4.450: Computational Structural Design and Optimization // Fall 2021
Instructor: Caitlin Mueller // TA: Yijiang Huang

Homework 01 // Beam Shaping

Assigned: Wednesday, 15 September 2021
Due: Wednesday, 29 September 2021 as PDF on Canvas
Instructions: This assignment asks you to consider the classical optimization problem of shaping a beam along its length to save material. You will explore, implement, and compare two methods: Galileo's approach and a NURBS-based computational design tool. You should find the Lecture 02 slides and the Week 02 readings useful for completing Problem 1.

1. Galileo's approach (2 points each, 12 points total)

Recall that Galileo proved the important result that the ratio of structural demand to capacity does not scale linearly with size. In other words, as a beam structure is scaled up uniformly in all directions, its volume and so the moment applied by its weight increase cubically, but its moment capacity only increases quadratically. This means that structures cannot be arbitrarily scaled up or down, and a limit exists for the size structures can attain.

1.1 Demonstrate that this is true for a cantilevered prismatic beam supporting its own weight, given a beam height *h*, width *b*, length *L*, allowable stress σ_0, and density ρ, when *h*, *b*, and *L* are scaled uniformly and σ_0 and ρ kept constant.

1.2 Now consider a cantilevering beam of varying height (*b*, *L*, ρ, and σ_0 constant), and assume the self-weight is negligible compared to a large point load *P* at the cantilever tip. Galileo observed that this beam can be shaped, removing material where it is not fully stressed to reduce the weight of the beam. At every position *x* along the beam, find the moment demand and moment capacity of the section at that position. Deduce the minimum height *h*(*x*) for all positions *x*.

1.3 You can now compute the total weight of the shaped beam by integrating the changing area, $A(x)$, along its length. Give this expression. How does this value compare to the weight of a prismatic beam with the minimum constant height?

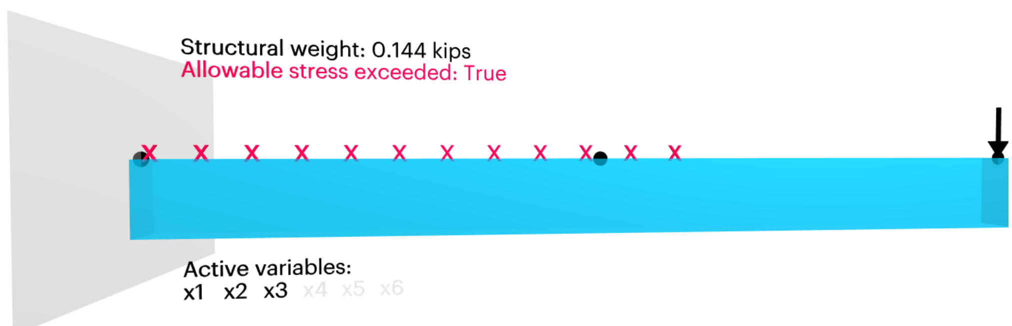
1.4 Repeat 1.2 and 1.3 for a constant height h and a variable width $b(x)$. Does shaping a beam's height or width lead to greater volume savings?

1.5 Now let's consider this problem with real values. Take b = 6 in, L = 120 in, σ_0 = 1 ksi, $\rho = 2 \times 10^{-5}$ kip/in^3, P = 2 kips. What are the values of the heights at the support and tip of the beam, $h(0)$ and $h(L)$? Plot the full profile along the beam's length. What is the total volume (in in^3) and weight (in kips, or "kilo-pounds") of the shaped beam?

1.6 The material properties given in 1.4 are similar to those found in timber. Comment on the holistic advantages and disadvantages of making an actual beam with this profile in this material.

2. NURBS approach (points given below, 8 points total)

Now let's consider the same problem, but address it with modern computational tools. [This tool]() allows you to generate shaped beam geometries and check whether the material is overstressed, and uses the same values given in 1.4. "Allowable stress exceeded: True" and the appearance of pink X's indicate that the current geometry is not valid:



By changing the values of variables x1 through x6, which set the height of control points generating a NURBS curve for the top profile of the beam, you can adjust the beam's shape until the pink X's disappear. Note that you can also change the number of control points (which also changes the variables that are active), and whether the beam profile is a curve or polyline. Changing the number of analysis points improves the resolution of the analysis.

2.1 (1 point) Using the original settings for Control Points (3), Polyline (False), and Analysis Points (20), find the best design you are able to with the lowest structural weight without exceeding the allowable stress (e.g. no pink X's), by changing the variables x1, x2, etc. How does this structural weight compare to the value you found in 1.4? Give any observations you have about the discrepancy.

2.2 (2 points) Repeat 2.1 for the following additional numbers of Control Points: 2, 4, 5, 6. Make a graph of the best structural weight you can find (y axis) vs.

number of control points (x axis). Comment on your observations.

2.3 (2 points) Repeat 2.1 and 2.2, but this time change Polyline to True; this will result in a segmented profile rather than a curve. Comment on your observations.

2.4 (2 points) Using any settings you want, find the best beam (in your opinion) from the perspective of manufacturability, and then from the perspective of aesthetics (without any pink X's in either case). Comment on your observations.

2.5 (1 point) In a few sentences, reflect on how this process differs from the one pursued by Galileo in 1638. What are the relative strengths and weaknesses of the two approaches? Can you think of any scenarios in which the computational approach would be definitively better?