Comparing Barnes-Hut Tree structures with Physical and Computational Metrics

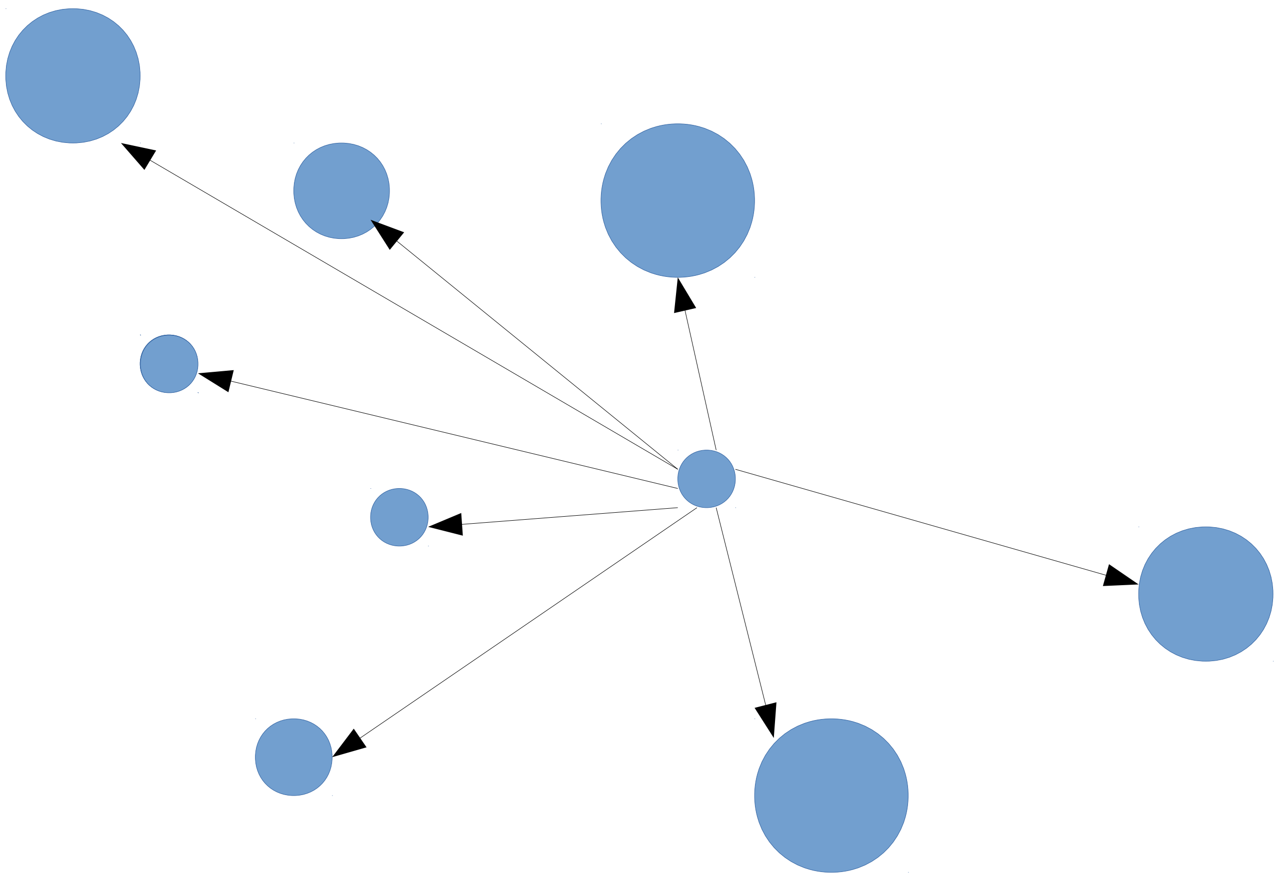
TIME

What These Dazzling James Webb Te
Images Mean for Space

How Do We Predict the Movement of the Planets, Stars, and Galaxies?

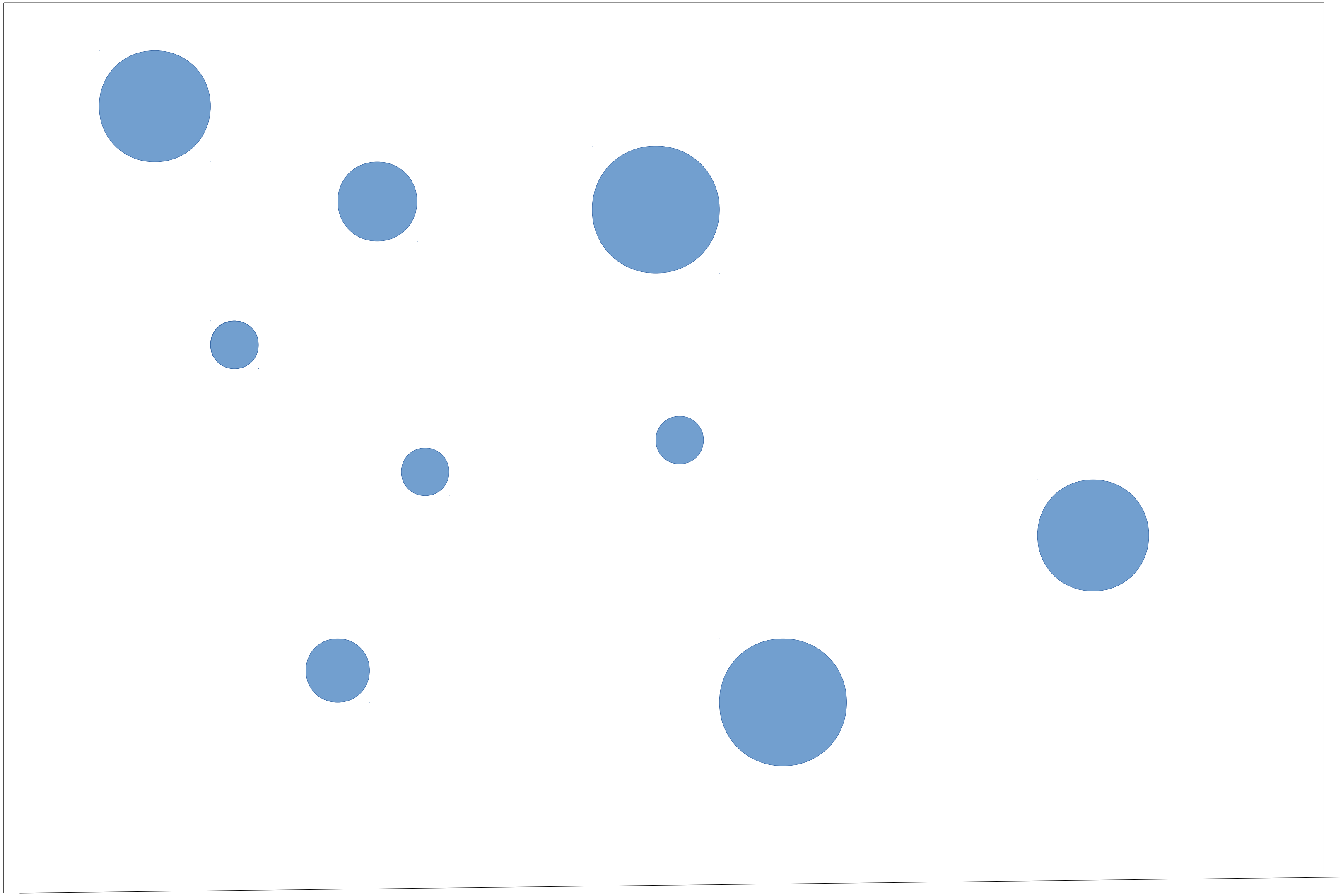
$$F = G\frac{m_1 m_2}{r^2}$$

Direct Summation



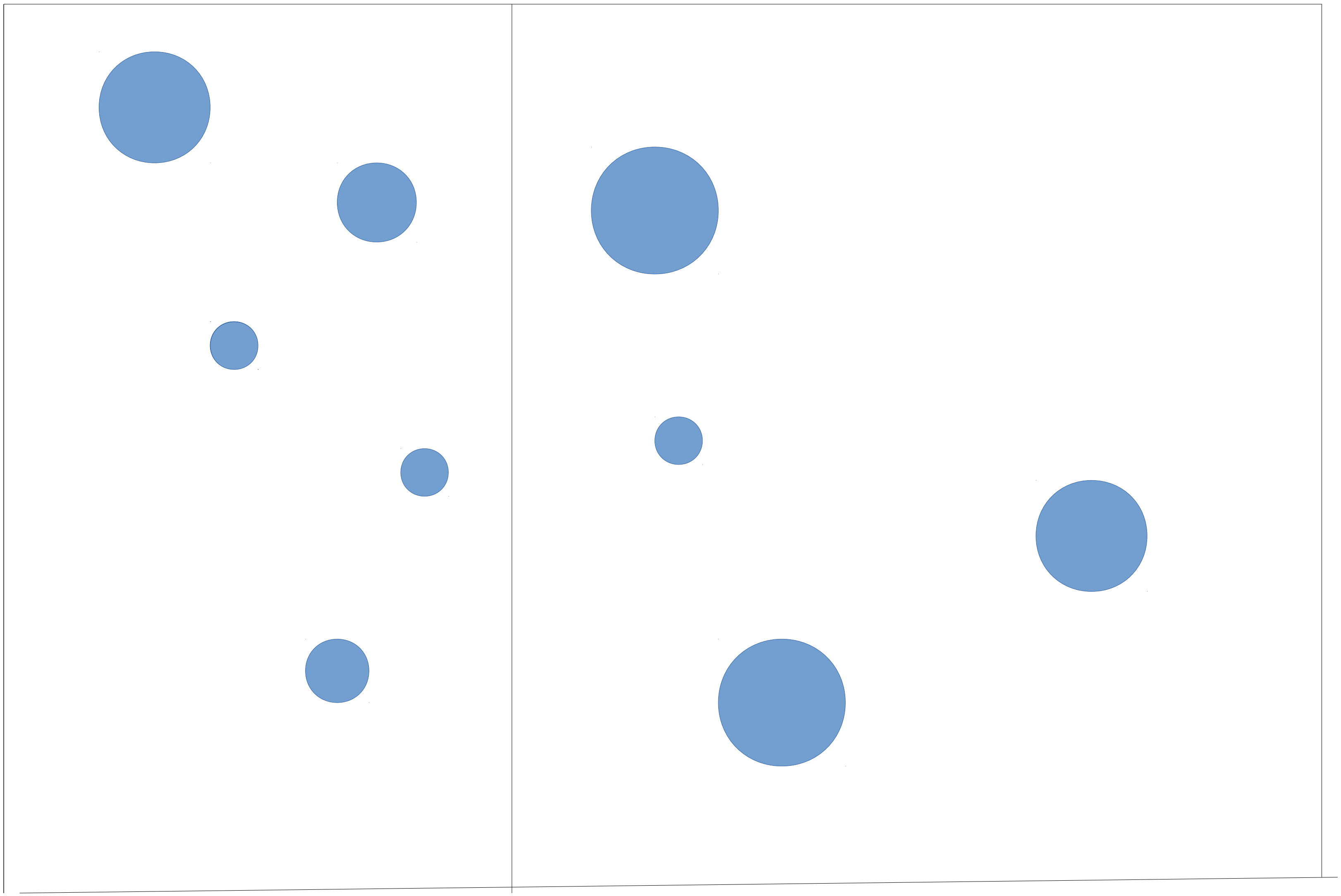
For every particle calculate the force between that particle and every other particle.

