Building a Robot Judge:
Data Science for Decision-Making

7. Instrumental Variables

Q&A Padlet

http://bit.ly/BRJ_Padlet7

Recap: Reading Response Essays

- Critical reading is an important skill:
  - useful for writing/reading reports
  - understanding the structure/code behind a paper – why have papers and not textbooks?

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- Some common patterns in the responses:
  - great summaries
  - more mixed on the evaluation

Another nice guide (now on HW Assignments page):
`https://www.icpsr.umich.edu/files/instructors/How_to_Read_a_Journal_Article.pdf`

Recap: Study on Trustworthiness in Artworks

http://bit.ly/BRJ-W6-A3-padlet

Learning Objectives

1. Implement and evaluate machine learning pipelines.
2. **Implement and evaluate causal inference designs.**
   - Today: Instrumental Variables
3. Understand how (not) to use data science tools (ML and CI) to support expert decision-making.

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3. Econometrics:
   - Articulate a research design and the identification assumptions for procuring causal estimates.
   - Run regressions to produce the estimates.
   - Run identification checks and specification checks to enhance confidence in results.

Outline

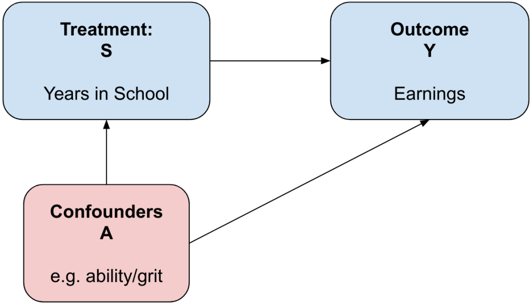
Instrumental Variables

IV with Machine Learning

Deep IV

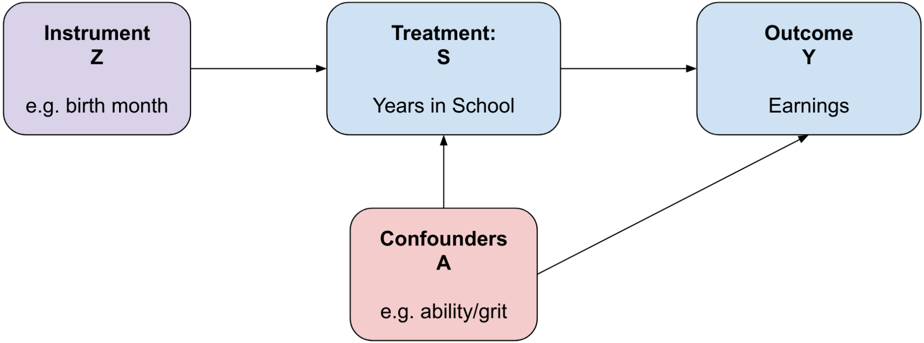
- Example from Week 3: Causal effect of schooling S_i on earnings Y_i.
- There is an unobserved confounder (say ability A_i) correlated with schooling and earnings

$$Y_i = \alpha + \rho S_i \underbrace{(+\phi A_i)}_{\text{unobserved}} + \eta_i$$



- OLS estimates for $\hat{\rho}$ will be biased.

Instrumental Variable (IV): a variable Z_i, that is correlated with S_i, but not correlated with anything else affecting Y_i.



