In [142…

```
import numpy as np
from scipy import stats

# for pi
from mpmath import mp
import mpmath
```

In [8]:

```
import matplotlib.pyplot as plt
```

In [22]:

```
def plot_complex_eigenvalues(w, ax):
    """Plots `w` on the complex plane."""
    w_r, w_c = w.real, w.imag
    ax.scatter(w_r, w_c)
```

Choose n

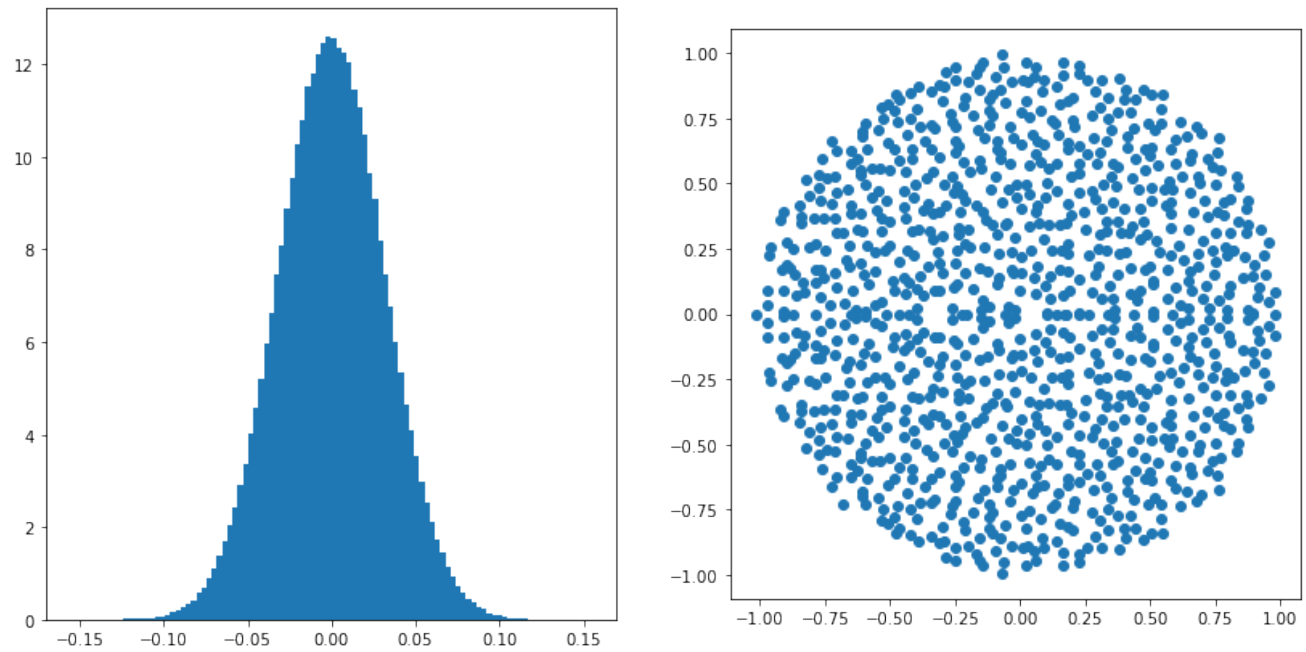
In [114…

```
n = 1000
```

1.1 i.i.d Gaussian entries.