Barnes-Hut: Binary Tree



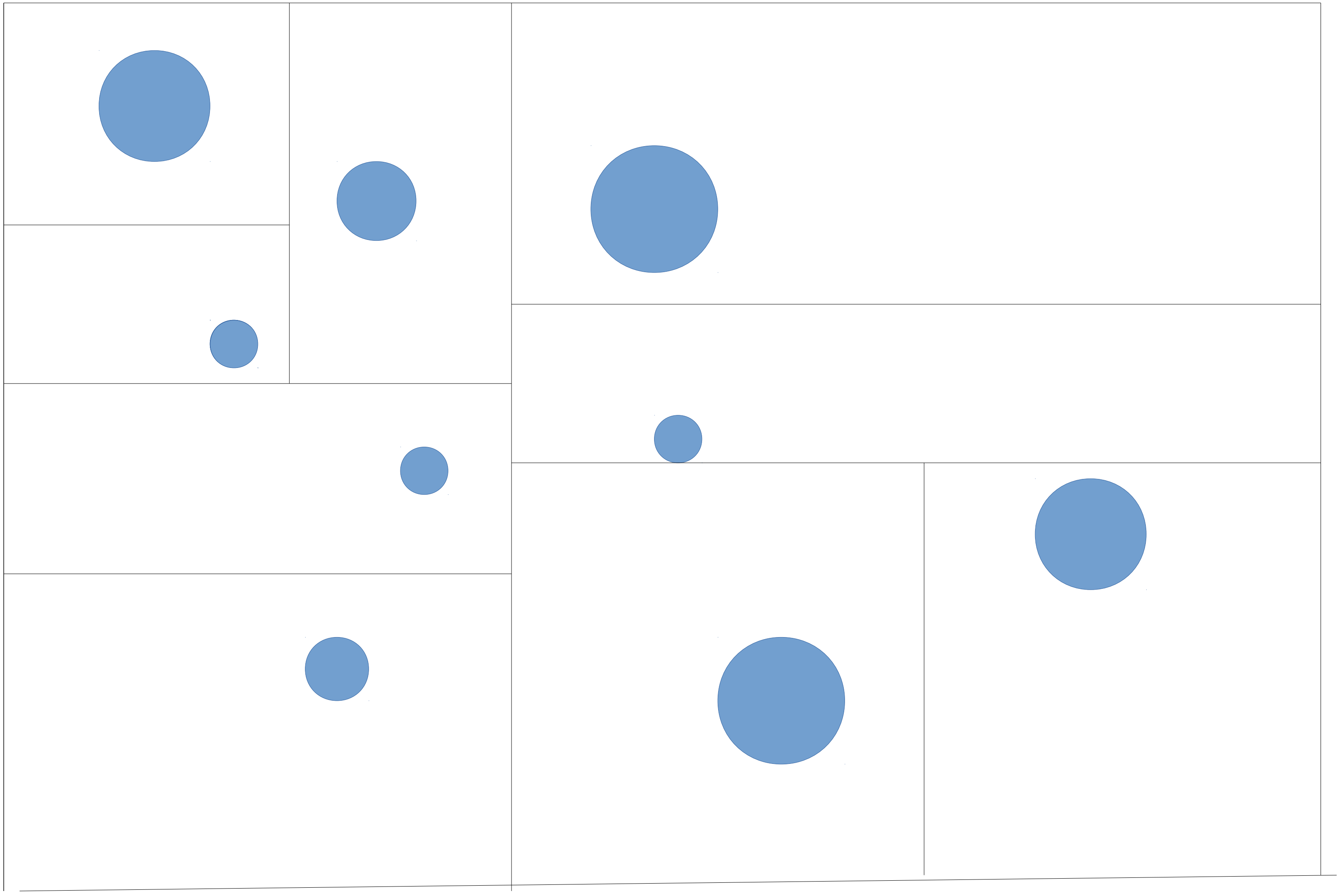
Put a box around all the particles

Barnes-Hut: Binary Tree



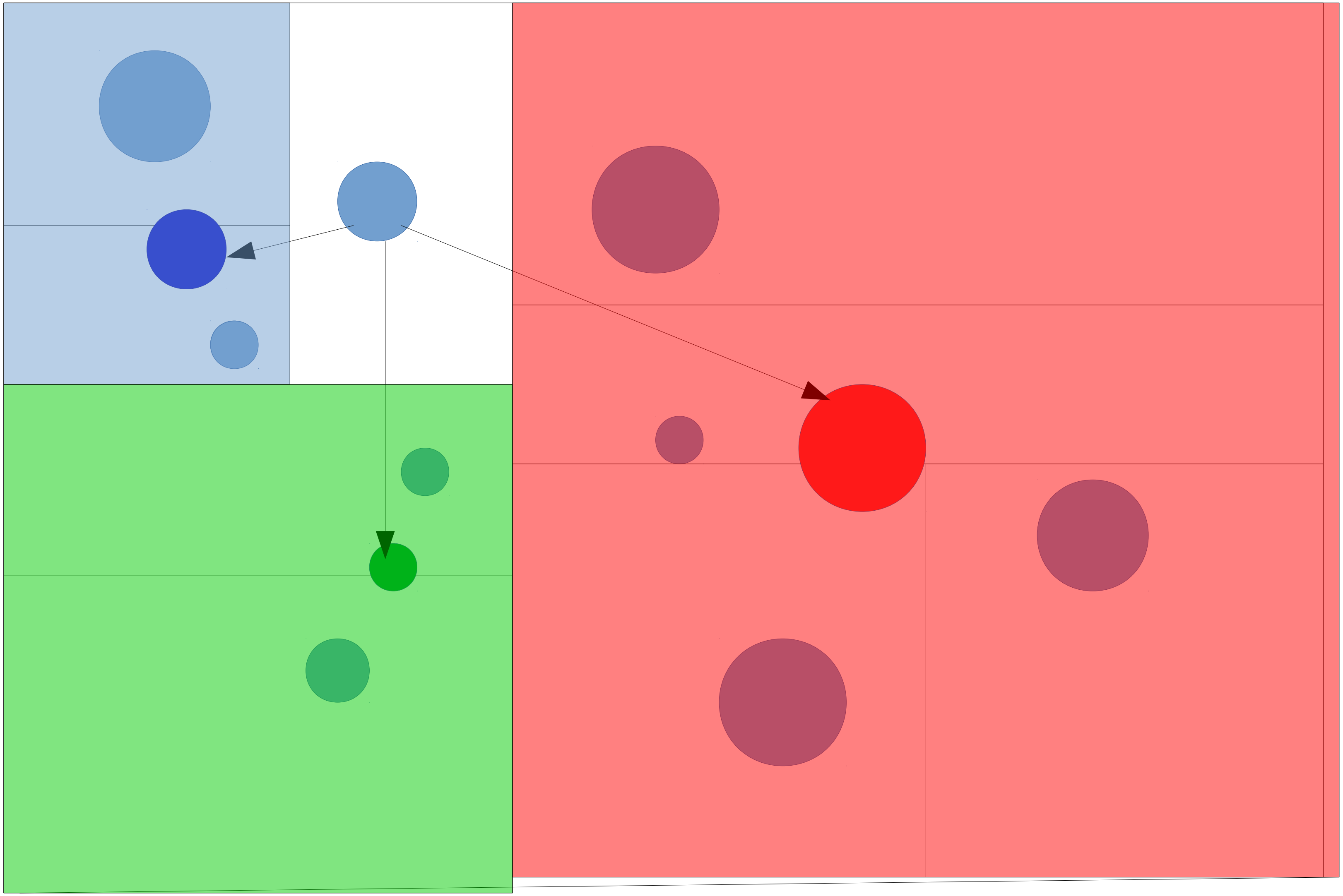
Cut it in half such that each new box has around the same amount of particles in it

Barnes-Hut: Binary Tree



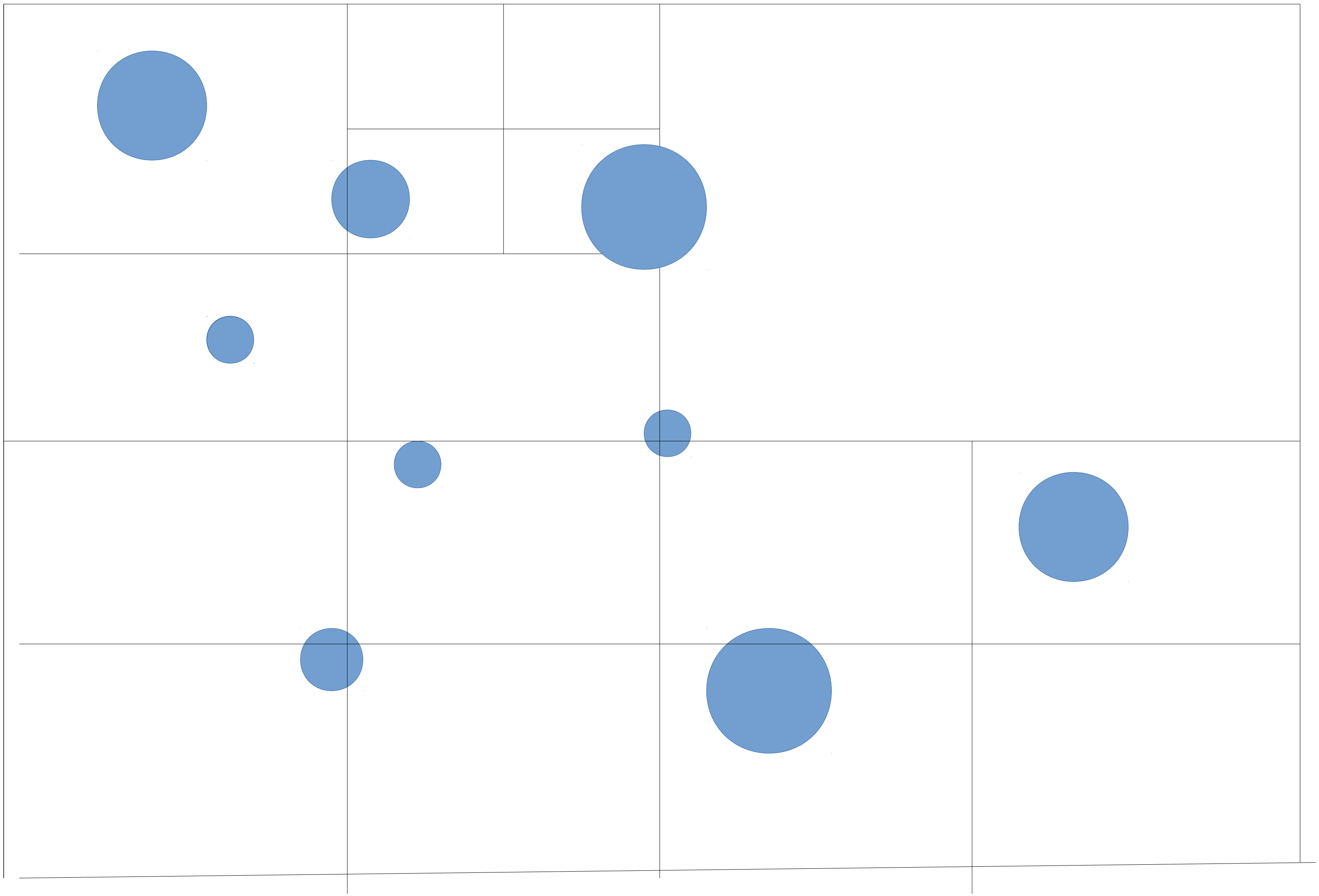
Keep cutting up the boxes until there is one particle per box

