THE ANDREW & ERNA VITERBI FACULTY OF ELECTRICAL ENGINEERING

Technion

COMMUNICATION LABORATORY
המעבדה לתקשורת

Design of error correcting codes for storage devices

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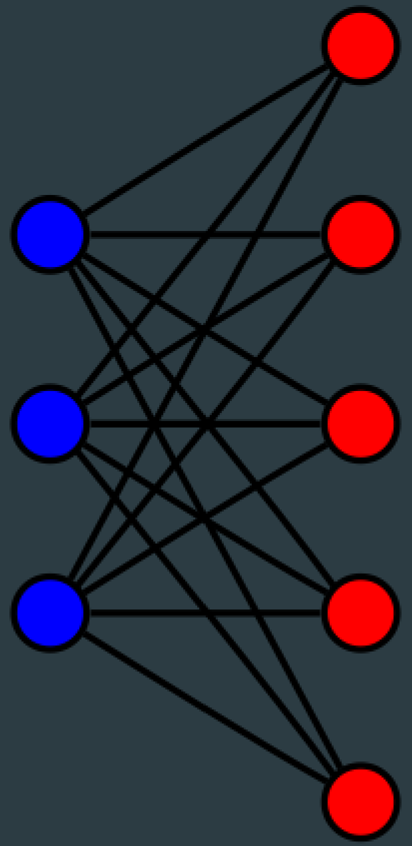
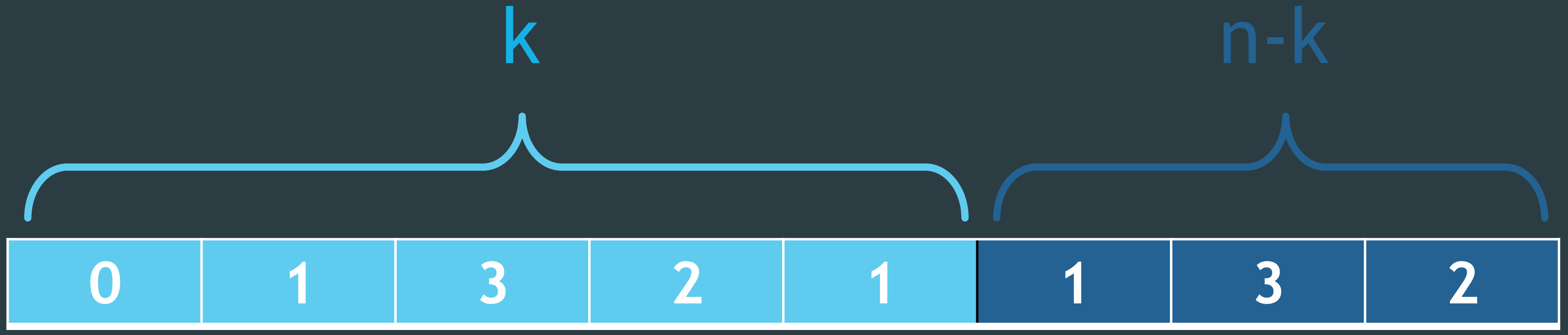


Outline

- Low Density Parity Check codes
- Message passing algorithm
- q-ary Bit Measurement Channel
- Empirical and theoretical results
- Approximation models for the problem

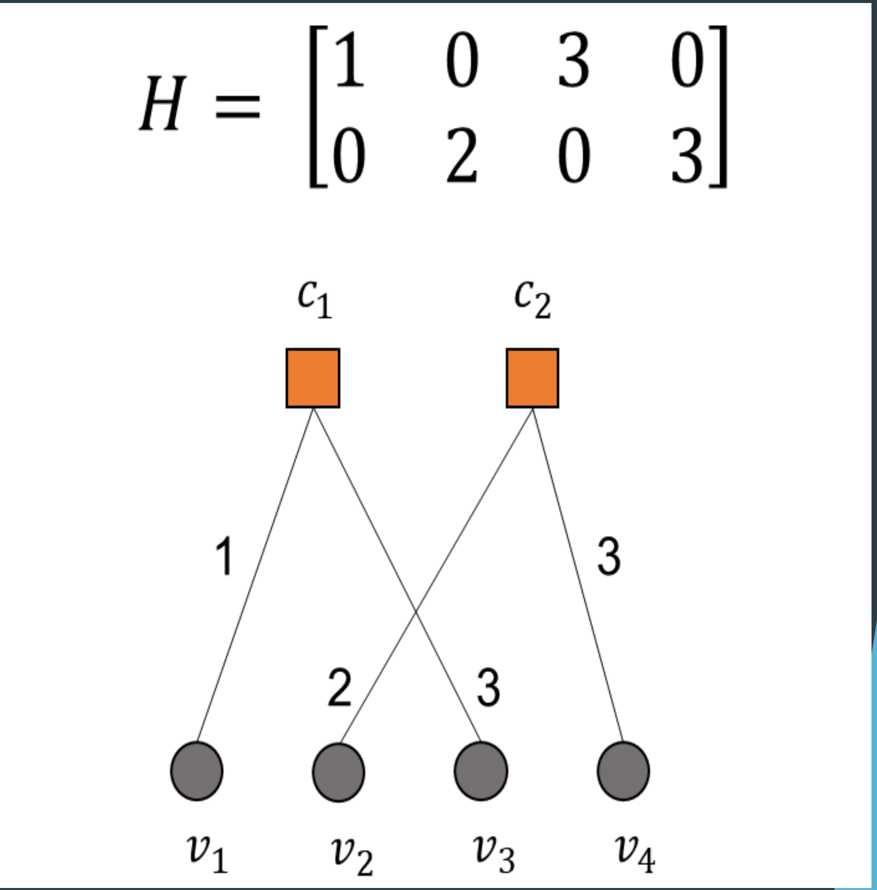
Linear GF_q Block Codes

- k information symbols ⟶ n code symbols
- Code word is generated from a generator matrix $G \in GF_q^{n \times k}$
- Dual matrix $H \in GF_q^{(n-k) \times n}$ - Parity check matrix



