frac{P(a_1, a_2 \ldots a_n | v_j) P(v_j)}{P(a_1, a_2 \ldots a_n)}$$

$$= \underset{v_j \in V}{\text{argmax}} P(a_1, a_2 \ldots a_n | v_j) P(v_j)$$

Naive Bayes assumption:

$$P(a_1, a_2 \ldots a_n | v_j) = \prod_i P(a_i | v_j)$$

which gives

Naive Bayes classifier: $v_{NB} = \underset{v_j \in V}{\text{argmax}} P(v_j) \prod_i P(a_i | v_j)$

The Naive Bayes Classifier

Given a test example with attribute values $(a_1,\ldots,a_n)$ assign x to the class c_i that maximizes $p(x|c_i)$

$$\text{pred}(x) = \text{argmax } p(c_i) * p(a_1|c_i) * p(a_2|c_i) * \ldots * p(a_n|c_i)$$

$p\_att\_given\_class_{0\ldots n} = 1 \vee 0$ depends on given data

In order to do this, we need to estimate, for every class c_i,
$p(c_i)$ and $p(a_j|c_i)$ for j in 1,...,n for every possible value of a_j

· Exercise # 3

c_0 c_1 c_2

p_class =[0.33,0.33,0.33]

a_0 a_1 a_2 a_3 a_4

p_att_given_class =[[0.72 0.21 0.89 0.47 0.64]
[0.32 0.82 0.54 0.82 0.17]
[0.76 0.65 0.74 0.31 0.75]]

where *p_class[i]* represents the probability that an example belongs to class i and p_att_given_class[i,j] represents the probability that attribute j in an example of class i is equal to 1.

3. How would the Naïve Bayes classifier classify example [1,1,1,0,0]?

which class has highest prob

which class has highest prob using given data?

[1, 1, 1, 0, 0]

Let $C_i = 0$

$P(C_0) = .33$ *

$P(a_0 | C_0) = g_{a_0} = .72$ *

$P(a_1 | C_0) = g_{a_1} = .21$ *

$P(a_2 | C_0) = g_{a_2} = .89$ *

$P(a_3 | C_0) = 1 - g_{a_3} = 1 - .47$ *

$P(a_4 | C_0) = 1 - g_{a_4} = 1 - .64$ **

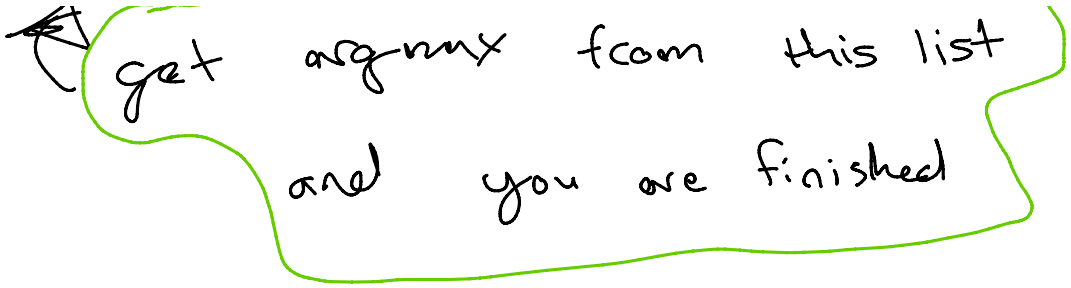
$P(\text{test point} \mid 0^{\text{th}} \text{ class})$

↳ $**_0$

Repeat for all classes (1 & 2) and get highest probability = value

[$**_0$, $**_1$, $**_2$]

get argmax from this list


get argmax from this list
and you are finished