Barnes-Hut: Binary Tree



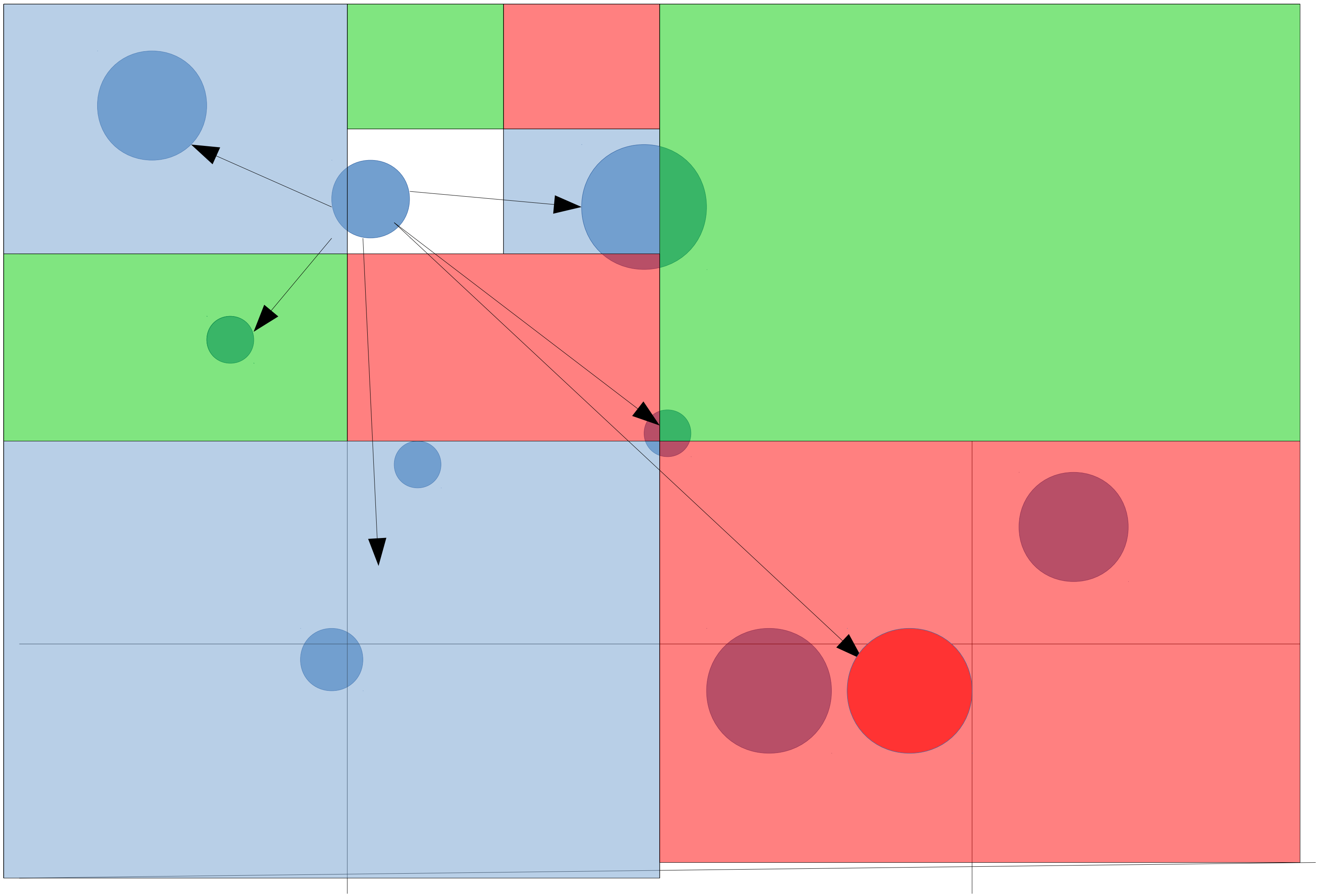
For each box that a particle is in, attract it to that box's neighbor's center of mass

Barnes-Hut: Octree



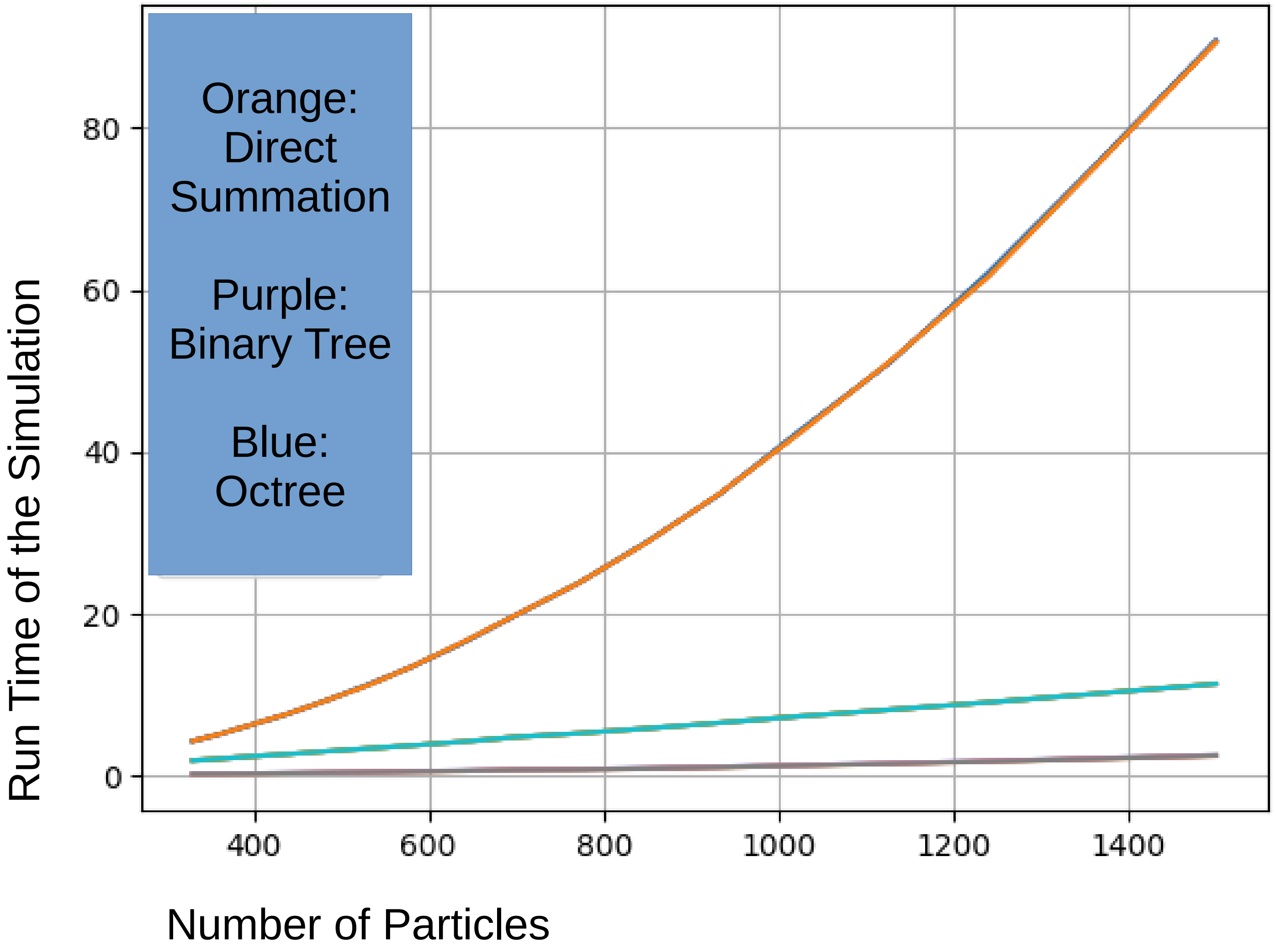
Instead of cutting the boxes by particles just
cut them in fourths

Barnes-Hut: Octree

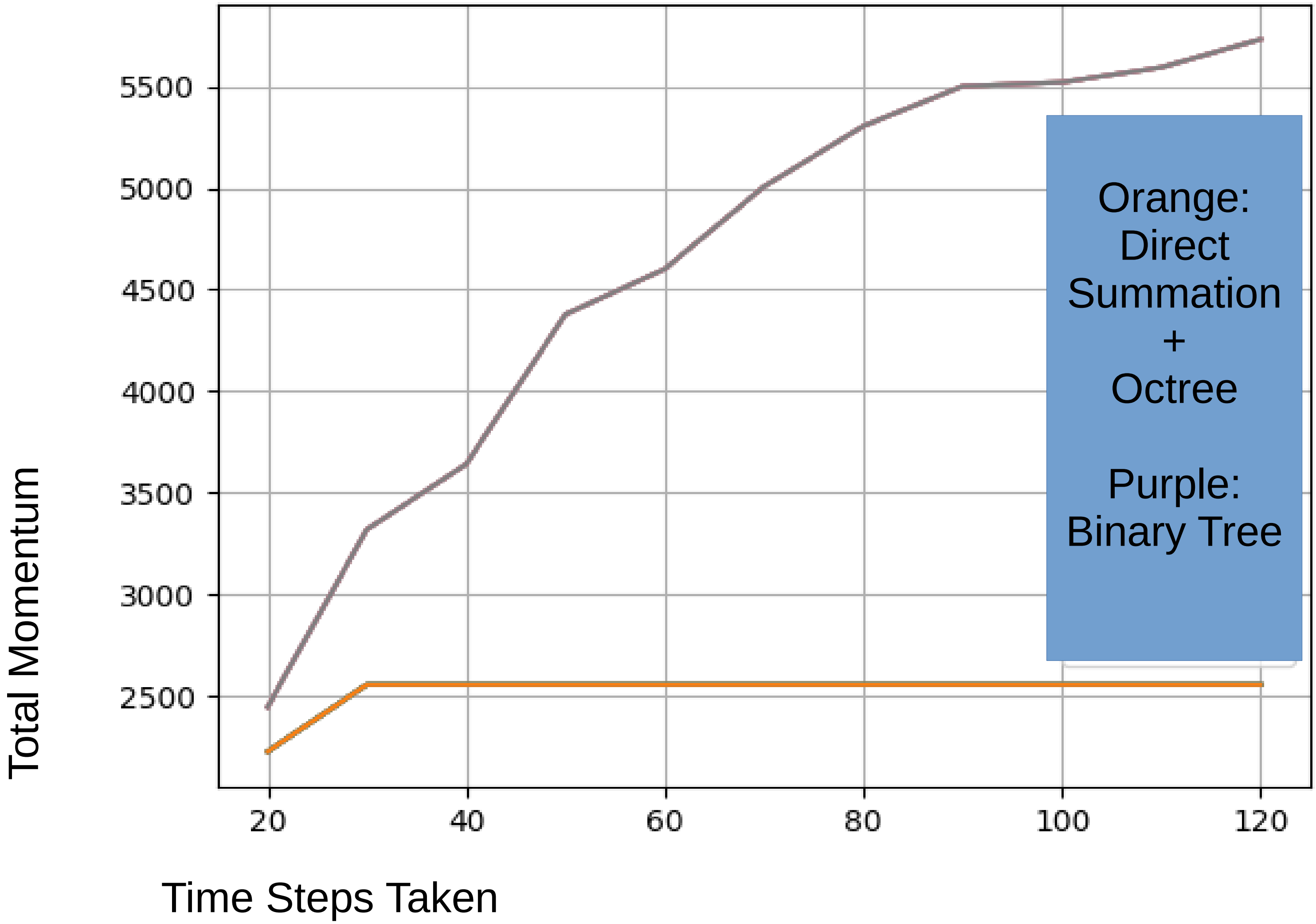


For each box a particle is in attract that particle
to all its neighbors' centers of mass

O(n^2) vs O(nlog(n))