Low Density Parity Check GF_q Codes

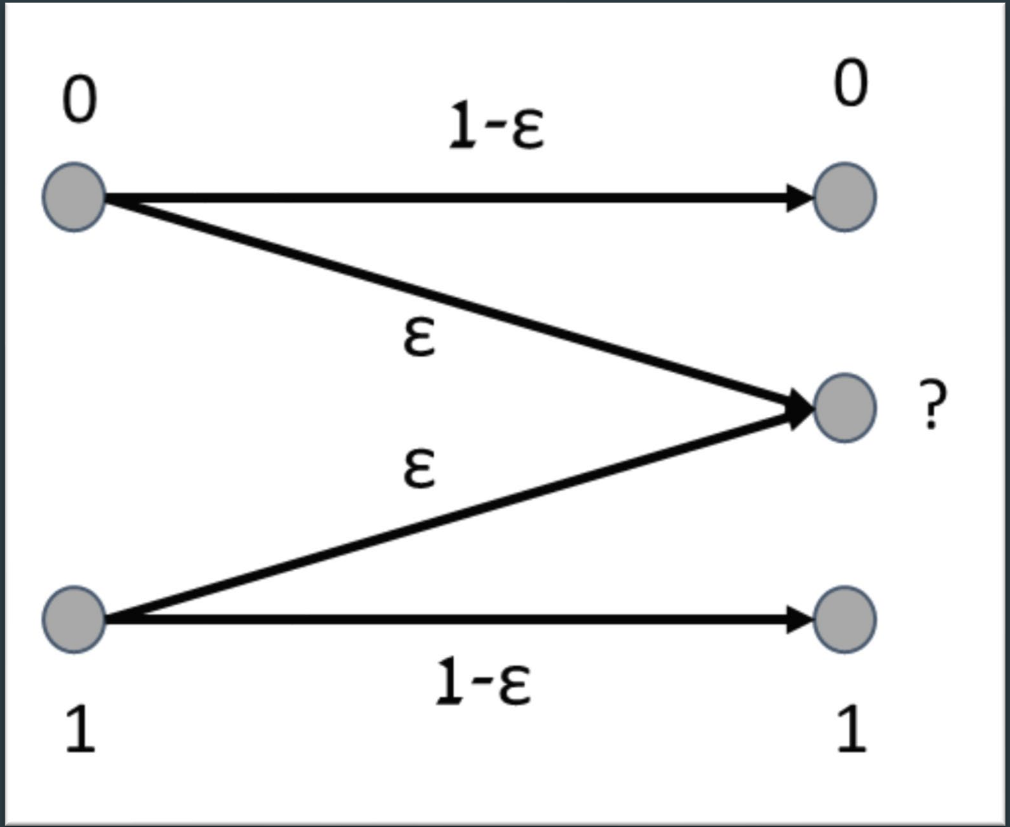
- Sparse matrix for parity check H
- Regular code - d_v, d_c constants
- Parity Check:
  - $1 \cdot v_1 + 3 \cdot v_3 = 0$
  - $2 \cdot v_2 + 3 \cdot v_4 = 0$



Binary Erasure Channel (BEC)

- Binary Alphabet (0/1)
- Symmetric erasure probability - ε
- Erasure (?) is an unknown state

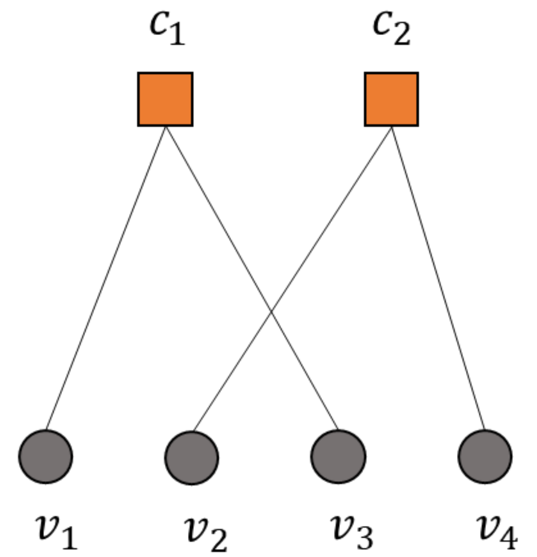
$$[1\ 0\ 1\ 0] \rightarrow [\ 1\ ?\ ?\ \ 0]$$



Low Density Parity Check Binary Codes

- Parity Check:
  - $v_1 \oplus v_3 = 0$
  - $v_2 \oplus v_4 = 0$
- Iterative decoding of [? ? 1 0] -
  - $v_1 = 0 \oplus 1 = 1$
  - $v_2 = 0 \oplus 0 = 0$

$$H = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$



Implementation

- MATLAB
  - Convenient matrix operations
  - GF_q add-on for calculations
  - Concurrent simulations

