

Asymptotic notations

Big-Oh

Big-Omega

Big-theta

Big-Oh # asymptotic upper bound

$f(n) = O(g(n))$: there exists constant $c > 0$ and n_0 such that $f(n) \leq cg(n)$ for $n \geq n_0$

$$\log_{10} n = \frac{\log_2 n}{\log_2 10} = O(\log_2 n) = \log n$$

$$\sum_{i=1}^n i \leq n \cdot n = O(n^2)$$

$$\log(n!) = \log(n) + \dots + \log 1 = O(n \log n)$$

Harmonic Series:

$$\sum_{i=1}^n \frac{1}{i} = O(\log n)$$

Big-Omega # asymptotic lower bound

$f(n) = \Omega(g(n))$: there exists constant $c > 0$ and n_0 such that $f(n) \geq cg(n)$ for $n \geq n_0$

$$\log(n!) = \log(n) + \log(n-1) + \dots + \log 1$$

$$\geq \log(n) + \log(n-1) + \dots + \log\left(\frac{n}{2}\right)$$

$$\geq \frac{n}{2} \cdot \log\left(\frac{n}{2}\right)$$

$$= \frac{n}{2} \cdot (\log n - 1) = \Omega(n \log n)$$

Harmonic Series:

$$\sum_{i=1}^n \frac{1}{i} = \Omega(\log n)$$

Big-Theta # asymptotic tight bound

$f(n) = \Theta(g(n))$: $f(n) = O(g(n))$ and $f(n) = \Omega(g(n))$

An interesting fact about logarithm:

$$\log_{b_1} n = O(\log_{b_2} n)$$

For any constant $b_1 > 1$ and $b_2 > 1$

Obviously, Ω and Θ also have this property