So Why not Just use the Binary tree



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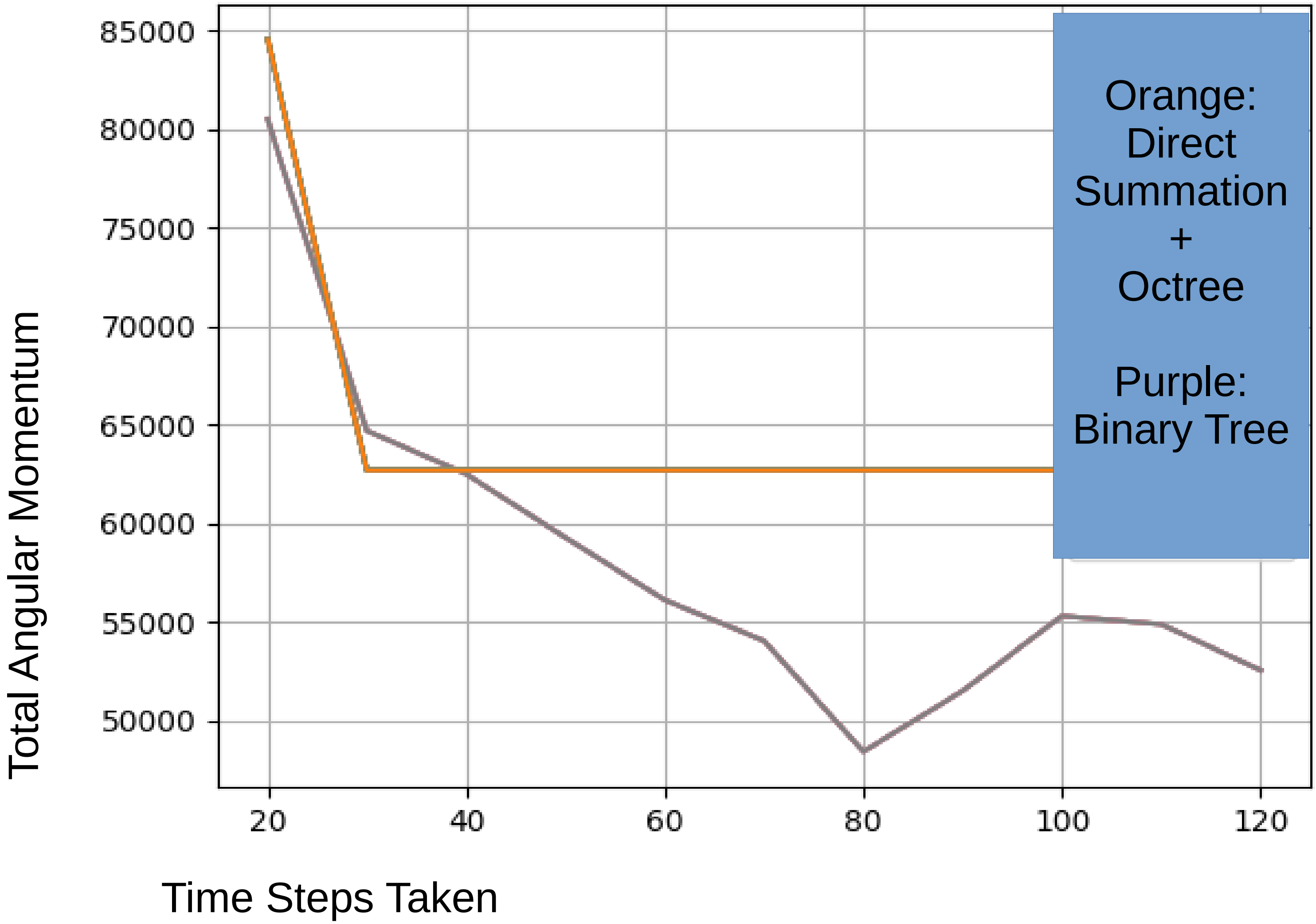

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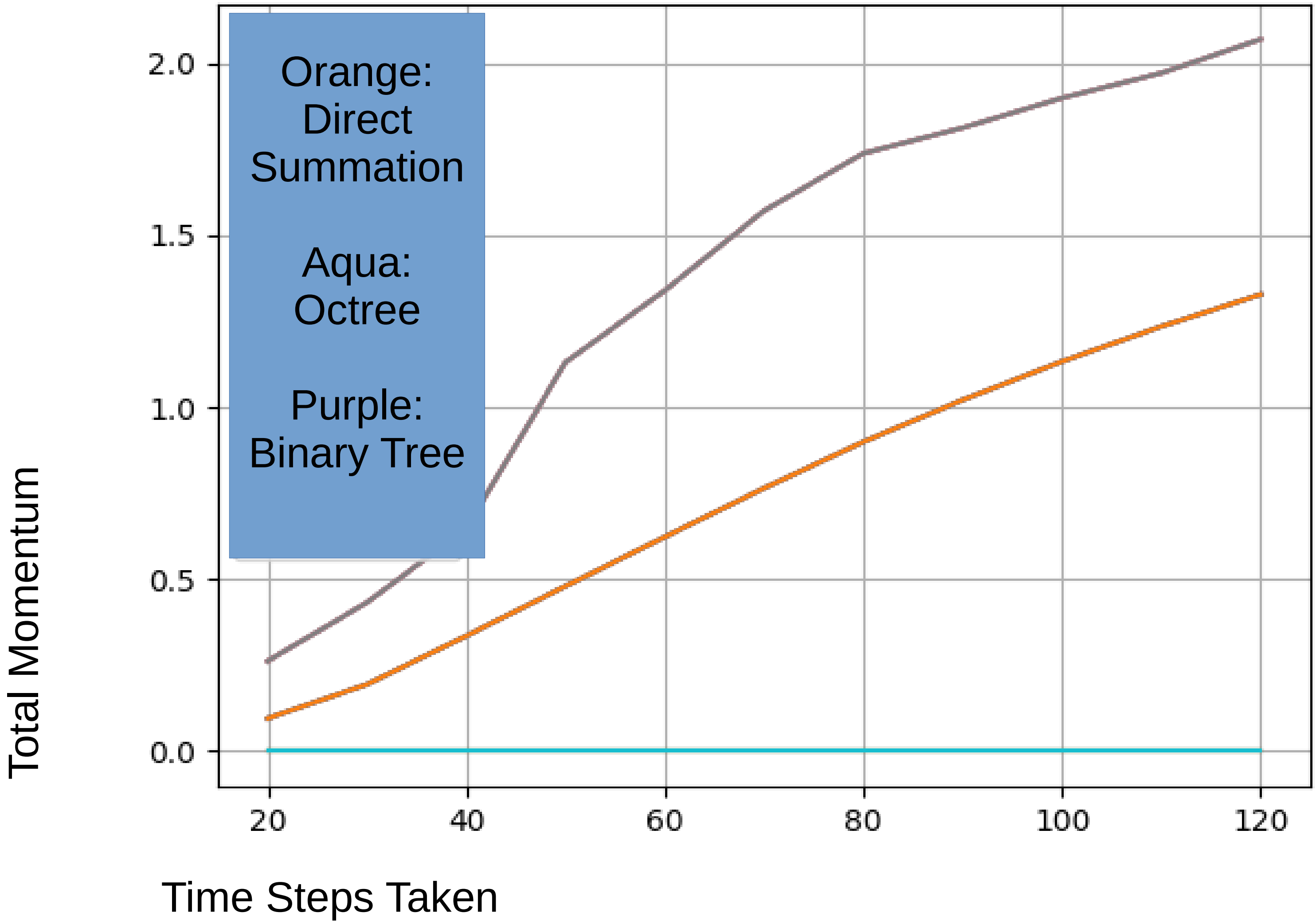
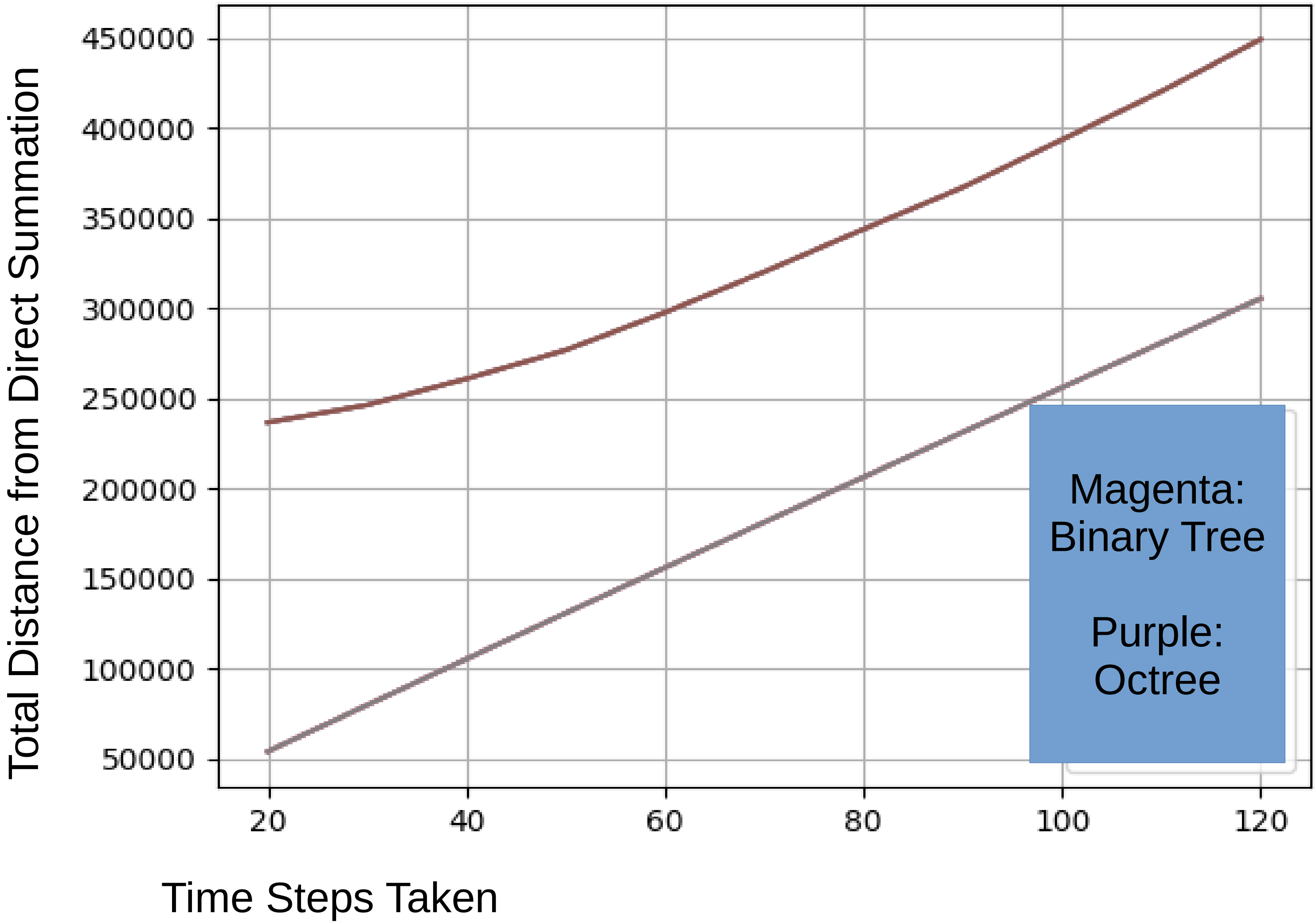


Why are There Only Two Lines?



How do we use the forces to calculate the movement?