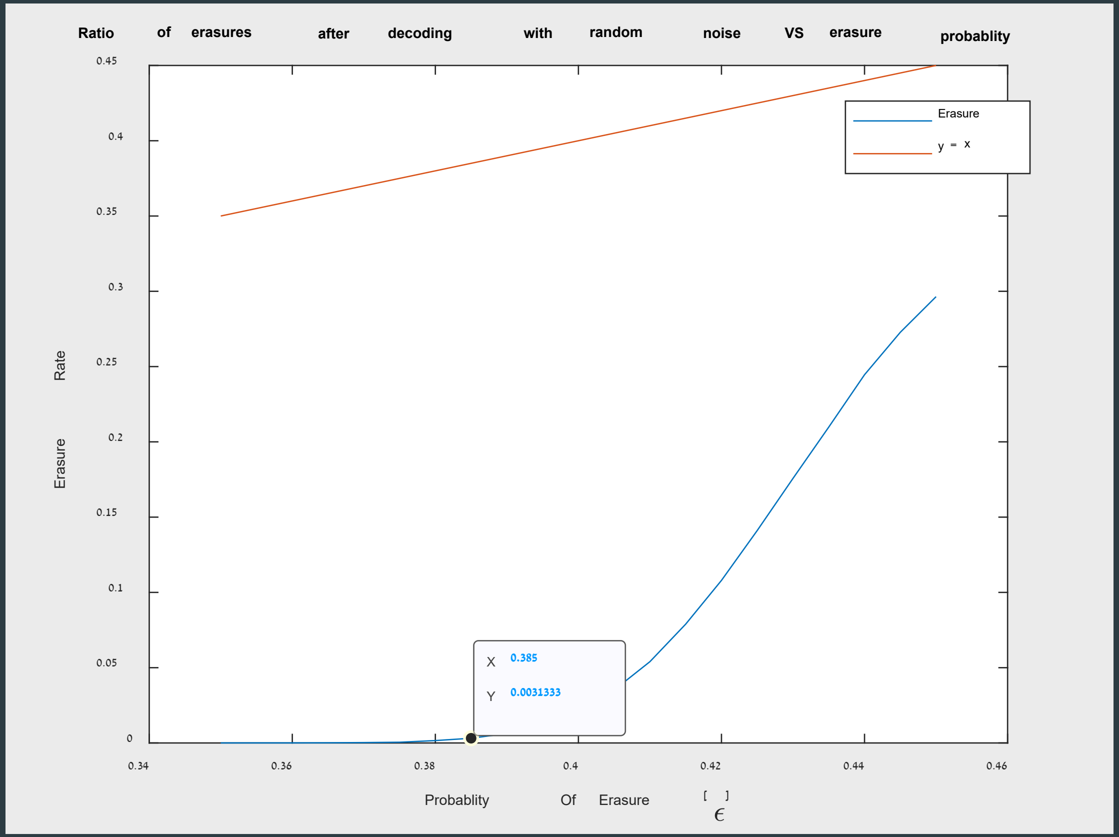
Communications Toolbox™

Parallel Computing Toolbox™

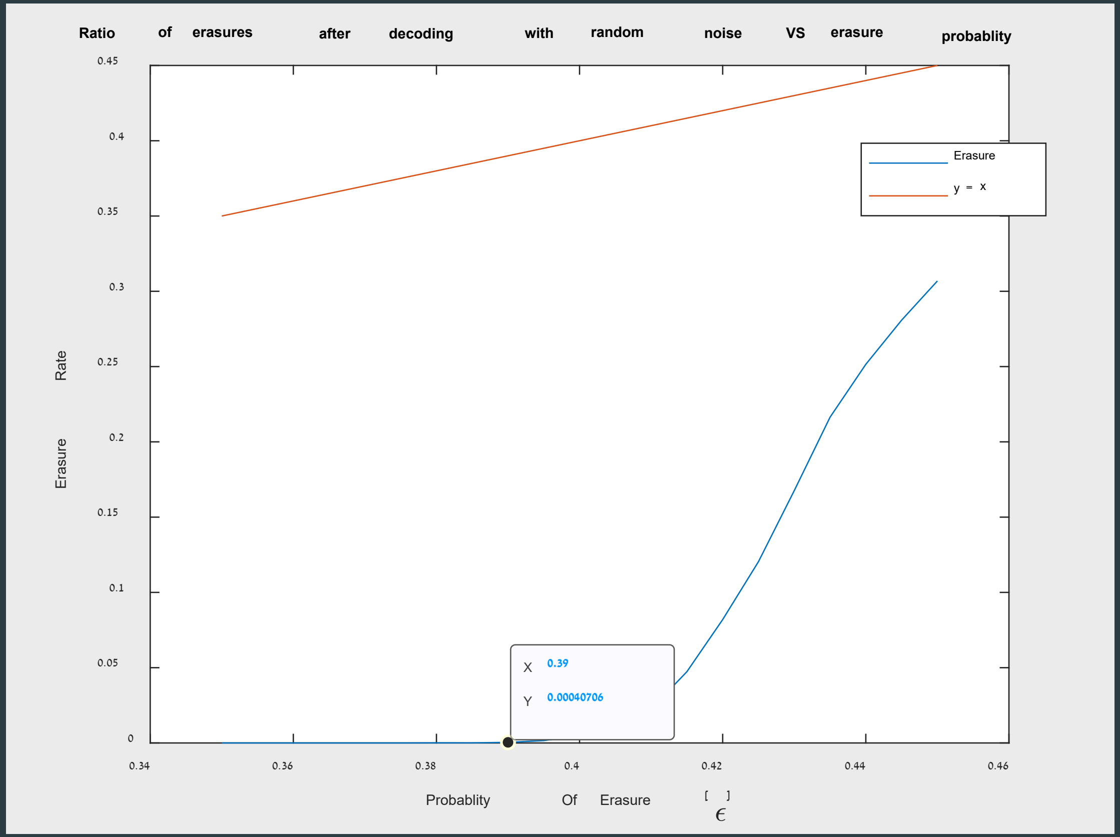
Running Time ≈ 1 hour

BEC - Results

Code length = 1002

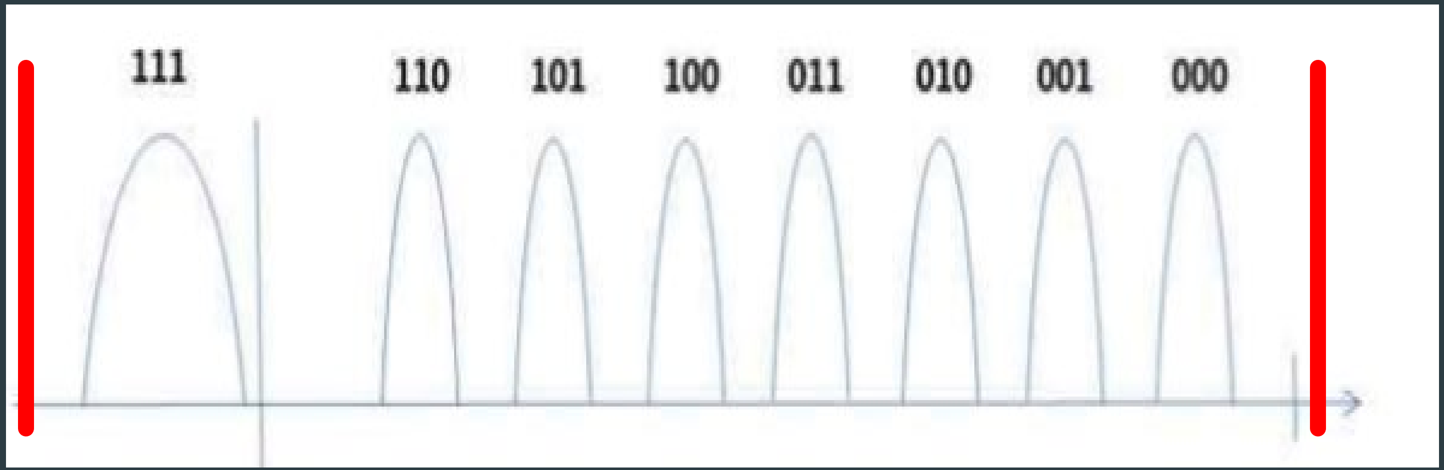


Code length = 2004



Reading from Flash Storage Device

Stored word in cell 011
Reading MSB to LSB



q-ary Bit-Measurement Channel (QBMC)

- $q = 2^s$ symbols, each represented by s bits
- Partial erasure = some of the bits are unknown
  - Each partial erasure has its probability