In [135…

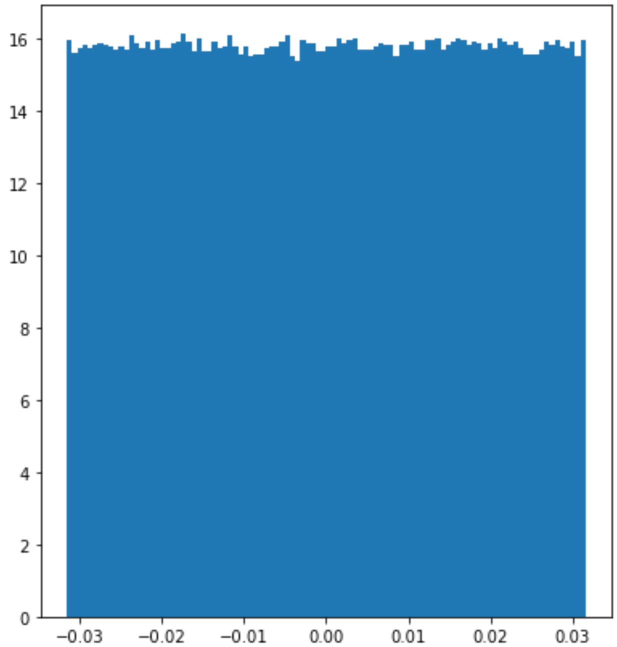
```
H = np.random.normal(scale=1, size=(n,n)) / np.sqrt(n)

# Extract eigenvalues.
w, v = np.linalg.eig(H)

# Plot.
fig, ax = plt.subplots(1,2,figsize=(14,7))

# Plot distribution.
ax[0].hist(H.flatten(), bins=100, density=True, stacked=True)

# Plot circular law.
plot_complex_eigenvalues(w[1:], ax[1])
ax[1].set_aspect('equal')
```

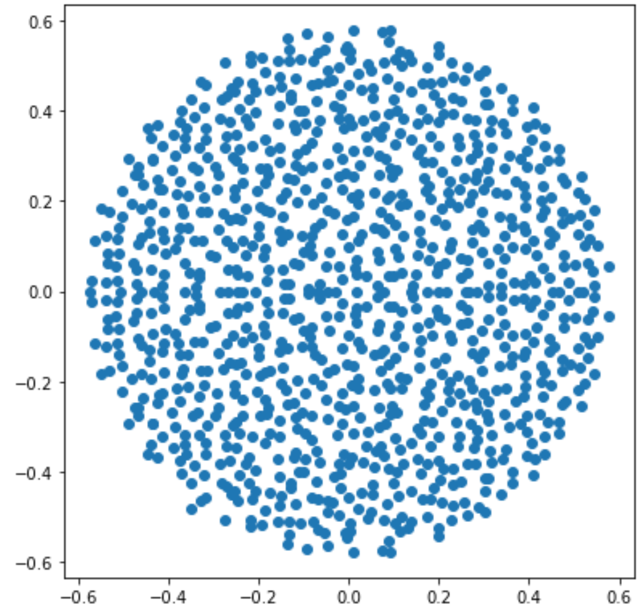


Discrete random w/ equal probability.

In [176...

```
# Random values [0, 1]
H = np.random.rand(n,n)

# Rescale H to appropriate bounds (+-1/sqrt(n))
H = (H - .5) * 2 / np.sqrt(n)

w, v = np.linalg.eig(H)
fig, ax = plt.subplots(1,2,figsize=(14,7))

# Plot distribution.
ax[0].hist(H.flatten(), bins=100, density=True, stacked=True)

# Plot circular law.
plot_complex_eigenvalues(w[1:], ax[1])
ax[1].set_aspect('equal')
```





Create matrix from digits of number (c)