Leapfrog Integrator:

O(dt^2)

$$x_{i+1} = x_i + v_{i+1/2}\,\delta t,$$

$$v_{i+3/2} = v_{i+1/2} + f(x_{i+1})\,\delta t,$$

Time reversible meaning
it is very stable

Runge Kutta Integrator:

O(dt^4)

$$\frac{dy}{dt} = f(t, y), \quad y(t_0) = y_0.$$

$$y_{n+1} = y_n + \frac{1}{6}\left(k_1 + 2k_2 + 2k_3 + k_4\right)h,$$
$$t_{n+1} = t_n + h$$

$$k_1 = f(t_n, y_n),$$
$$k_2 = f\left(t_n + \frac{h}{2}, y_n + h\frac{k_1}{2}\right),$$
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Averages 4 slopes
together to get very
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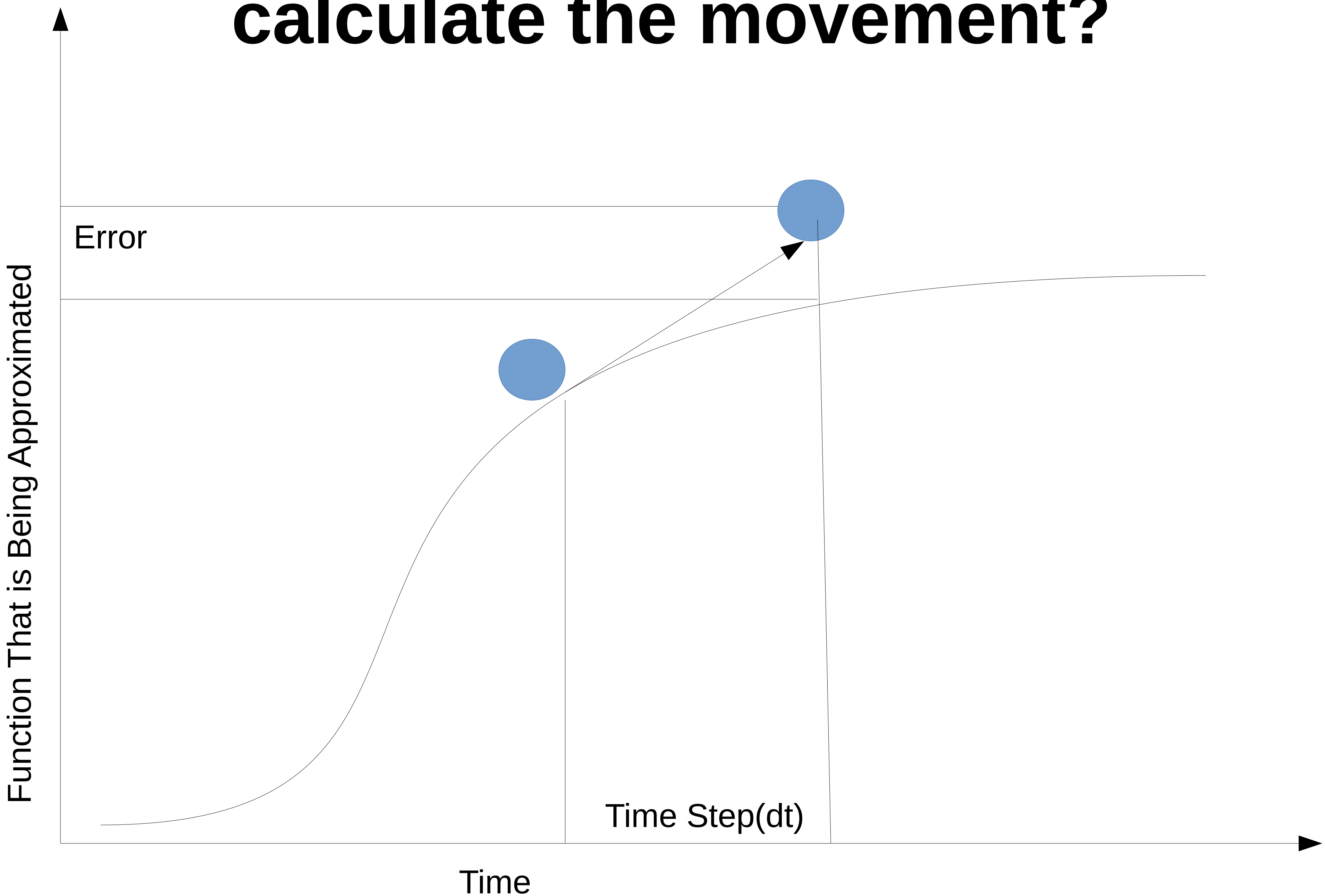

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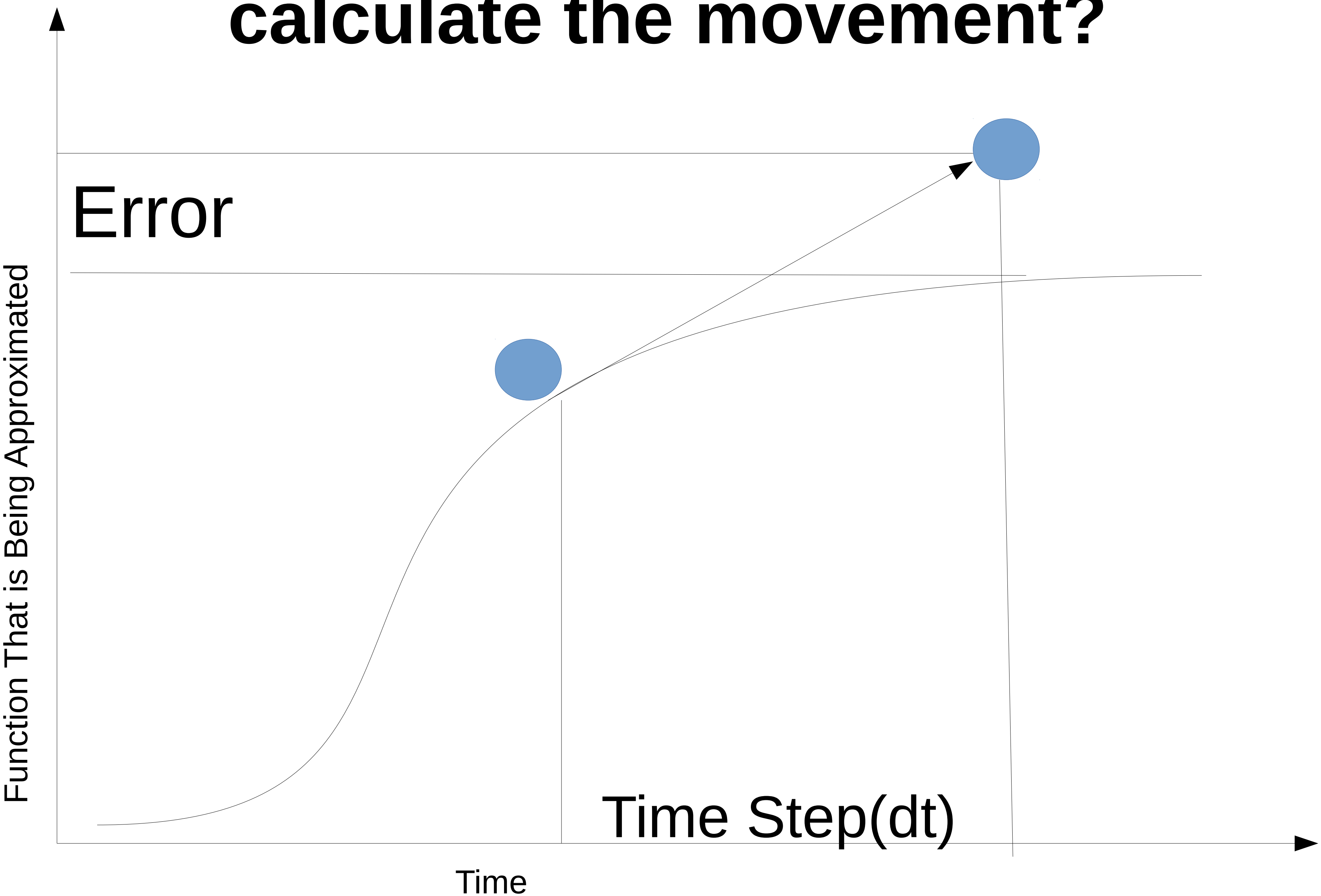

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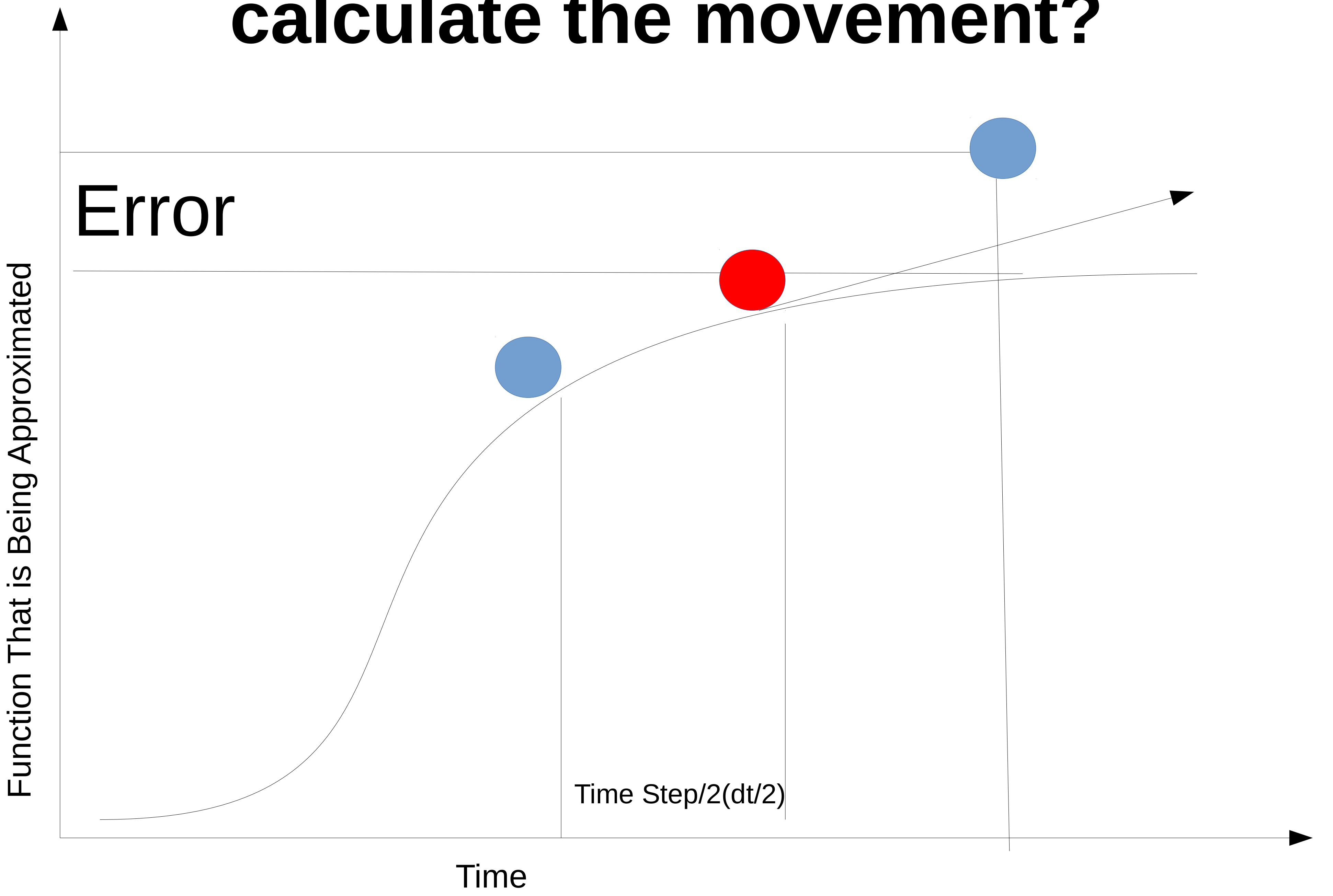