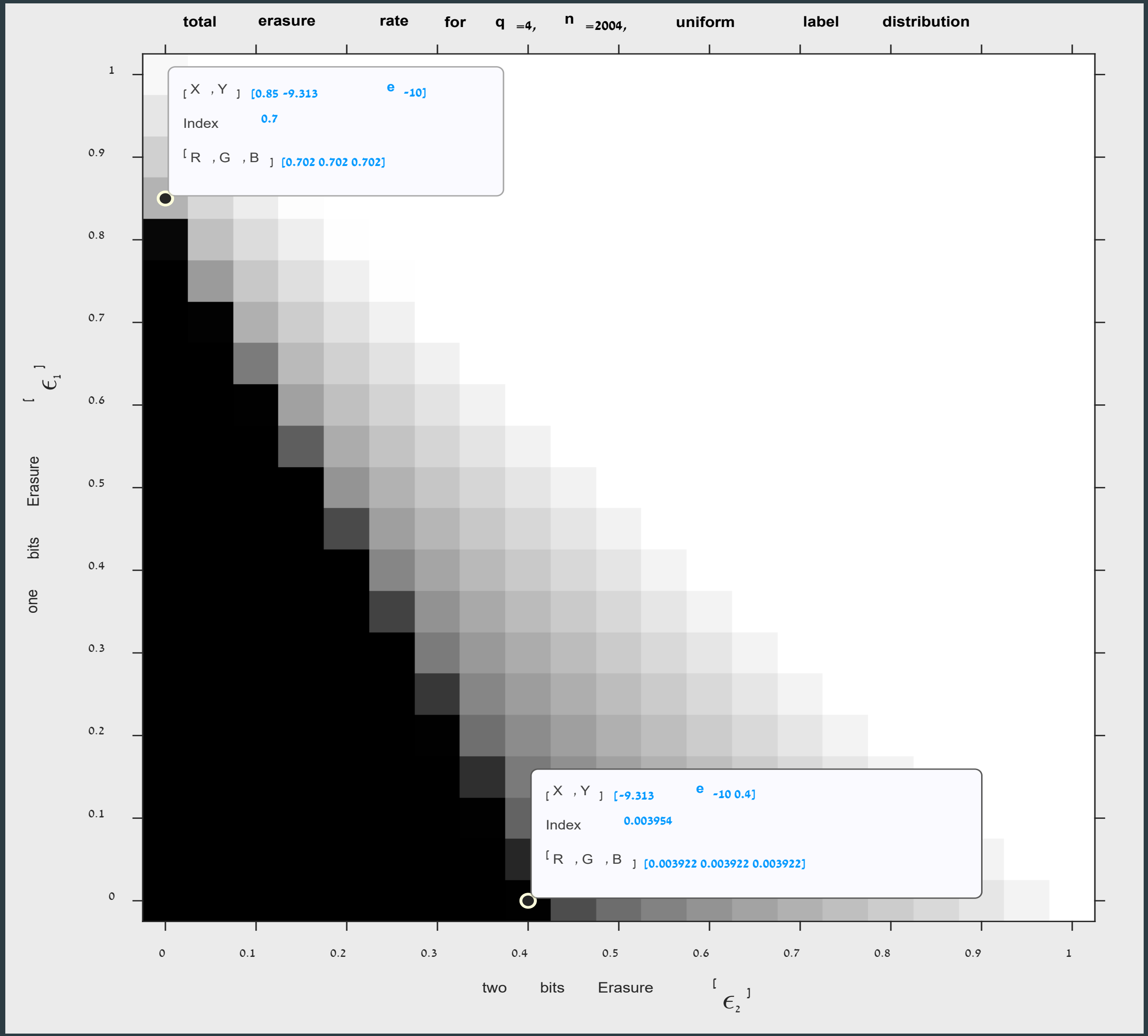
$$\underbrace{[1\ 0\ 1\ 0] \rightarrow [\ 1\ ?\ ?\ ?\ ]}_{3\ bits\ erasure} \qquad \underbrace{[1\ 0\ 1\ 0] \rightarrow [\ 1\ 0\ 1\ ?\ ]}_{1\ bits\ erasure}$$

Message Passing for LDPC over QBMC

- Generalized binary iterative decoding of BEC to q elements from GF_q
- Two main operations -
  - Sumset: $sumset(\{x, y\}, \{a, b\}) = \{x + a, x + b, y + a, y + b\}$
  - Intersection: $intersection(\{x, y, z\}, \{x, a, b\}) = \{x\}$