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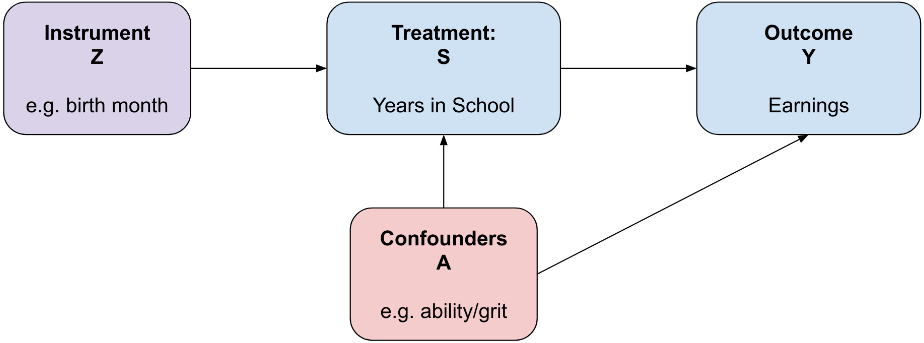
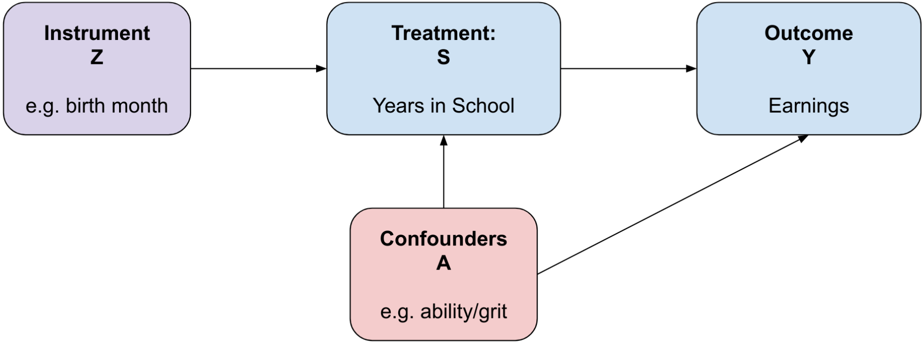


$$Y_i = \alpha + \rho S_i + \underbrace{(+\phi A_i)}_{\text{unobserved}} + \epsilon_i$$

$$\text{Cov}[Z_i, S_i] \neq 0, \text{Cov}[Z_i, A_i] = 0$$

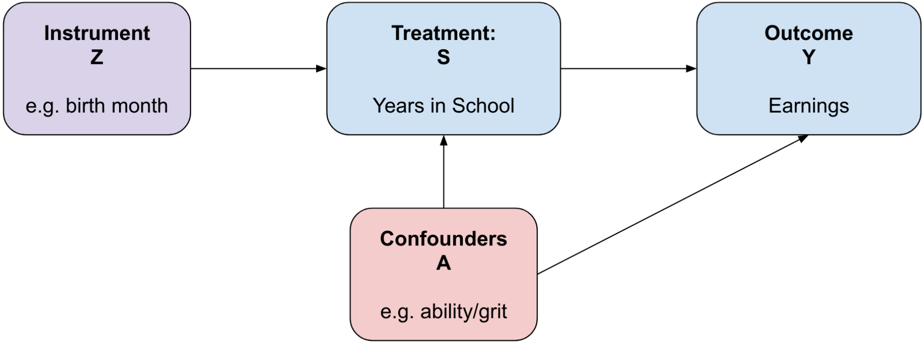
- With a valid instrument, can procure causal estimates for $\hat{\rho}$

Instrumental Variables: Main Intuition



- We identify a source of variation in treatment assignment that is as good as random – orthogonal to any relevant unobserved confounder.
- We compare individuals that, due to the instrument, are shifts between the control group and treatment group.

What is a valid instrumental variable?



1. Correlated with the causal variable, e.g.S_i:

$$\mathrm{Cov}[Z_i, S_i] \neq 0$$

2. Uncorrelated with any other determinants of outcome V:

$$\mathrm{Cov}[Z_i, \epsilon_i] = 0$$

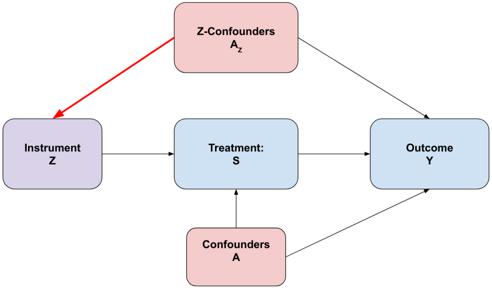
Identification requirement has two dimensions:

Exogeneity: None of the unobserved factors affects the instrument:

$$\epsilon_i \nrightarrow Z_i$$

- No "Z-confounders"

Violation of exogeneity:



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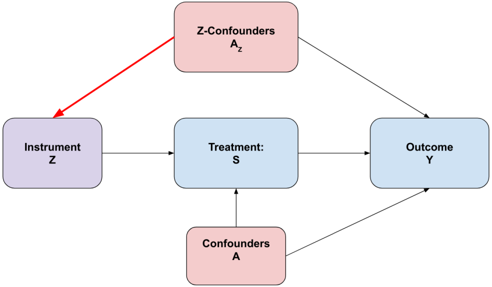
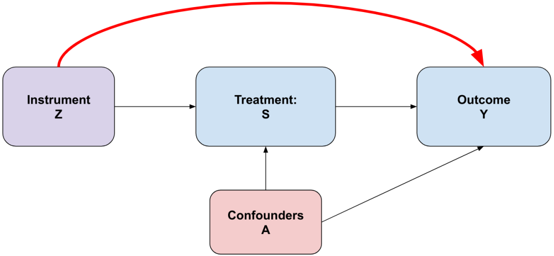


Exclusion: Instrument only affects outcome through treatment variable:

$$Z_i \nrightarrow \epsilon_i$$

Violation of exclusion:



Good instruments are hard to find

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  - Last but not least: Originality
- Some usual sources of instruments:
  - Nature (e.g. genes, weather)
  - Assignment rules (e.g. random assignment of judges to cases)
  - 'Natural' experiments (e.g. the quarter of birth, conscription lottery, electoral timing...)

Zoom Poll 7.1: Good instruments for schooling

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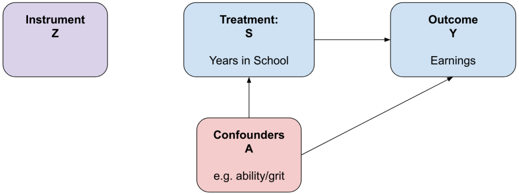
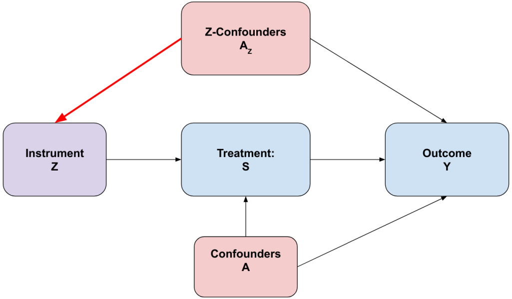
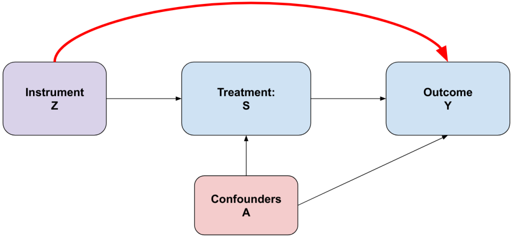
Violation of relevance:



Violation of exogeneity:



Violation of exclusion:



IV estimator

We have

$$Y_i = \alpha + \rho S_i + \epsilon_i$$

and an instrument Z_i where $\text{Cov}[Z_i, S_i] \neq 0$ and $\text{Cov}[Z_i, \epsilon_i] = 0$.