http://www.physics.drexel.edu/~steve/Courses/Comp_Phys/Integrators/leapfrog/

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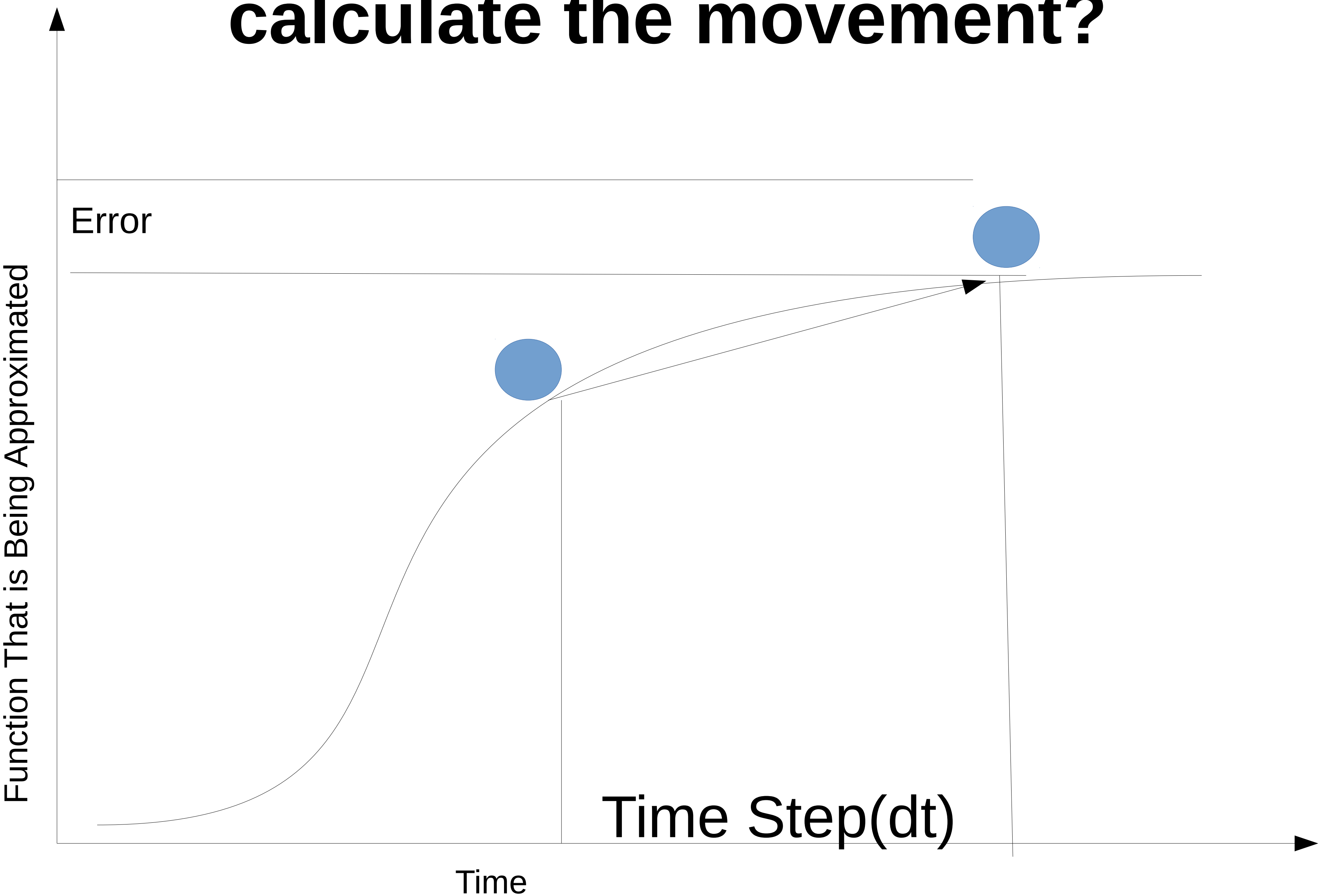

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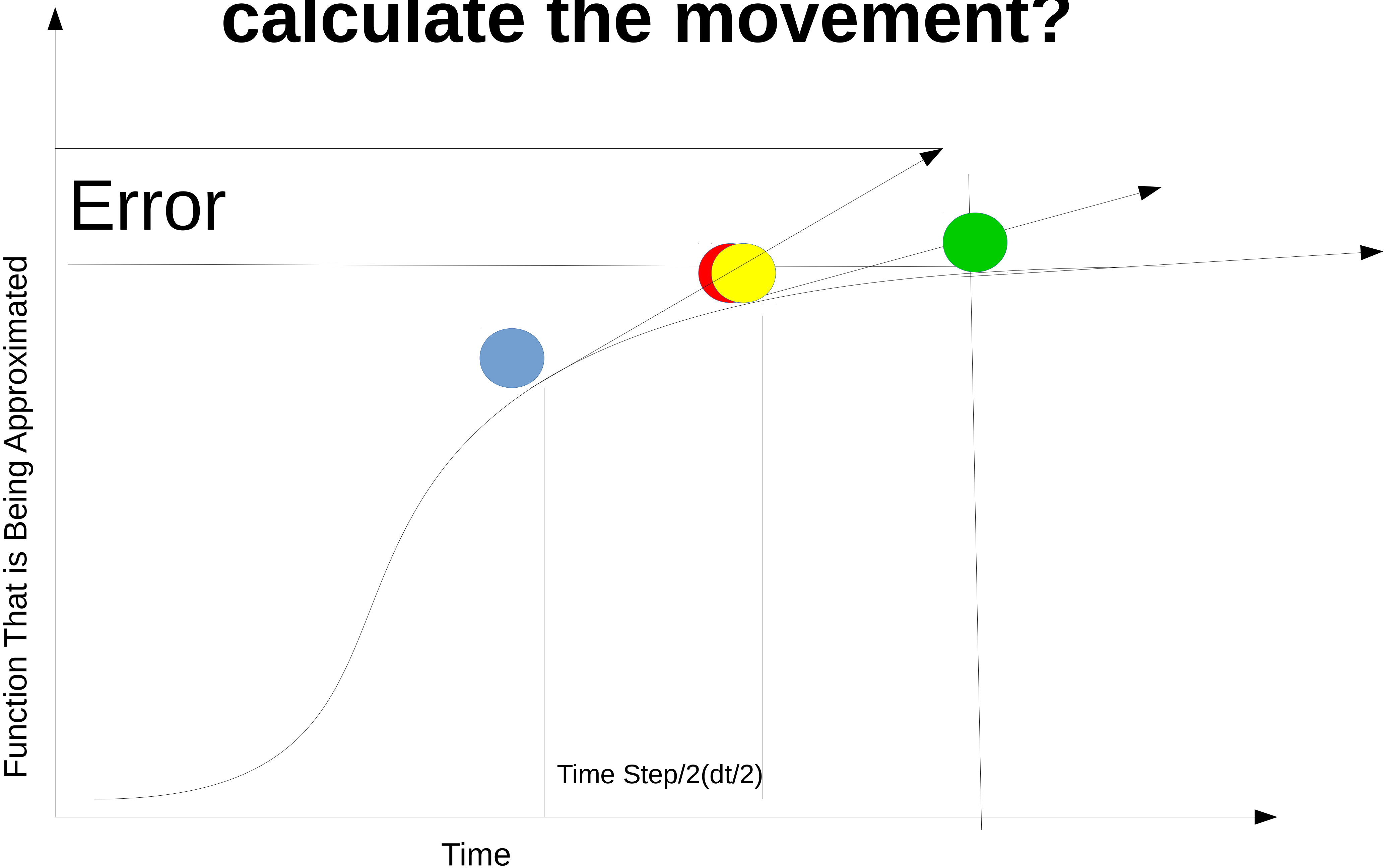

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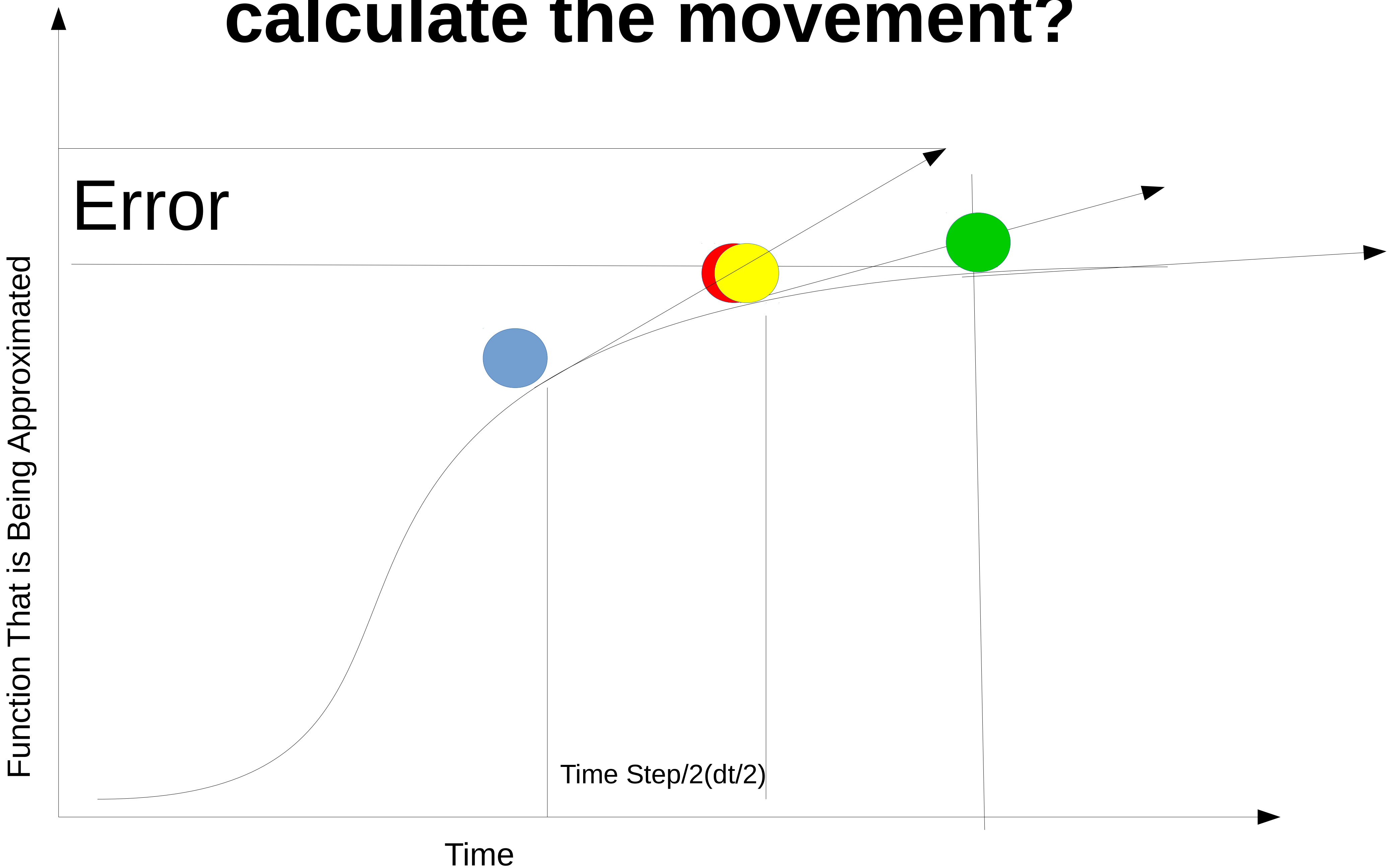