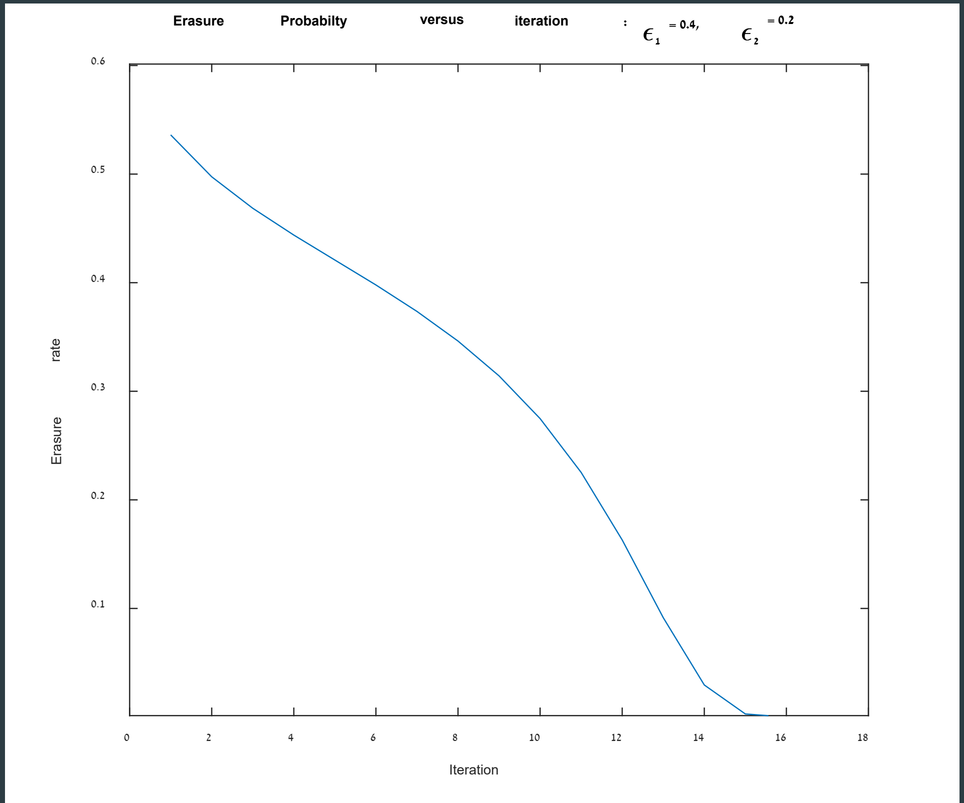
Running Time ≈ 10 hours

Message Passing over QBMC - Results



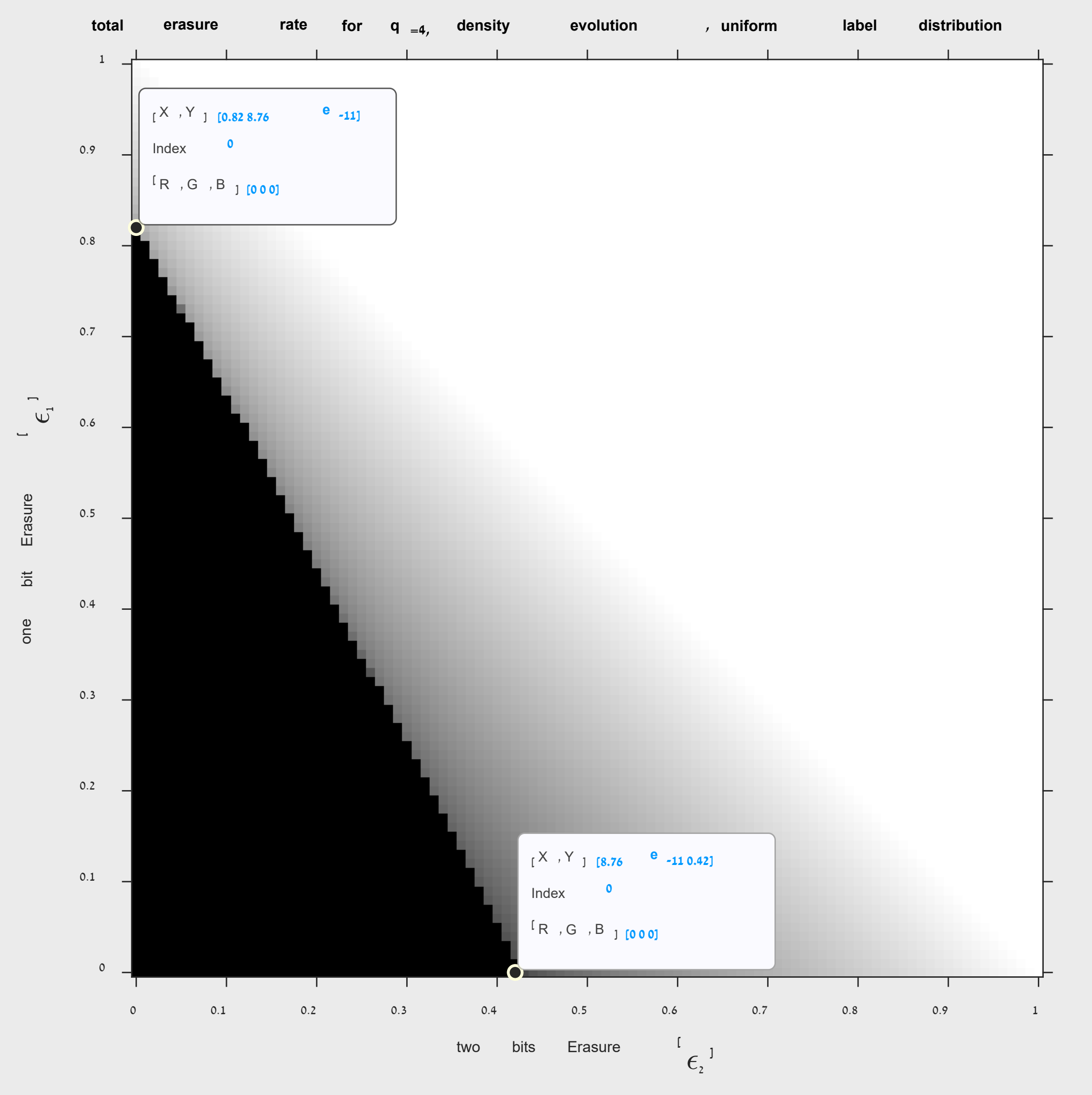
Analytic Performance (Density Evolution)

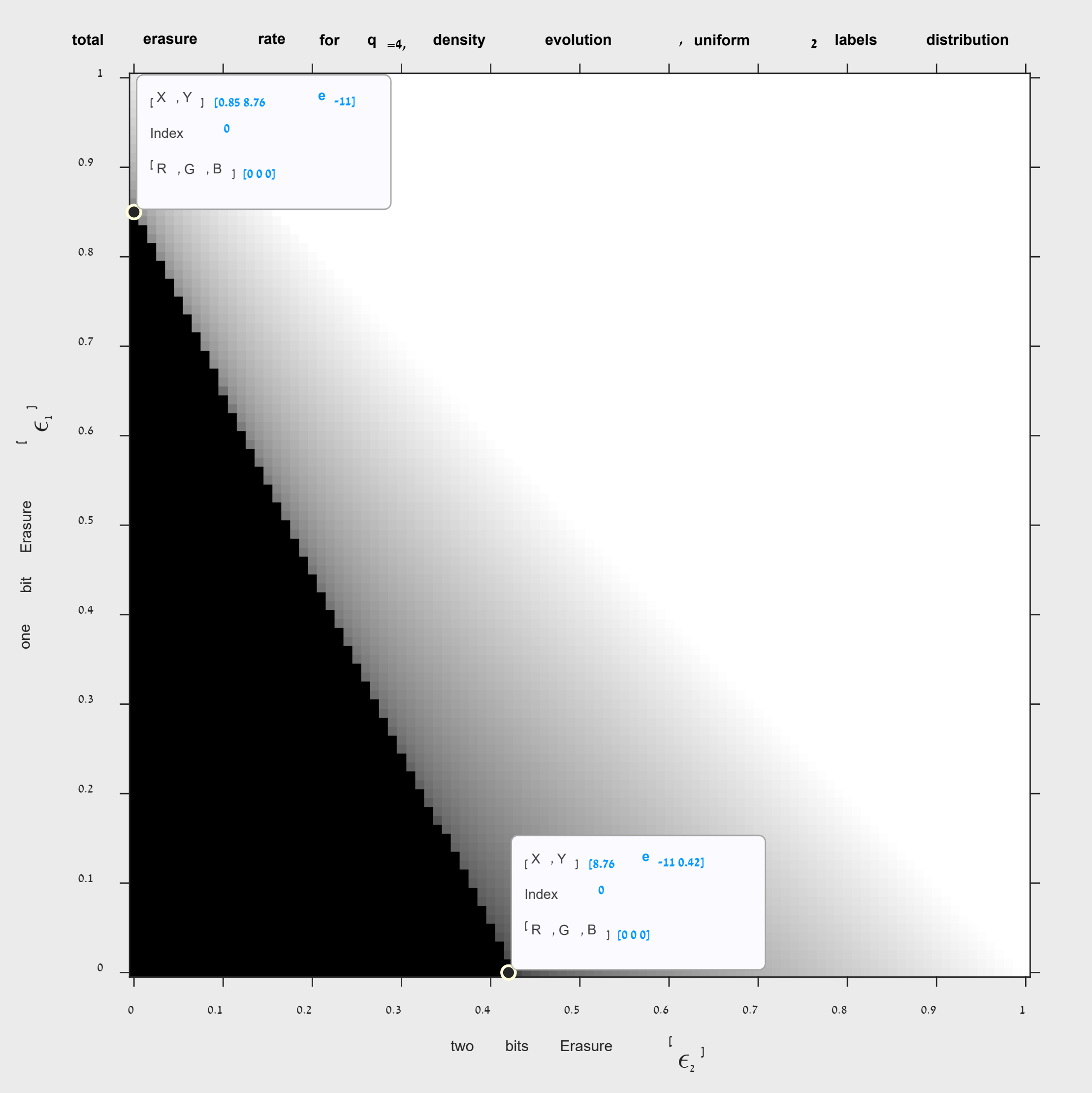
- Tracking the probability of partial erasures as a function of the decoding iteration
- Evolution of distribution at each iteration of message passing considering labels distribution
- Threshold determined by probability of erasure at output