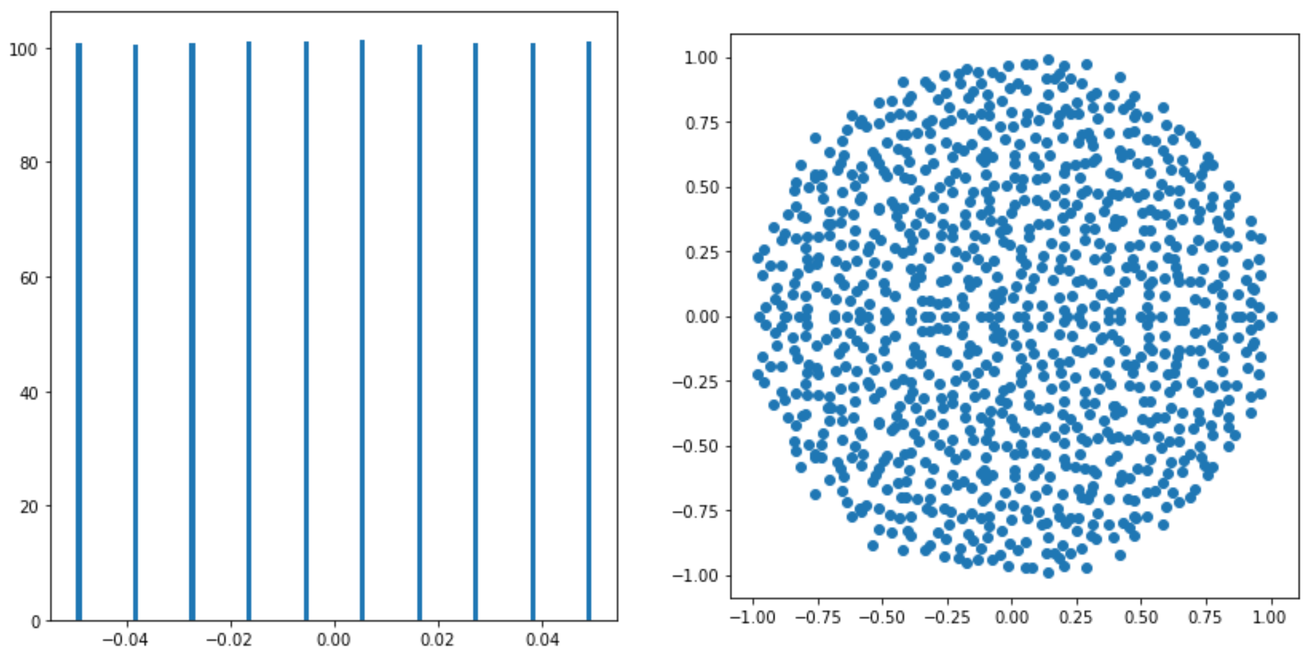
In [173…

```
def split_str_into_vector(number_as_str):
    return np.array([float(letter) for letter in number_as_str if letter.isdi

def construct_H_from_vector(v):
    b, var = np.mean(v), np.var(v)
    v_scale = 1/ np.sqrt(n * var)
    return (v.reshape(n, n) - b) * v_scale

mp.dps = n ** 2
```

c = π

In [184…

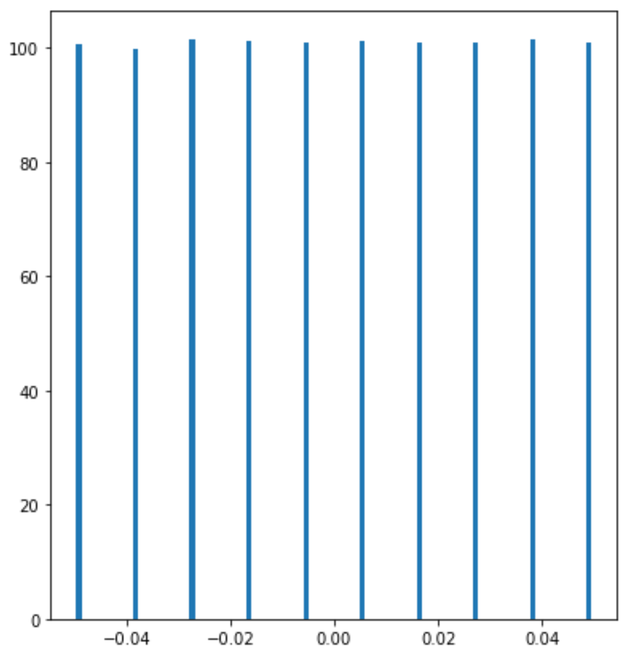
```
## PI
c = mp.pi

# Create string with appropriate number of digits.
c_str = mpmath.nstr(c, n = mp.dps)

# Split string into vector such that each element is a digit from original nu
c_vec = split_str_into_vector(c_str)

# Construct `H` matrix.
H = construct_H_from_vector(c_vec)

# Analyze.
w, v = np.linalg.eig(H)

# Plot.
fig, ax = plt.subplots(1,2,figsize=(14,7))

# Plot distribution.
ax[0].hist(H.flatten(), bins=100, density=True, stacked=True)

# Plot circular law.
plot_complex_eigenvalues(w[1:], ax[1])
ax[1].set_aspect('equal')
```

