Model skydive

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The following notation is used for constants of the skydiver

v^{plane}	the velocity of the aiplane.
h^{plane}	the altitude at which the airplane is flying.
h^{chute}	the height at which the skydiver opens his parachute.
m^{person}	the mass of the person.
m	the total mass of the skydiver and the parachute.
A^{person}_{front}	the area of the front of a person.
A^{person}_{side}	the area of the side of a person.
$A^{backpack}_{side}$	the sideways area of the backpack which holds the parachute.
A^{person}_{avg}	the average of A^{person}_{front} and A^{person}_{side}
C^{person}_{front}	the drag coefficient of a person facing the wind.
C^{person}_{side}	the drag coefficient of a person with wind coming from the side.
C^{person}_{avg}	the average of C^{person}_{front} and C^{person}_{side}
F_g	the gravitational force.

The following notation is used for constants of the T-10 parachute

m^{chute}	the mass of the parachute.
A^{chute}_{below}	the area of the parachute with viewpoint from below.
A^{chute}_{side}	the area of the parachute with viewpoint from the side.
C^{chute}_{below}	the drag coefficient of the parachute with wind coming from below.
C^{chute}_{side}	the drag coefficient of the parachute with wind coming from the side.

The following notation is used for variables (these are all a function of time and/or the height x_z)

$\rho(x_z)$	the air density.
$x_x(t)$	the position of the skydiver in the x-direction.
$x_y(t)$	the position of the skydiver in the y-direction.
$x_z(t)$	the position of the skydiver in the z-direction.
$v_x(t)$	the velocity of the skydiver in the x-direction.
$v_y(t)$	the velocity of the skydiver in the y-direction.
$v_z(t)$	the velocity of the skydiver in the z-direction.
$w_x(x_z)$	the velocity of the wind in the x-direction.
$w_y(x_z)$	the velocity of the wind in the y-direction.
$F_{D,x}(t, x_z)$	the drag force in the x-direction.
$F_{D,y}(t, x_z)$	the drag force in the y-direction.
$F_{D,z}(t, x_z)$	the drag force in the z-direction.
$F_{res,x}(t, x_z)$	the resulting force in the x-direction.
$F_{res,y}(t, x_z)$	the resulting force in the y-direction.
$F_{res,z}(t, x_z)$	the resulting force in the z-direction.

The setup of the skydiver

We assume the skydiver is an 'average' person, so it has an average mass, drag coefficient and surface body area. In [Penwarden et al., 1978] wind tunnel testing is done on persons facing the wind. This paper concluded that the average person has the following measurements

$$\begin{aligned} m^{person} &= 68.8kg, \\ A^{person}_{front} &= 0.55m^2, \\ A^{person}_{side} &= 0.38m^2, \\ C^{person}_{front} &= 1.18, \\ C^{person}_{side} &= 1.11. \end{aligned}$$

We will ignore the air resistance of air hitting the bottom of his feet and the top of his head, because this resistance is negligible since the skydiver is falling in a belly-to-earth orientation.

The setup of the parachute

The parachute of choice will be a T-10 parachute. This is a round parachute which shape is very similar to a hollow hemisphere. This is why we assume the drag coefficients of the parachute are equal to the drag coefficients of a hollow hemisphere. Furthermore, we will ignore the air resistance of the suspension lines because the air resistance of this component of the parachute will be negligible. Research with respect to the aerodynamics of a hollow hemisphere is done in [Annaloro et al., 2020]. The configuration in figure 1 illustrates the experiment in [Annaloro et al., 2020] to determine the drag coefficient of the hollow hemisphere under different angles of attack.

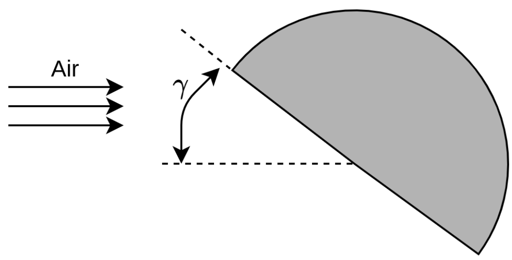

Figure 1: Convention used for the angle of attack.

Notice that the drag coefficient at angle $\gamma = 0$ is equal to C^{chute}_{side}. Moreover, the drag coefficient at angle $\gamma = \frac{1}{2}\pi$ is equal to C^{chute}_{below}. The paper conducted numerical experiments to obtain an approximate value of $C^{chute}_{side} = 0.35$ and $C^{chute}_{below} = 1.68$.