Running Time ≈ 10 hours

Density Evolution - Results



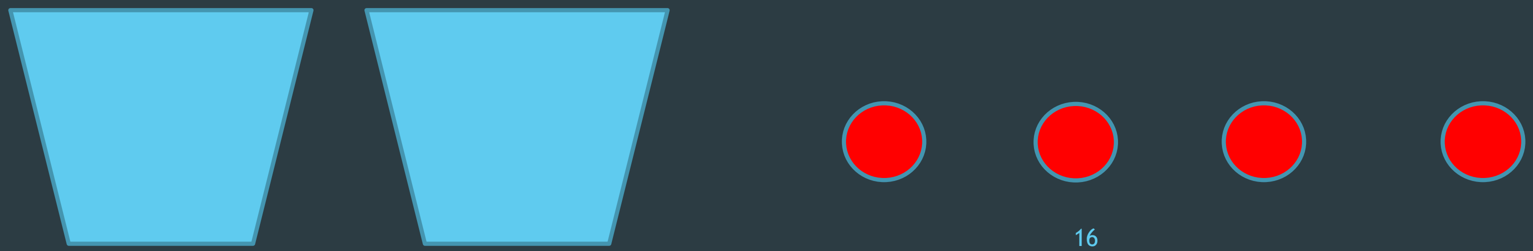


Approximation methods

- Tracking the exact probabilities is difficult as there are multiple messages
- Instead, we track the probabilities of certain subgroup sizes
  - Non erasure subgroup size is 1
- Using probabilistic models for uniform label distribution –
  - Balls and Bins
  - Union Model

