$$D_n = \begin{vmatrix} 1 & 1 & 1 & \cdots & 1 \\ x_1 & x_2 & x_3 & \cdots & x_n \\ x_1^2 & x_2^2 & x_3^2 & \cdots & x_n^2 \\ & \vdots & & & \\ x_1^{n-2} & x_2^{n-2} & x_3^{n-2} & \cdots & x_n^{n-2} \\ x_1^{n-1} & x_2^{n-1} & x_3^{n-1} & \cdots & x_n^{n-1} \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 & \cdots & 1 \\ 0 & x_2-x_1 & x_3-x_1 & \cdots & x_n-x_1 \\ 0 & x_2^1(x_2-x_1) & x_3^1(x_3-x_1) & \cdots & x_n^1(x_n-x_1) \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & x_2^{n-2}(x_2-x_1) & x_3^{n-2}(x_3-x_1) & \cdots & x_n^{n-2}(x_n-x_1) \end{vmatrix}$$

$$= (x_2-x_1)(x_3-x_1)\cdots(x_n-x_1) \begin{vmatrix} 1 & 1 & \cdots & 1 \\ x_2 & x_3 & \cdots & x_n \\ \vdots & \vdots & & \vdots \\ x_2^{n-2} & x_3^{n-2} & & x_n^{n-2} \end{vmatrix}_{(n-1)\times(n-1)}$$

$$= (x_2-x_1)(x_3-x_1)\cdots(x_n-x_1)(x_3-x_2)\cdots(x_n-x_2)\cdots(x_n-x_{n-1})$$

$$\begin{vmatrix} 1 & 1 & 1 & 1 \\ x_2 & x_3 & x_4 & x_5 \\ x_2^2 & x_3^2 & x_4^2 & x_5^2 \\ x_2^3 & x_3^3 & x_4^3 & \boxed{x_5^3} \end{vmatrix}$$

【例 2】［取自《张宇线性代数 9 讲》P30，习题 2.3］

已知 n 阶行列式 $|A| = a$，将 $|A|$ 中的每一列减去其余的各列得到的行列式记为 $|B|$，则 $|B| =$ _____．

【分析】

$$|B| = \left| \alpha_1 - \sum_{i\neq 1}\alpha_i, \quad \alpha_2 - \sum_{i\neq 2}\alpha_i, \quad \cdots, \quad \alpha_n - \sum_{i\neq n}\alpha_i \right|$$

$$= \left(\alpha_1 \; \alpha_2 \; \cdots \; \alpha_n \right) \begin{pmatrix} 1 & -1 & \cdots & -1 \\ -1 & 1 & \cdots & -1 \\ \vdots & \vdots & & \vdots \\ -1 & -1 & \cdots & 1 \end{pmatrix}$$

$$= \left| \alpha_1 \cdots \alpha_n \right| \cdot \begin{vmatrix} 1 & -1 & \cdots & -1 \\ -1 & 1 & \cdots & -1 \\ \vdots & \vdots & & \vdots \\ -1 & -1 & \cdots & 1 \end{vmatrix}_{n\times n}$$

$$\underset{\underset{a}{\parallel}}{}$$

$$= a(2-n) \begin{vmatrix} 1 & -1 & -1 & \cdots & -1 \\ 1 & 1 & -1 & \cdots & -1 \\ \vdots & \vdots & \vdots & & \vdots \\ 1 & -1 & \cdots & & 1 \end{vmatrix}^{(-1)}$$

$$= a(2-n) \begin{vmatrix} 1 & -1 & -1 & \cdots & -1 \\ 0 & 2 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \cdots & 2 \end{vmatrix} = a\cdot(2-n)\cdot 2^{n-1}$$