Chapter06   Span
1. Consistent
   a) A system of linear equations is called consistent if it has solution
   b) A system of linear equations is called inconsistent if it has no solution
2. Linear Combination
   a) Given a vector set $\{u^1, u^2, \ldots, u^k\}$, the linear combination of the vectors in the set: $v = c^1 u^1 + c^2 u^2 + \cdots + c^k u^k$, $c^1, c^2, \ldots, c^k$ are scalars
   b) Column Aspect: $A = [a^1 \; a^2 \; \ldots \; a^n]$, $x = [x^1 \; x^2 \; \ldots \; x^n]^T$
   $Ax = x^1 a^1 + x^2 a^2 + \cdots + x^n a^n$
   c) Has solution or not $\longleftrightarrow$ Is b the linear combination of columns of A?
   If u and v are any nonparallel vectors in R^2, then every vector in R^2 is a linear combination of u and v
   Nonparallel: u and v are non-zero vectors, and $u \; != cv$
3. Span
   a) Given a vector set $S = \{u^1, u^2, \ldots, u^k\}$, Span of S is the vector set of all linear combinations of $u^1, u^2, \ldots, u^k$
   b) $Span\; S = \{c^1 u^1 + c^2 u^2 + \cdots + c^k u^k \mid for\; all\; c^1, c^2, \ldots, c^k\}$
   c) If S contains a non-zero vector, then $Span\; S$ has infinitely many vectors
   d) Has solution or not $\longleftrightarrow$ Is b in the span of the columns of A?

Chapter07   Independent
1. Unique Solution
   a) The columns of A are independent
   b) Rank $A = n$ Nullity $A = 0$
2. Infinite Solution
   a) The columns of A are dependent
   b) Rank $A < n$ Nullity $A > 0$
3. Dependent and Independent
   a) A set of vectors $\{u^1, u^2, \ldots, u^k\}$ is dependent
   If there exists scalars $c^1, c^2, \ldots, c^k$, not all zero, such that
   $c^1 u^1 + c^2 u^2 + \cdots + c^k u^k = 0$
   b) A set of vectors $\{u^1, u^2, \ldots, u^k\}$ is independent
   $c^1 u^1 + c^2 u^2 + \cdots + c^k u^k = 0$
   Only if $c^1 = c^2 = \cdots = c^k = 0$
   c) Zero vector is the linear combination of any other vectors
4. Columns of A are dependent, if $Ax = b$ has solution $\rightarrow$ infinite solutions
5. Homogeneous Linear Equations
   a) Constant term b = 0
   b) A set of vectors is dependent: $Ax = 0$ have infinite non-zero solutions
   c) A set of vectors is independent: $Ax = 0$ only have zero solution
6. Rank and Nullity
   a) Rank is defined as the maximum number of linearly independent columns
   b) Nullity = number of columns – rank