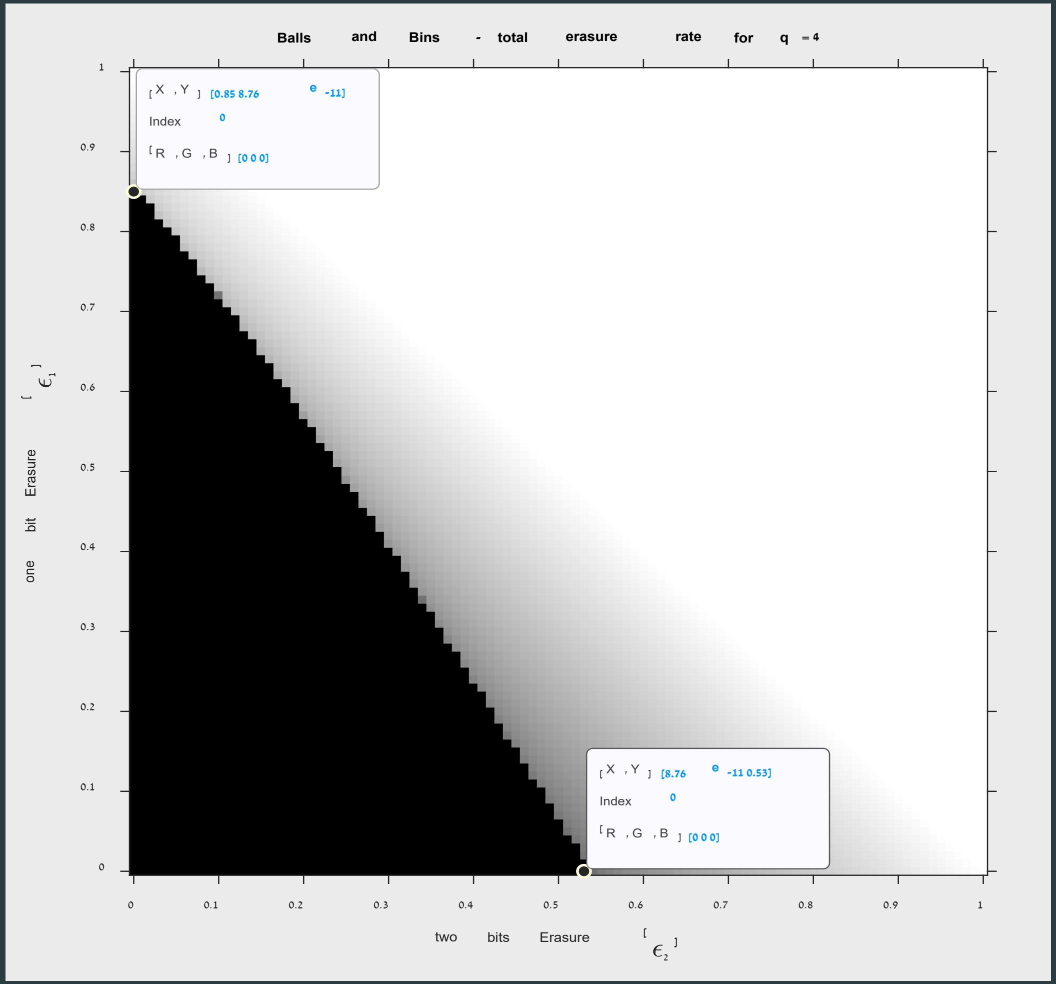
Balls and Bins Model

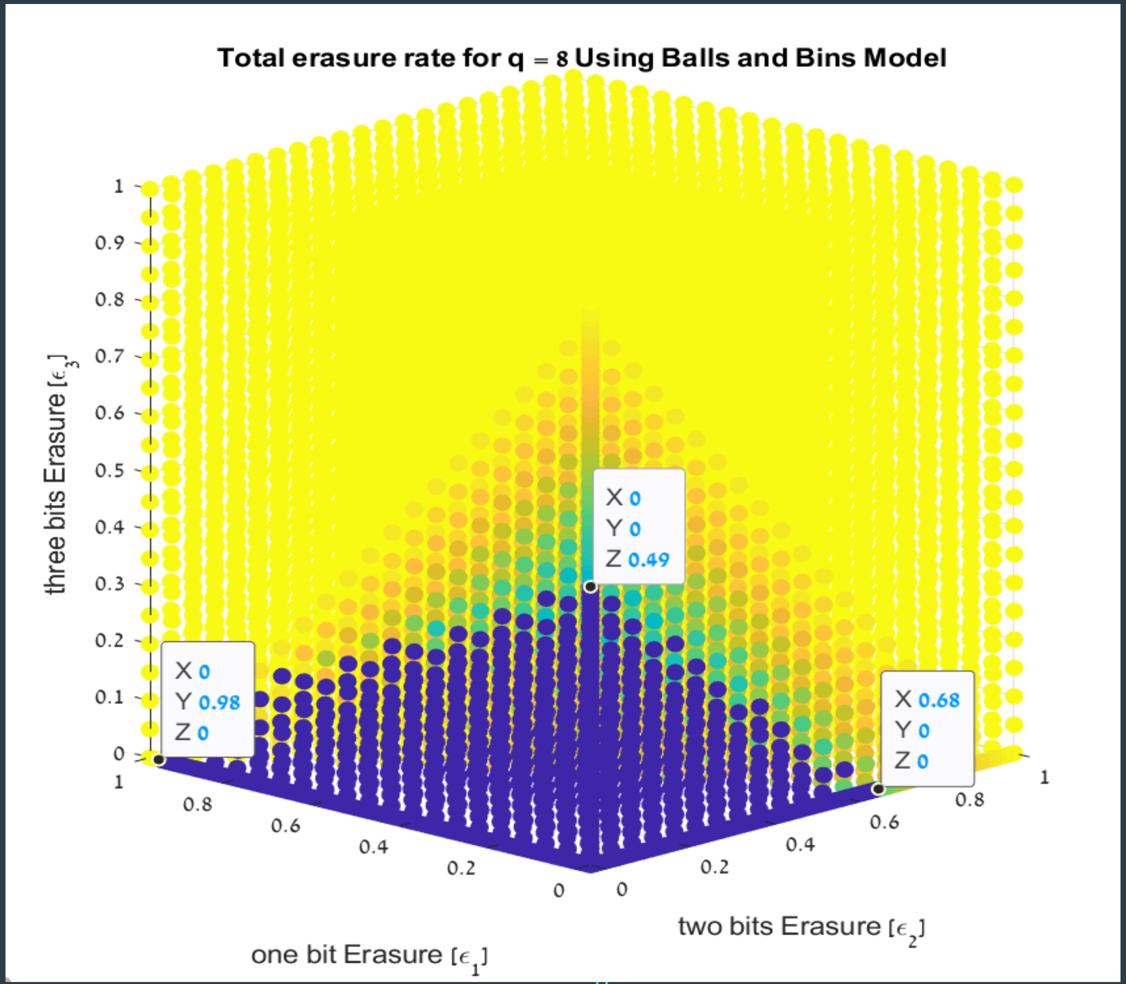
- Throwing balls independently and uniformly to bins
- Number of balls – product of the subgroups' sizes
- Number of bins – q
- Number of non-empty bins is the size of the subgroup at output



Balls and Bins - Results

Running Time ≈ 15 hours





Running Time ≈ 10 hours

Union Model

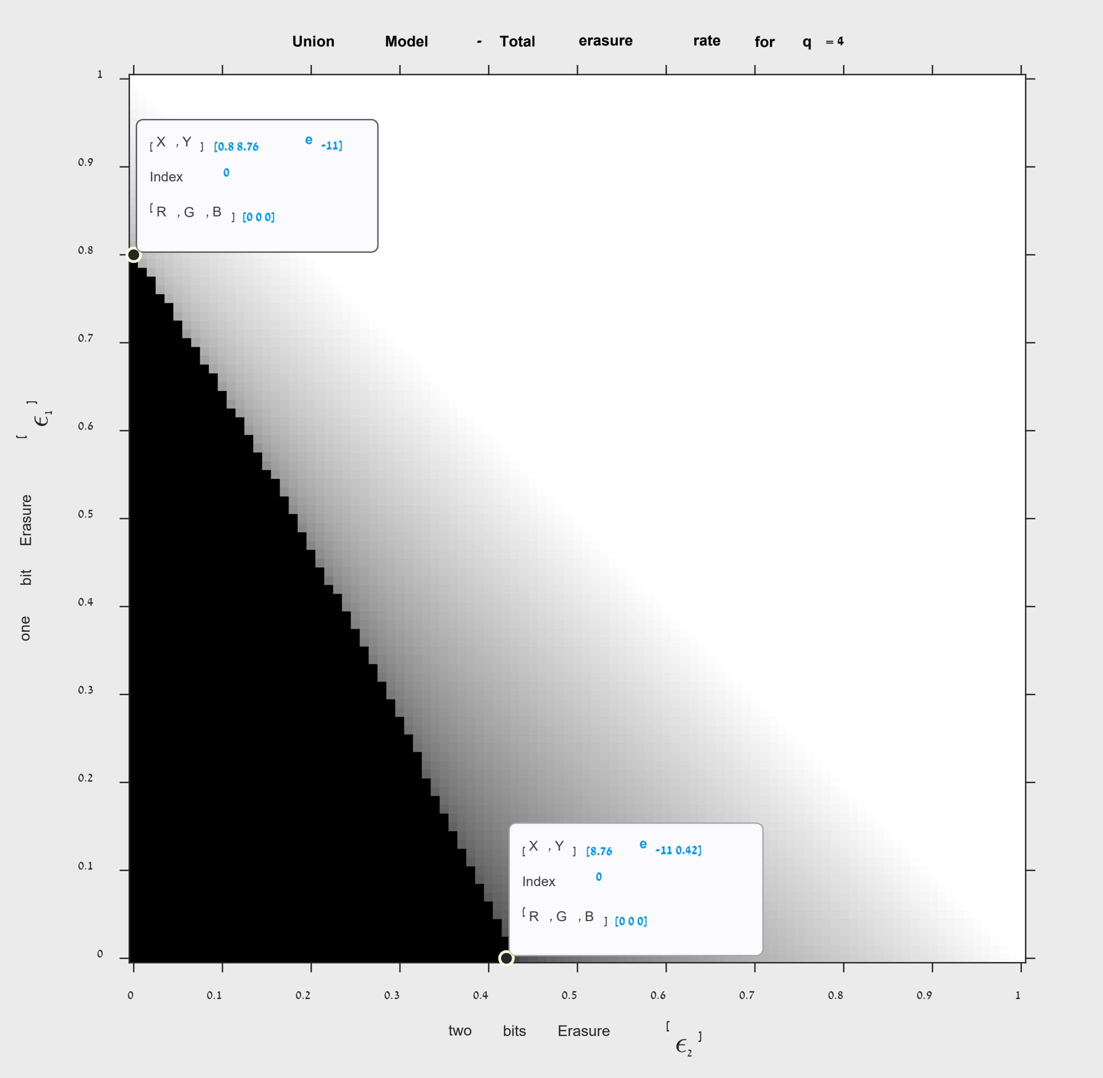
- Similar to balls and bins, except independence assumption
- Given 3 elements $x, y, z \in GF_q$ such that $y \neq z$