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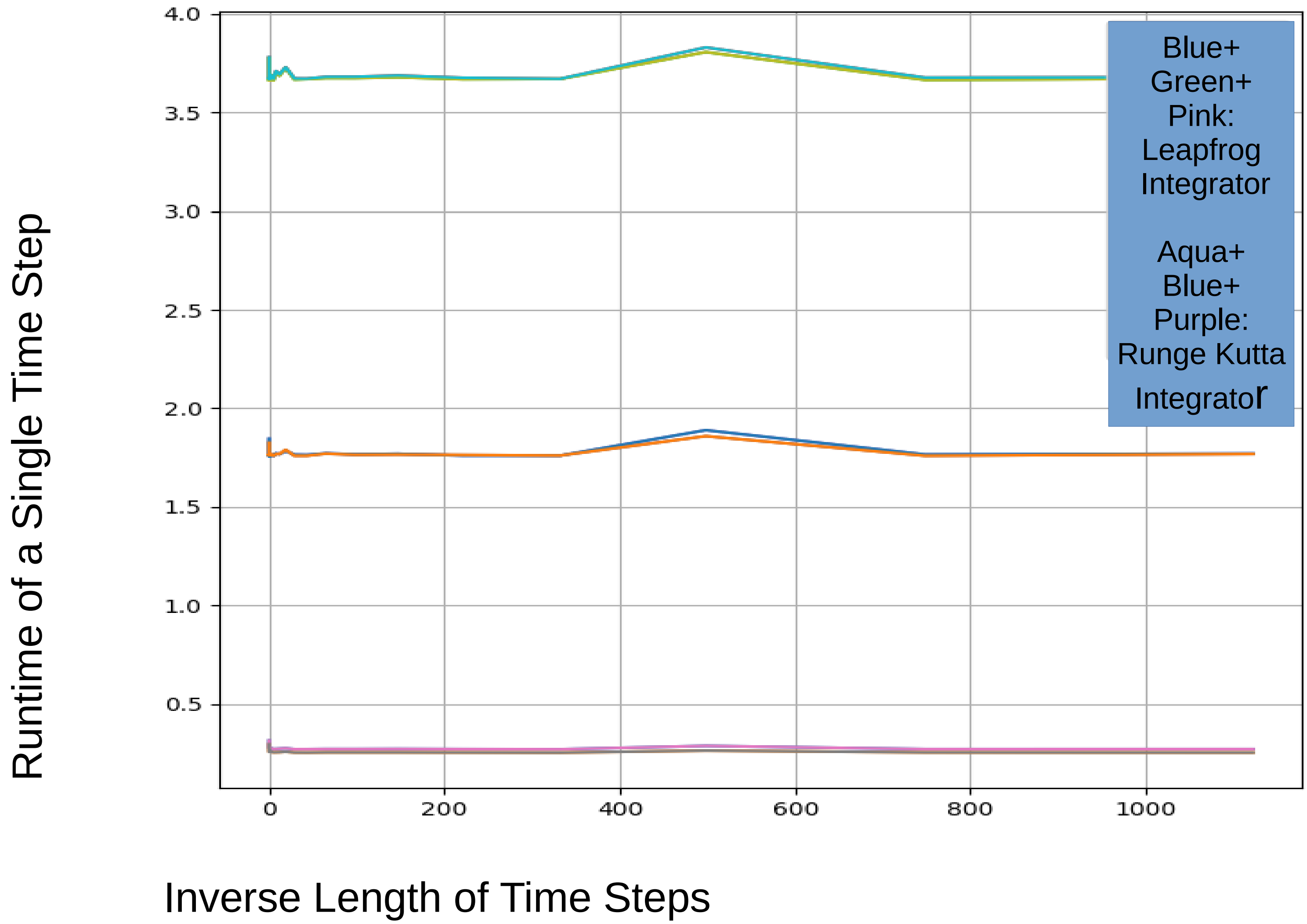
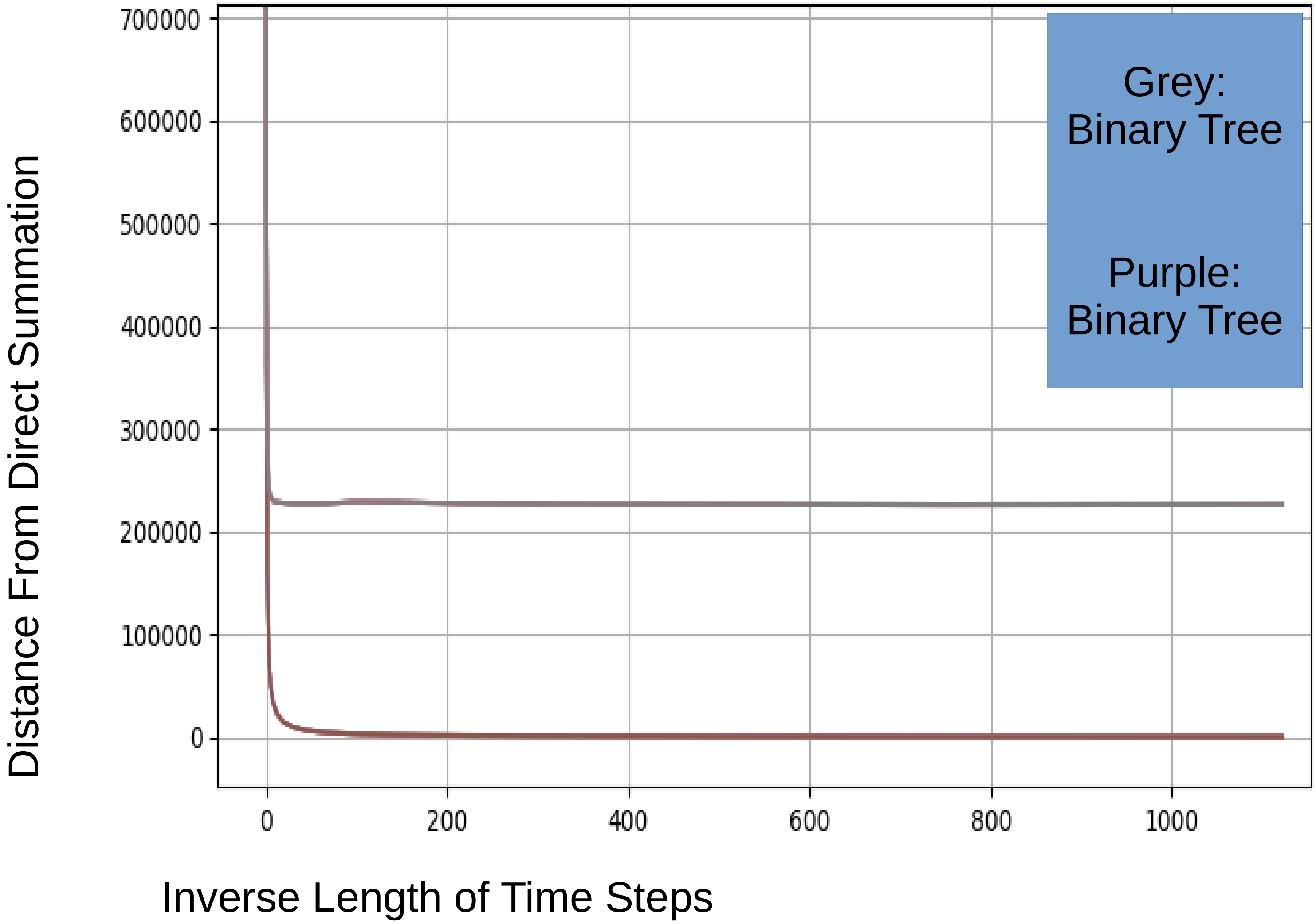
How do These Integration Methods Work in Practice?



What about accuracy?

All the previous graphs have shown both integration methods but there has been no variation in accuracy

How Does Length of the Time Step Affect the Simulation?



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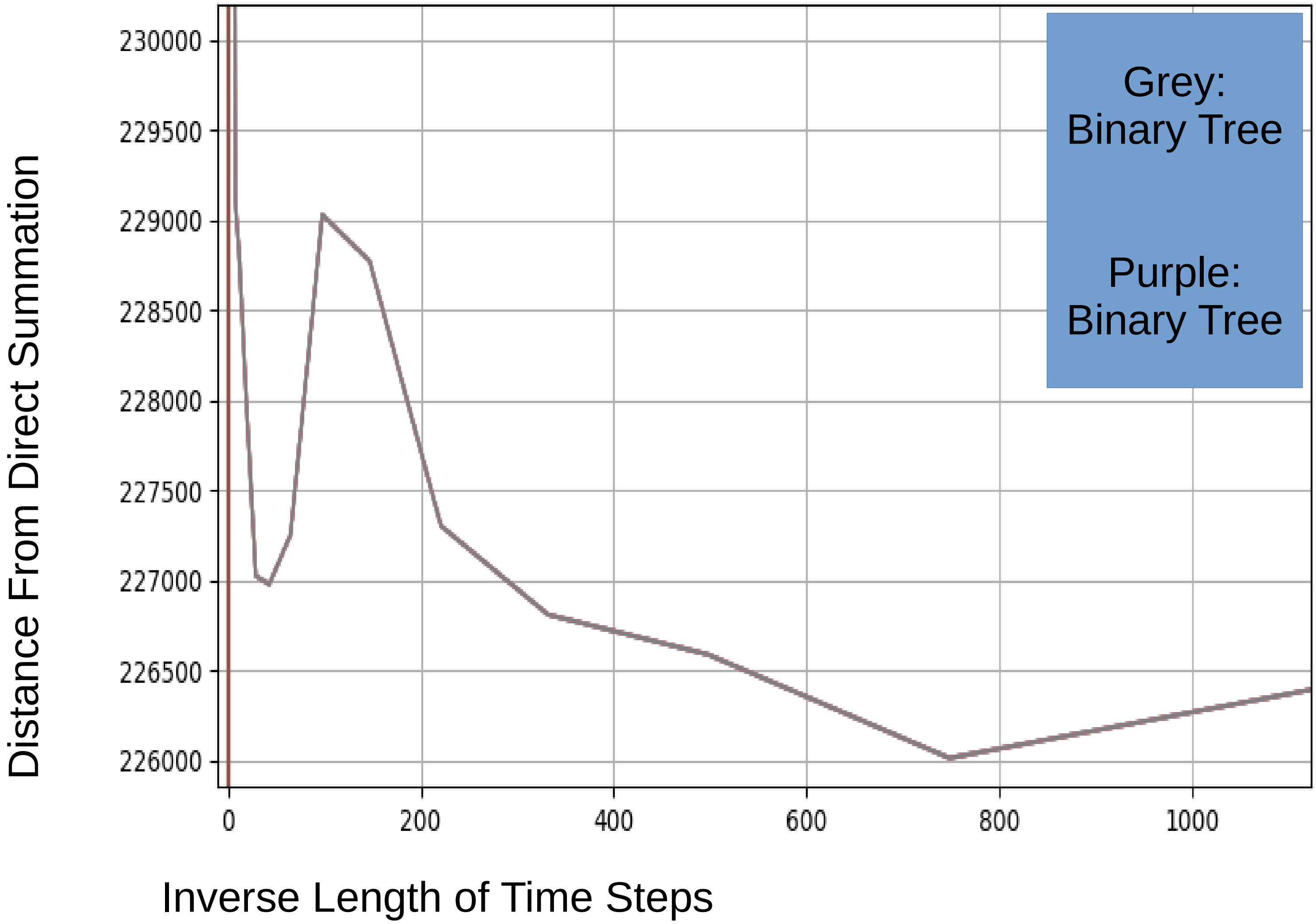

