# Homework 1. Probs 1-4

1. Show by Induction
   1. Show 5^2n + 3n – 1 is divisible by 9
      1. Hypothesis: 5^2n + 3n – 1 is divisible by 9, n is integer
      2. Base case: n = 1 => 5^2 +3 – 1 = 25 + 2 = 27
         1. 27 is divisible by 9 ( 27 / 9 = 3)
      3. Induction proof: Assume 5^2n + 3n – 1 is divisible by 9 (= 9m), show this holds for (n + 1) also
         1. 5^2(n+1) + 3(n+1) -1 = 5^(2n+2) + 3n + 3 -1
         2. = 5^2 \* 5^2n +3n +2 = (9m -3n +1)\*25 + 3n + 2
         3. = 9m\*25 -75n +25 + 3n +2 = 9\*25\*m -72n + 27
         4. 72 and 27 ar multiples of 9 so can pull 9 out the whole thing
         5. = 9 \* (25m – 8n + 3 )
         6. Since our final equation is a multipe of 9, the equation as a whole will always be divisible by 9 and therefore we have proved 5^2(n+1) + 3(n+1) -1 is divisible by 9
   2. Show n! > 3^n for n >= 7 and n is integer
      1. Hypothesis: n! > 3^n for n >= 7 and n is integer
      2. Base case: n = 7. => 7! >3^7 = 5040 >2187
         1. This is true 5040 > 2187
      3. Induction proof: Assume n! > 3^n for n >=7 and n is integer, show this holds for n+1
         1. (n+1)! > 3^(n+1) = (n+1)\*n! > 3\*3^n
         2. With n starting at 7, n+1 is a minimum of 8
         3. With the assumption of n!>3^n, multiplying the already larger side by 8 and the smaller side by 3 will yield the same results therefore (n+1)! > 3^(n+1) is true
   3. Show = n / (n+1) for positive integers n
      1. Hypothesis: = n / (n+1) for positive integers n is true
      2. Base case: n = 1 => 1 / 1(2) = 1 / (1+1) => ½ = ½
         1. This is true ½ = ½
      3. Induction proof: Assume = n / (n+1) for positive integers n, is true. Show it holds for n+1
         1. = (n+1) / (n+1+1)
         2. Left side
            1. = 1/(n+1)(n+2)+
            2. Based on assumption
            3. = 1/(n+1)(n+2) + n/(n+1)
            4. = (1 + n(n+2)) / (n+1)(n+2)
            5. = n^2 + 2n + 1 / (n+1)(n+2)
            6. (n+1)^2 / (n+1)(n+2) = (n+1)/(n+2)
      4. Since the left side now equals the right, the assumed holds for n+1 and we have proved = n / (n+1)
   4. Show = (n\*(n+1))/2
      1. Hypothesis: = (n\*(n+1))/2 is true
      2. Base case: n = 1 => 1 = 1(2)/2 => 1= 1
         1. This is true
      3. Induction Proof: Assume = (n\*(n+1))/2 is true, show this holds for n+1
         1. = (n+1)\*(n+2))/2
            1. = n+1 +
            2. Using assumed
            3. (n+1) + (n\*(n+1))/2 = ½ 2(n+1) +n^2 +n
            4. = ½ (2n + 2 + n^2 + n) = ½ (n^2 +3n + 2)
            5. = (n+1)(n+2) / 2
         2. Since the left side now equals the right side, the case holds for n+1 and we have proved = (n\*(n+1))/2 is true for all positive n.
2. Compute the sums
   1. = (n+1 – 3 +1)1 = n-1
      1. Second simple series from class notes
   2. = n+1 + = n+1 + -

= n+1 + – 6 = n+1 + n(n+1)/2 – 6

=1/2 (2n + 2 + n^2 + n – 12) = ½ (n^2 + 3n – 10)

* + 1. Simple series from 1d above is close so got there
  1. = = (n\*(n+1)/2)

= (n\*(n+1)/2) = (n\*(n+1)/2) \* (n\*(n+1)/2)

= n^2 (n+1)^2 / 4

* + 1. 2 of that same 1d simple series, pull out “constants”

1. Determine order of growth (prove)
   1. (n^2 + 1)^10 = n^20 + … + 1
      1. O (n^20) by the polynomial theorem (#3)
   2. (10n^2 + 7n + 3)^1/2
      1. Limit test with n
      2. Lim n>inf of (10n^2 + 7n + 3)^1/2 / n >> l’h
      3. Lim n>inf of 20n + 7 / (10n^2 + 7n + 3)^1/2 >> l’h
      4. Lim n>inf of 20(20n + 7)/ (10n^2 + 7n + 3)^3/2 >>
      5. This is approaching 1/inf = 0
      6. Therefore (10n^2 + 7n + 3)^1/2 is in O(n)
   3. 2n log ((n+2)^2) + (n+2)^2 log (n/2)
      1. = 4n log(n+2)+ (n+2)^2(log(n) – log(2))
      2. = 4n log(n+2) + (n+2)^2 log(n) – (n+2)^2
      3. Limit test with n log(n)
      4. Lim n>inf of (4n log(n+2) + (n+2)^2 log(n) – (n+2)^2) / (n log(n)) >> l’h
      5. After looking at the limit (it is very big and ugly) I am unsure I am doing this right.
   4. 2^(n+1) + 3^(n-1)
      1. 2^n or 3^n is a complexity class given
      2. As n approaches infinity the modifier to either will be negligible and so this will be big theta of 3^n
      3. Lim n > inf of 2^n+1 + 3 ^ n-1 / 3^n
      4. = Lim n > inf of (2^n+1 / 3^n ) + 1/3
      5. = 1/3 ? therefore 2^(n+1) + 3^(n-1) is in big theta of 3^n
   5. Log(n)
      1. Log n is a complexity class given so this would simply be Big theta(log(n))?
         1. This doesn’t seem right (too easy) not sure what I am missing
2. Order of growth (justify)
   * 1. N is not I or j so is constant and pulled out front
     2. n = n =
     3. n ( - + ) = n(n^2 – n(n+1)/2 +n)
     4. n^3 – n(n+1)/2 + n
     5. This will have order of growth of n^3 by the polynomial theorem
     6. + =
     7. + =
     8. + = n(n+1)(2n+1)/6 + 1/2
     9. n(n+1)(2n+1)/6 + ½ (n(n+1)(2n+1)/6 + n(n+1)/2
     10. This will have order of growth of n^3 by the polynomial theorem
     11. =
     12. (n(n+1)(2n+1)/6)^2 + 2(n(n+1)(2n+1)/6) + n
     13. This will have order of growth of n^6 by the polynomial theorem
     15. Since no other variables all other summations result in n
     16. N^3 ( n(n+1)/2 )
     17. This will have order of growth of n^6 by the polynomial theorem
     18. =
     19. = n^2(n+1)/2 – n(n+1) – n(n+1)(2n+1)/6
     20. This will have order of growth of -n^2 by the polynomial theorem