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- We can write ρ in terms of the population moments

$$\text{Cov}[Z_i, Y_i] = \rho \text{Cov}[Z_i, S_i] + \underbrace{\text{Cov}[Z_i, \epsilon_i]}_{=0}$$

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- Thus:

$$\rho = \frac{\mathrm{Cov}[Z_i, Y_i]}{\mathrm{Cov}[Z_i, S_i]}$$

with sample estimate

$$\hat{\rho}_{\mathrm{IV}} = \frac{\sum_{i=1}^{n} Z_i Y_i}{\sum_{i=1}^{n} Z_i S_i}$$

```
// stata
ivreghdfe wages (schooling = instrument), absorb(FE) cluster(FE)
```

Examples

Have to look at papers if curious

- Immigration
  - Networks of immigrants (Card 1991)
- Does police decrease crime?
  - Electoral cycles (Levitt 1997)
- The impact of violent movies on crime
  - Blockbuster movies (Dahl and DellaVigna 2009)
- The effect of preschool television exposure on standardized test scores during adolescence:
  - Gentzkow and Shapiro 2008
- The Potato's Contribution to Population and Urbanization:
  - Nunn and Nancy Qian 2011
- Influence of mass media on U.S. government response to natural disasters
  - Eisensee and Strömberg 2007

Two-Stage Least Squares (2SLS)

IV estimates are equivalent to running two separate OLS regressions:

1. Estimate"first stage", regressing treatment on instrument:

$$S_i = \gamma Z_i + \nu_i$$

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IV estimates are equivalent to running two separate OLS regressions:

1. Estimate"first stage", regressing treatment on instrument:

$$S_i = \gamma Z_i + \nu_i$$

2. Form prediction$\hat{S}_i = \hat{\gamma} Z_i$and estimate the"second stage", regressing outcome on first-stage-predicted treatment:

$$Y_i = \rho \hat{S}_i + \epsilon_i$$

Can we test validity of IV?

- Is Z_i correlated with causal variable of interest, S_i?
  - YES: check for significance of first stage (first-stage F-statistic)

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- Is Z_i correlated with causal variable of interest, S_i?
  - YES: check for significance of first stage (first-stage F-statistic)
- Is Z_i uncorrelated with any other determinants of Y_i?
  - Not directly testable – relies on institutional knowledge
  - but often indirect ways to probe exogeneity and exclusion

Weak Instruments

The bias of 2SLS can be written as:

$$\text{plim}\hat{\rho} = \rho + \frac{\text{Corr}[Z,\epsilon]}{\text{Cov}[S,Z]} \cdot \frac{\sigma_\epsilon}{\sigma_S}$$

- When the instrument is weakly correlated with the endogenous regressor, the bias increases.
- Kleibergen-Paap First-stage F-statistic (reported automatically by ivreghdfe with cluster( ) option) should be higher than 10.

Reduced Form

"Reduced Form" (RF) means regressing the outcome directly on the instrument:

$$Y_i = \alpha + \phi Z_i + \epsilon_i$$

- papers will normally report this along with 2SLS estimates.
- for causal interpretation, RF requires exogeneity but not exclusion.

Instruments with Observed Confounders

- Recall that with OLS, observed confounders are not a problem because we can adjust for them.

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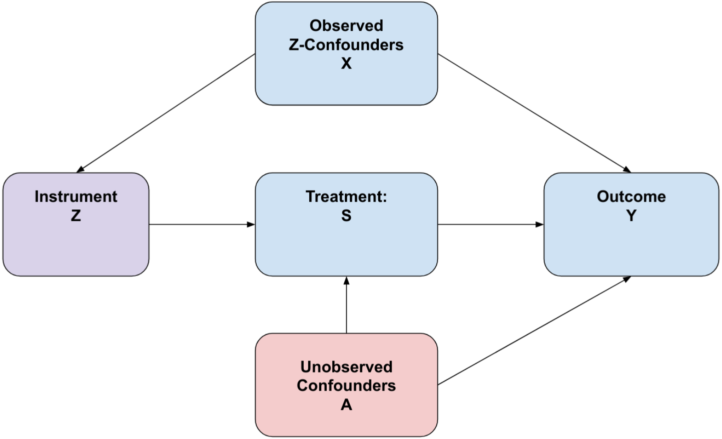
- Recall that with OLS, observed confounders are not a problem because we can adjust for them.
- With Z-confounders, we have the same property.



- IV independence assumption can be written as $\text{Cov}[Z_i, \epsilon_i | X] = 0$.

Practice: Effect of Fox News on COVID-19 Social Distancing

http://bit.ly/BRJ-W7-FNC-doc

Outline

Lasso IV with Weak Instruments

Consider the problem of a sparse first stage:

$$S_i = \alpha + \boldsymbol{Z}_i' \boldsymbol{\phi} + \nu_i$$

- $\boldsymbol{Z}_i$ is a high-dimensional vector
- many elements of $\boldsymbol{\phi} = (\phi_1, ...\phi_{n_z})$ are zero, $\phi_k \approx 0$
- but we don't know which.

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Solution:

- Train lasso (or elastic net), $S \sim$ Lasso($\boldsymbol{Z}$)
  - use CV grid search across the whole dataset to select L1 penalty
  - get subset of instruments with non-zero coefficients, Z_{Lasso}.
- Run 2SLS with Z_{Lasso} as instrument(s).
- This is the optimal set of instruments under sparsity (Belloni et al 2014).

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    - e.g., some people won't go to school even if they win a scholarship.
    - first stage is driven by"compliers" (responders to instrument).
- Standard 2SLS estimates give a"local average treatment effect" on the complier population.

Estimating Heterogeneous First Stage

- Can use machine learning to estimate treatment effect heterogeneity in the first stage:
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  - Learn$\eta_0(X) = \mathbb{E}(S|X, Z = 0)$
  - Learn$\eta_1(X) = \mathbb{E}(S|X, Z = 1)$
- Conditional first stage effect estimate is$\hat{\gamma}(X) = \eta_1(X) - \eta_0(X)$.

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- Conditional first stage effect estimate is $\hat{\gamma}(X) = \eta_1(X) - \eta_0(X)$.
- Can be used to analyze complier population, or to re-weight regressions to get closer to an average treatment effect.

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Deep Instrumental Variables

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- *Deep IV: A Flexible Approach for Counterfactual Prediction*
  - Hartford, Lewis, Leyton-Brown, and Taddy (2017)
  - use deep learning to extend 2SLS to high-dimensional settings

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- *Deep IV: A Flexible Approach for Counterfactual Prediction*
  - Hartford, Lewis, Leyton-Brown, and Taddy (2017)
  - use deep learning to extend 2SLS to high-dimensional settings
- Causal effect of interest:

$$f(S;\theta) = \mathbb{E}\{Y|S\}$$

where$w$ could be high-dimensional and$f(\cdot)$ could be highly non-linear.

First stage

In first stage, approximate $g(S|\gamma(Z))$, the distribution of S:

- assume that $g(\cdot)$ is a mixture density network (a mixture of gaussian distributions) where the parameter vector $\gamma(\cdot)$ includes the weights, means, and variances (Bishop 2006).
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  - $\gamma(Z)$ is a modeled as a feed-forward neural network.
- $g(\cdot)$ has to be a parametrized distribution because Deep IV requires that the distribution be integrated in the second stage.
- validate first-stage relevance in in held-out test set.

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- The integral in $\mathcal{L}(\theta)$ is approximated by

$$\int \hat{Y}(S;\theta) d\hat{g}(S|\gamma(Z_i)) \approx \frac{1}{m} \sum_j^m \hat{Y}(\tilde{S}(Z_i);\theta)$$

where you make m draws from the estimated treatment distribution given Z_i (the instruments for observation i).

  - Like 2SLS, a prediction for the endogenous regressor with the instruments is used during second-stage estimation.

Practice: Adding Instruments to Custom Causal Graphs

`http://bit.ly/BRJ-W7-graphs-doc`

Outline

Appendix Slides

2SLS Matrix Notation compared to OLS

- With model $Y = X'\beta + U$ and instrument Z, we have

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$$\mathbb{E}[(X'X)^{-1}(X'U)] \gtrless \mathbb{E}[(Z'X)^{-1}(Z'U)]?$$