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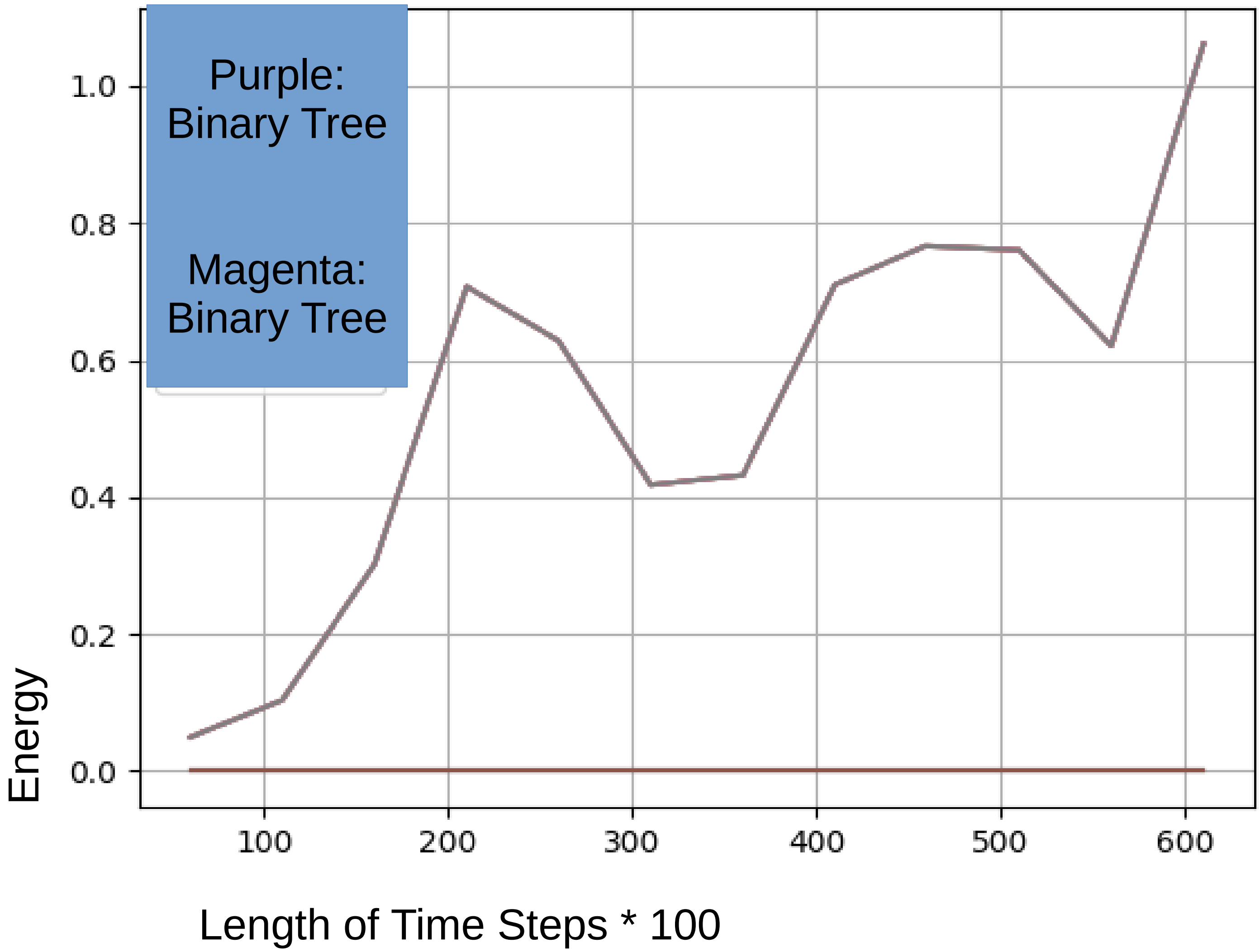

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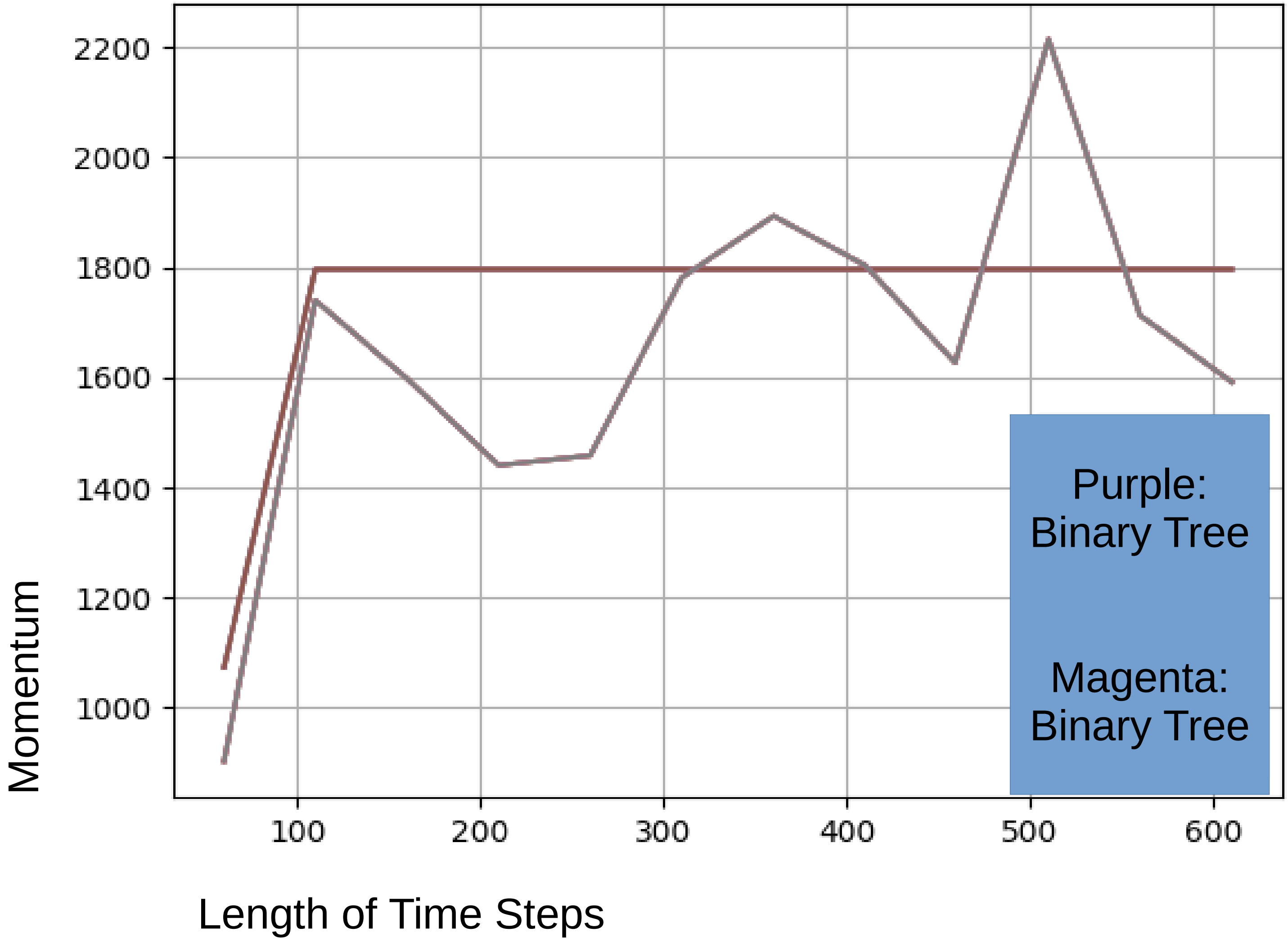
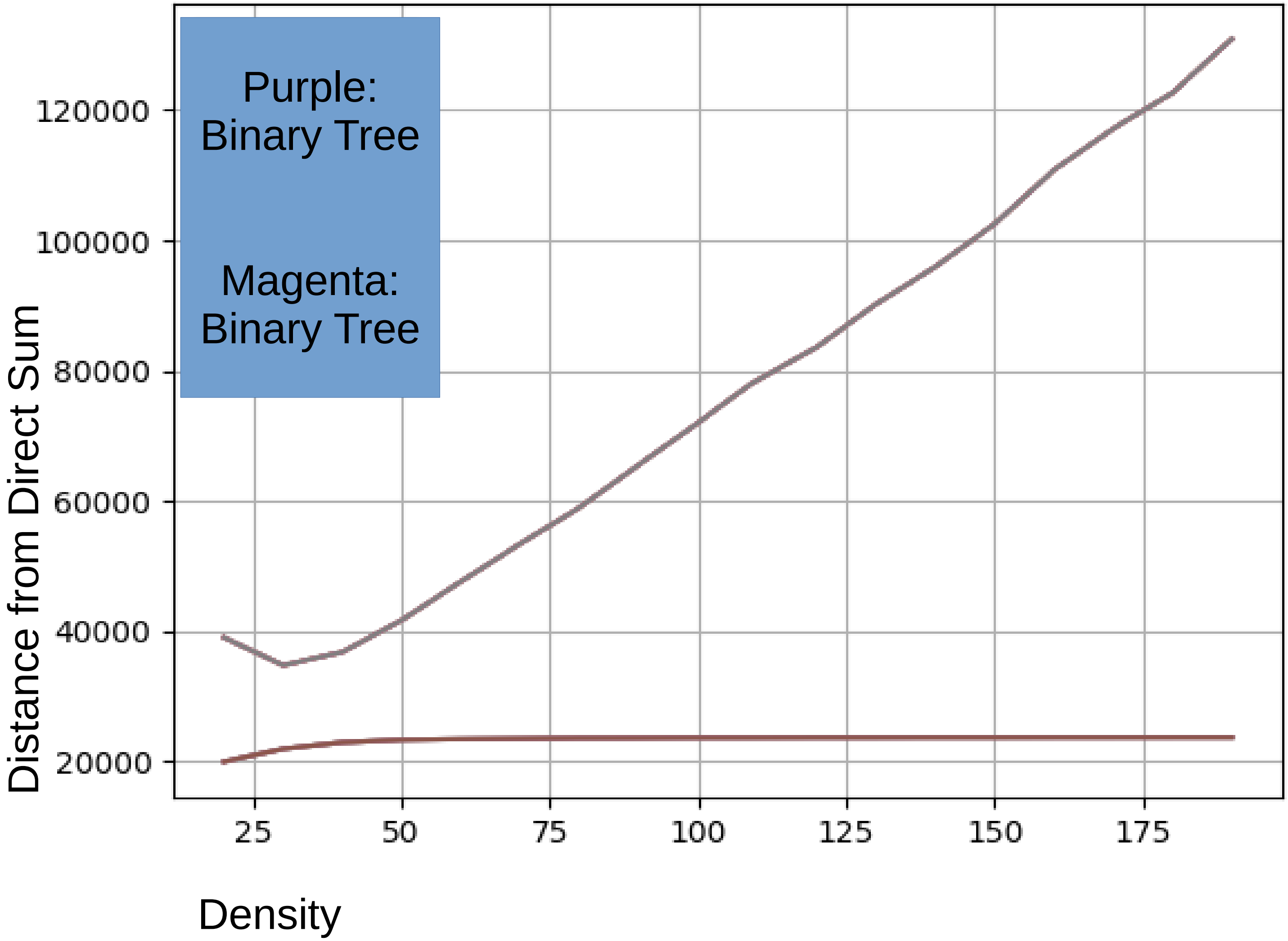


Does a Higher Initial Density of the Particles change the accuracy?



Acknowledgements:

Owen Young

Johnny Powell