Subspace

1. Properties of subspace
   a) The zero vector 0 belongs to V
   b) Closed under addition: If u and w belong to V, then $u + w$ belongs to V
   c) Closed scalar multiplication: if u belongs to V, c is a scalar, then cu belongs to V
2. The span of a vector set is a subspace
3. Null Space
   a) The null space of a matrix A is the solution set of $Ax = 0$, denoted as $Null\ A$
   b) $Null\ A$ is a subspace
4. Column Space
   a) Column space of a matrix A is the span of its columns, denoted as $Col\ A$
   b) If matrix A represents a function, $Col\ A$ is the range of the function
5. Row Space
   a) Row space of a matrix A is the span of its rows, denoted as $Row\ A$
   b) $Row\ A = Col\ A^T$

Basis

1. Basis
   a) Let V be a nonzero subspace of R^n, A basis B for V is a linearly independent generation set of V
   b) The pivot columns of a matrix form a basis for its subspace
2. Property of Basis
   a) S is contained in Span S
   b) If a finite set S’ is contained in Span S, then Span S’ is also contained in Span S
   c) For any vector z, Span S = Span S∪{z} if and only if z belongs to the Span S
3. Theorem of Basis
   a) A basis is the smallest generation set
   b) A basis is the largest independent vector set in the subspace
   c) Any two bases for a subspace contain the same number of vectors
   d) The number of vectors in a basis for a nonzero subspace V is called dimension of V
4. Reduction Theorem
   a) There is a basis containing in any generation set S
   b) S can be reduced to a basis V by removing some vectors
5. Extension Theorem
   a) Given a independent vector set S in the space, S can be extended to a basis by adding more vectors
6. Confirming a set is a basis
   a) Given a subspace V, assume that we already know that dim V = k. Suppose S is a subset of V with k vectors

      If S is independent → S is a basis

      If S is a generation set → S is a basis