The parachute has a mass of $m^{chute} = 14kg$ and an inflated radius of $r = 3.9m$. Since the parachute from the top has the shape of a circe, we obtain that

$$A^{chute}_{below} = \pi \cdot r^2 = \pi \cdot 3.9^2 = 47.8m^2,$$

and notice that the parachute from the side has the shape of semicircle of radius r. This implies that

$$A^{chute}_{side} = \frac{1}{2}\pi \cdot r^2 = 23.9m^2.$$

The stages of skydiving

The skydiver jumps at time 0 out of the airplane. We assume the skydiver starts at position

$$x(0) := \begin{pmatrix} x_x(0) \\ x_y(0) \\ x_z(0) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ h^{plane}, \end{pmatrix}$$

and has velocity

$$v(0) := \begin{pmatrix} v_x(0) \\ v_y(0) \\ v_z(0) \end{pmatrix} = \begin{pmatrix} v^{plane} \\ 0 \\ 0 \end{pmatrix}.$$

The skydiver falls on his belly as pictured in Figure 2a. The skydiver opens his parachute whenever his z-position reaches a height of h^{chute} (Figure 2b). We will call the freefall stage 1 and stage 2 is the stage in which the skydiver uses a parachute.

(a) Stage 1: the freefall.

(b) Stage 2: the parachute.

Figure 2: The different stages of skydiving.

The wind

We assume that air only travels horizontally, because the winddata can only measure wind in horizontal direction. This means that the wind has a certain velocity in the x-direction, which is w_x, and a velocity in the y-direction, which is w_y. We further assume that the wind velocities are only dependent on the height of the skydiver x_z and not on the x- or y-position of the skydiver.

The air density is different for different heights. We can calculate the air density using the temperature and air pressure by

$$\rho = \frac{pm}{k_B T},$$

where

p	is the air pressure,
m	is the molecular mass of dry air, approximately $4.81 \times 10^{-26} kg$,
k_B	is the Boltzmann constant, $1.380 \times 10^{-23} m^2 kg s^{-2} K{-1}$,
T	is the temperature.

The mathematical model

Stage 1

The gravitational constant on the surface of earth is equal to $g = 9.81$ and we assume that this constant also applies to the skydiver during flight. During the whole flight, we have a gravitational force of

$$F_g = \begin{pmatrix} 0 \\ 0 \\ -mg \end{pmatrix},$$

since gravity only pulls you down. Remember that the drag force of an object is equal to

$$F_D = \frac{1}{2}\rho C A v^2,$$

where ρ is the air density, C is the drag coefficient of the object, A is the surface area of the object and v is the velocity relative to the skydiver. During stage 1 we have that the front of the skydiver experiences a drag force. This drag force is equal to

$$F_{D,z} = -\frac{1}{2}\rho C_{front}^{person} A_{front}^{person} v_z^2.$$

In Figure 3 a skydiver during freefall can be seen from below. During the fall the skydiver might rotate in the x, y-plane. This seems to be a problem at first, because we only know the area A_{side}^{person} of a person from the side. This is the area you see when you have viewpoint A in Figure 3. In Figure 3 are also 2 other viewpoints illustrated, viewpoint B and C. But since skydivers

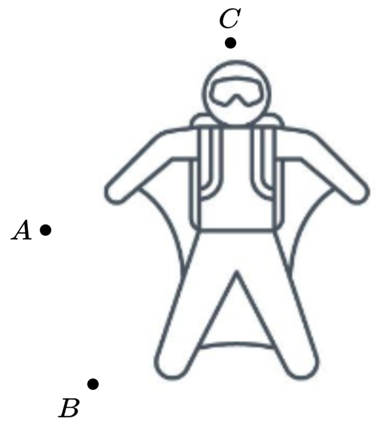

Figure 3: A skydiver during freefall from below.

will always stretch out their arms and legs, the area of the skydiver as seen from viewpoint B and C doesn't seem to be very different as from the area of the skydiver from viewpoint A.

This leads us to assume that during the freefall stage, the drag coefficient and the area of the skydiver in both the x- and y-direction are equal to C_{side}^{person} and A_{side}^{person}. Furthermore, we only care about the velocity of the skydiver relative to the wind, so we should use $v_x - w_x$, $v_y - w_y$ and v_z for the velocities. Moreover, the drag force should be positive if $w_x > v_x$. This is equivalent to multiplying the drag force by $\operatorname{sign}(v_x - w_x)$. By combining this information, we get that during stage 1 we have a drag force of

$$F_D := \begin{pmatrix} F_{D,x} \\ F_{D,y} \\ F_{D,z} \end{pmatrix} = -\frac{1}{2}\rho \begin{pmatrix} C_{side}^{person} A_{side}^{person} (v_x - w_x)^2 \operatorname{sign}(v_x - w_x) \\ C_{side}^{person} A_{side}^{person} (v_y - w_y)^2 \operatorname{sign}(v_y - w_y) \\ C_{front}^{person} A_{front}^{person} v_z^2 \operatorname{sign}(v_z) \end{pmatrix}.$$