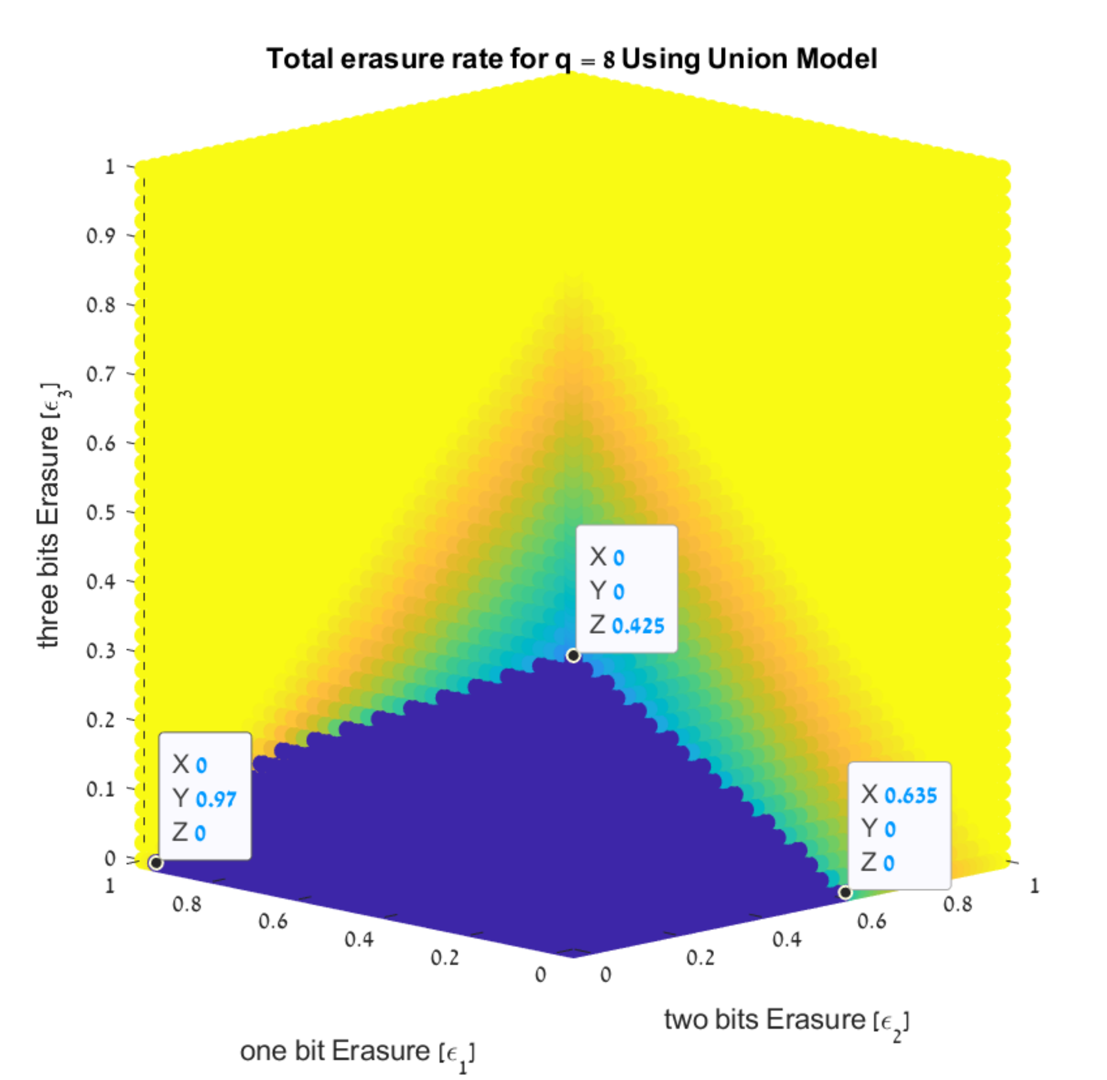
$$\rightarrow x + y \neq x + z$$

- Different elements in the same subgroup must go to different bins

Union Model - Results

Running Time ≈ 20 hours





Running Time ≈ 10 hours

Summary

- GF_q LDPC codes show good results under iterative decoding
- Better iterative-decoding performance as q increase
- The iterative method of “Message Passing” is fast, but empirical and analytical calculations are complex
- The Union model is a good approximation of the problem that can help us compute the thresholds efficiently

Future Research

- Optimize the precise algorithm for faster results
- Find a more accurate threshold by using higher resolution
- Better approximations than the Union Model with different label distribution

References

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3. R. C. a. Y. Cassuto, “LDPC Codes for the q-ary Bit-Measurement Channel,” Technion - Israel Institute of Technology, Haifa, 2016.
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6. S. K. T. L. J. Nana Traore, “Message Passing Algortihm and Linear Programming Decoding for LDPC and Linear Block Codes,” Aalborg University, Aalborg, 2007.
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Thank you for listening