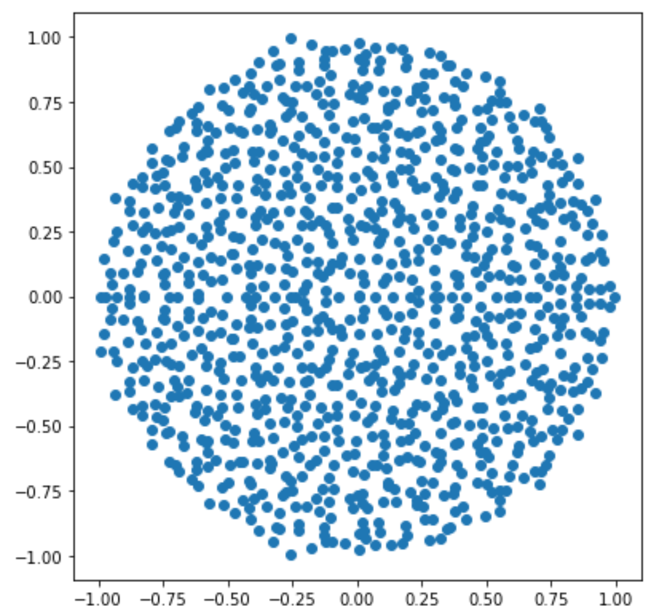


c = $\sqrt{2}$

In [182…

```
c = mp.sqrt(2.)

# Create string with appropriate number of digits.
c_str = mpmath.nstr(c, n = mp.dps)

# Split string into vector such that each element is a digit from original nu
c_vec = split_str_into_vector(c_str)

# Construct `H` matrix.
H = construct_H_from_vector(c_vec)

# Analyze.
w, v = np.linalg.eig(H)

# Plot.
fig, ax = plt.subplots(1,2,figsize=(14,7))

# Plot distribution.
ax[0].hist(H.flatten(), bins=100, density=True, stacked=True)

# Plot circular law.
plot_complex_eigenvalues(w[1:], ax[1])
ax[1].set_aspect('equal')
```

