Chapter09 RREF

1. RREF v.s. Linear Combination
   a) Column Correspondence Theorem
      i) If a_j is a linear combination of other columns of A → r_j is a linear combination of the corresponding columns of R with the same coefficient
      j) The RREF of augmented matrix [A b] is [R b']
         $Ax = b$ and $Rx = b'$ have the same solution set
         The RREF of matrix A is R
         $Ax = 0$ and $Rx = 0$ have the same solution set
      k) The relations between the rows are changed
   b) Span
      i) The span of the rows are the same
      j) The span of the columns are different
2. RREF v.s. Independent
   a) The pivot columns are independent
   b) The non-pivot columns are the linear combination of the pivot columns
   c) All columns are independent, every column in RREF(A) is standard vector
   d) More than m vectors in R^m must be dependent
3. RREF v.s. Rank
   a) Num of independent columns = num of nonzero rows
   b) Rank(A) $\leqslant$ min(m, n)
   c) If m < n, the columns of A are dependent
   d) Rank = num of basic variables
   e) Nullity = num of free variables
4. RREF v.s. Span
   a) Rank A $\neq$ Rank [A b] → Ax = b is inconsistent
   b) Ax = b is consistent for every b → Rank A = num of rows
      Every b is in the span of the columns of A
      Every b belongs to $Span\{a_1, a_2, \dots, a_n\}$ $Span\{a_1, a_2, \dots, a_n\} = R^m$
      m independent vectors can span R^m