This gives us a resulting force of

$$F_{res} := \begin{pmatrix} F_{res,x} \\ F_{res,y} \\ F_{res,z} \end{pmatrix} = \begin{pmatrix} F_{D,x} \\ F_{D,y} \\ F_{D,z} + F_g \end{pmatrix}.$$

By Newton's second law, we have that $F_{res} = ma$, so we have that the acceleration is equal to

$$\dot{v} := \begin{pmatrix} \dot{v}_x \\ \dot{v}_y \\ \dot{v}_z \end{pmatrix} = \frac{1}{m} F_{res},$$

because $\dot{x} = v$ and $\dot{v} = a$. The dot here indicates the derivative. We can combine all the equations into one system to obtain that

$$\begin{pmatrix} \dot{x_x} \\ \dot{x_y} \\ \dot{x_z} \\ \dot{v_x} \\ \dot{v_y} \\ \dot{v_z} \end{pmatrix} = \begin{pmatrix} v_x \\ v_y \\ v_z \\ -\frac{1}{2m}\rho C^{person}_{side} A^{person}_{side} (v_x - w_x)^2 \operatorname{sign}(v_x - w_x) \\ -\frac{1}{2m}\rho C^{person}_{side} A^{person}_{side} (v_y - w_y)^2 \operatorname{sign}(v_y - w_y) \\ -\frac{1}{2m}\rho C^{person}_{front} A^{person}_{front} v_z^2 \operatorname{sign}(v_z) - g \end{pmatrix}.$$

This system can now numerically be solved using for example runga-kutta 4.

Stage 2

The only difference between stage 1 and 2 is the drag force. During stage 2, both the skydiver and the parachute will create a drag force. The parachute is much larger than the person, so the drag force of the person will be negligible in comparison to the drag force of the parachute. Therefore, we will ignore the drag force of the person. The drag force in the x- and y-direction are caused by drag on the side of the parachute and the drag force in the z-direction is caused by the drag created from below. We therefore have that

$$\begin{pmatrix} F_{D,x} \\ F_{D,y} \\ F_{D,z} \end{pmatrix} = \begin{pmatrix} -\frac{1}{2}\rho C^{chute}_{side} A^{chute}_{side} (v_x - w_x)^2 \operatorname{sign}(v_x - w_x) \\ -\frac{1}{2}\rho C^{chute}_{side} A^{chute}_{side} (v_y - w_y)^2 \operatorname{sign}(v_y - w_y) \\ -\frac{1}{2}\rho C^{chute}_{below} A^{chute}_{below} v_z^2 \operatorname{sign}(v_z) \end{pmatrix}.$$

Stage 1 and 2 combined

Since the parachute is opened at h^{chute}, we then have that

$$\begin{pmatrix} \dot{x_x} \\ \dot{x_y} \\ \dot{x_z} \\ \dot{v_x} \\ \dot{v_y} \\ \dot{v_z} \end{pmatrix} = \begin{cases} \begin{pmatrix} v_x \\ v_y \\ v_z \\ -\frac{1}{2m}\rho C^{person}_{side} A^{person}_{side} (v_x - w_x)^2 \operatorname{sign}(v_x - w_x) \\ -\frac{1}{2m}\rho C^{person}_{side} A^{person}_{side} (v_y - w_y)^2 \operatorname{sign}(v_y - w_y) \\ -\frac{1}{2m}\rho C^{person}_{front} A^{person}_{front} v_z^2 \operatorname{sign}(v_z) - g \end{pmatrix} & : x_z > h^{chute} \\ \begin{pmatrix} v_x \\ v_y \\ v_z \\ -\frac{1}{2m}\rho C^{chute}_{side} A^{chute}_{side} (v_x - w_x)^2 \operatorname{sign}(v_x - w_x) \\ -\frac{1}{2m}\rho C^{chute}_{side} A^{chute}_{side} (v_y - w_y)^2 \operatorname{sign}(v_y - w_y) \\ -\frac{1}{2m}\rho C^{chute}_{below} A^{chute}_{below} v_z^2 \operatorname{sign}(v_z) - g \end{pmatrix} & : x_z \leq h^{chute} \end{cases}.$$

References

[Annaloro et al., 2020] Annaloro, J., Galera, S., Thiebaut, C., Spel, M., Van Hauwaert, P., Grossir, G., Paris, S., Chazot, O., and Omaly, P. (2020). Aerothermodynamics modelling of complex shapes in the debrisk atmospheric reentry tool: methodology and validation. *Acta Astronautica*, 171:388–402.

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