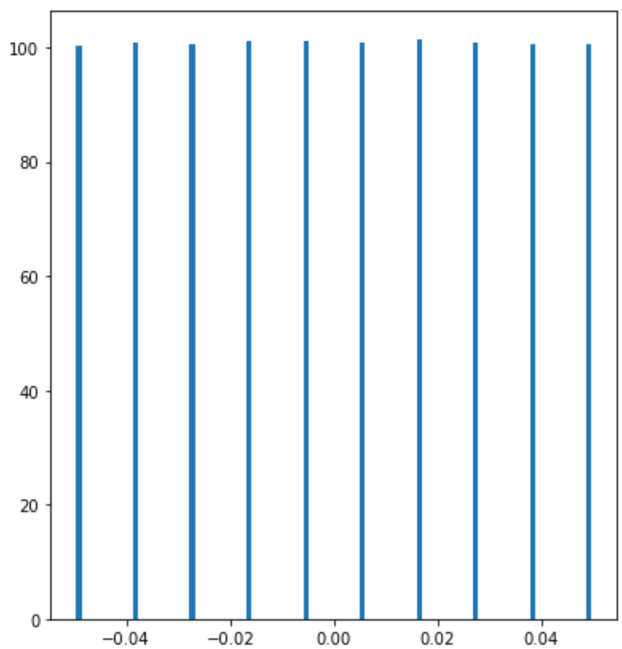


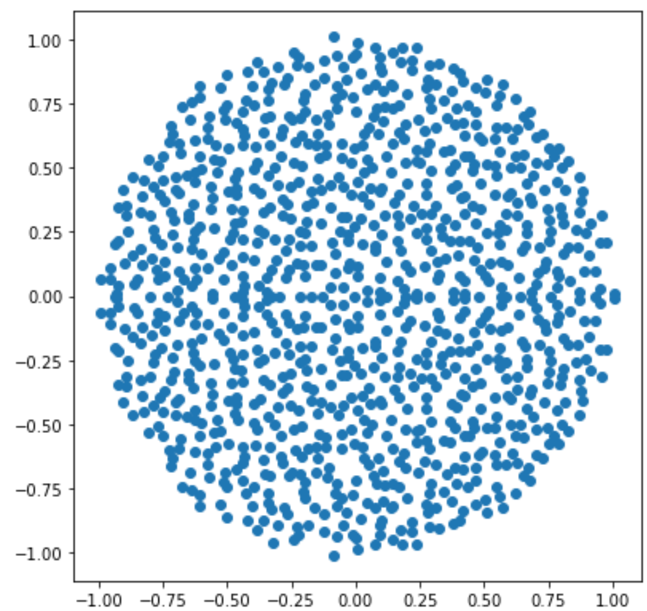


c = e

In [183…

```
c = mp.e

# Create string with appropriate number of digits.
c_str = mpmath.nstr(c, n = mp.dps)

# Split string into vector such that each element is a digit from original nu
c_vec = split_str_into_vector(c_str)

# Construct `H` matrix.
H = construct_H_from_vector(c_vec)

# Analyze.
w, v = np.linalg.eig(H)

# Plot.
fig, ax = plt.subplots(1,2,figsize=(14,7))

# Plot distribution.
ax[0].hist(H.flatten(), bins=100, density=True, stacked=True)

# Plot circular law.
plot_complex_eigenvalues(w[1:], ax[1])
ax[1].set_aspect('equal')
```





Problem 3

In [205…

```
theta = 4
n = 1000

def sigma(theta):
    return None

def construct_W(n):
    W = np.random.normal(scale=1/np.sqrt(n), size=(n,n))
    return 1/np.sqrt(2) * (W + np.transpose(W))
```

In [206…

```
m = 1000
largest_eigenvector=[]

# Construct signal matrix.
signal_matrix = np.zeros((n,n))
signal_matrix[0,0] = theta

for _ in range(m):
    # Construct `W`
    W = construct_W(n)

    # Construct `Y`
    Y = signal_matrix + W

    w = np.linalg.eigvalsh(Y)

    # Get eigenvectors.
#     w, v = np.linalg.eig(H)

    # Sort by size (smallest to largest).
#     w = sorted(w)

    largest_eigenvector.append(w[-1])

largest_eigenvector = np.array(largest_eigenvector)
```

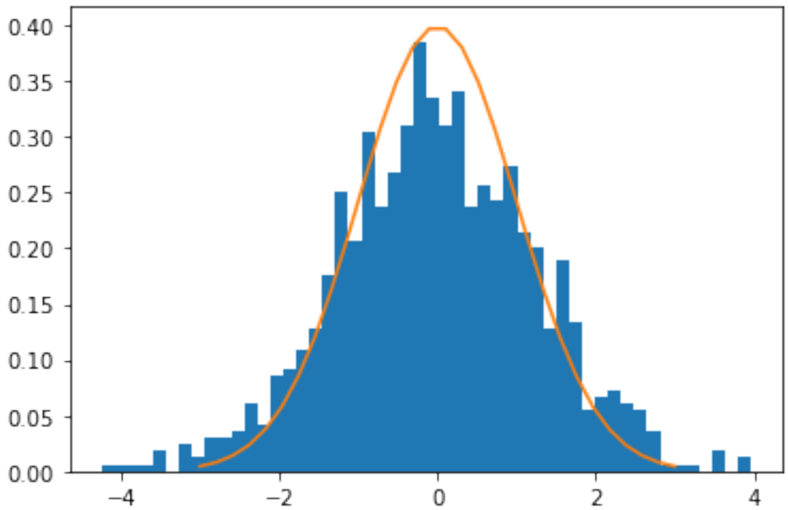
In [207…

```
# Define big Lambda.
Lambda = theta + 1 / theta

# Define variance.
analytic_variance =2 * ((1/2) + Lambda / np.sqrt(Lambda **2 - 4)) ** -1

# Scale eigenvectors
scaled_eigenvectors = np.sqrt(n) * (largest_eigenvector - Lambda) / np.sqrt(a
```

In [208…

```
x_plot = np.linspace(-3,3,30)
plt.hist(scaled_eigenvectors, bins=50, density=True, stacked=True)
plt.plot(x_plot, stats.norm.pdf(x_plot))
```

Out[208… [<matplotlib.lines.Line2D at 0x7f7818909550>]



In [ ]: