④ 内容:

行列式　矩阵　→ 基础篇

向量组　方程组　→ 主题篇

特征值　二次型　→ 应用篇

② 行列式的计算

1° 具体型计算

2° 抽象型计算

3° 关于展开式法的运用

第一讲 行列式

综述

①行列式的定义与性质 $\begin{cases} \text{三大定义} \\ \text{七大性质} \end{cases}$

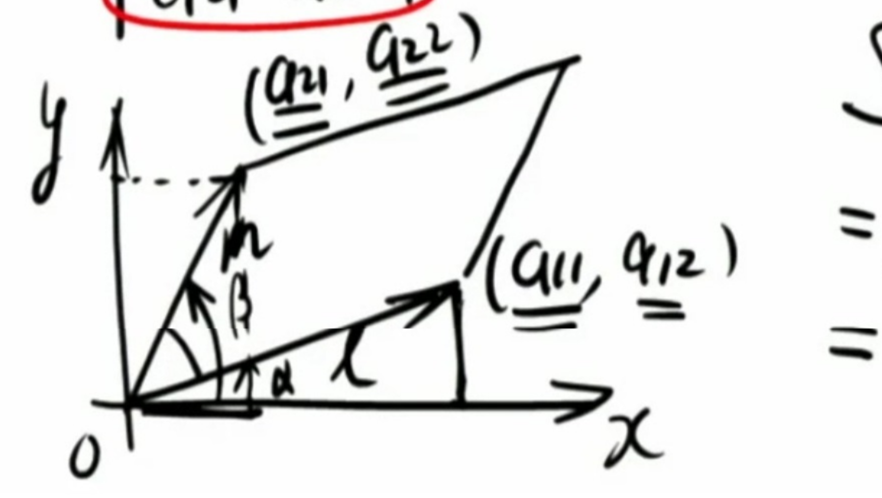
一、行列式的定义与性质

莱布尼茨(1693)

关孝和(1683)

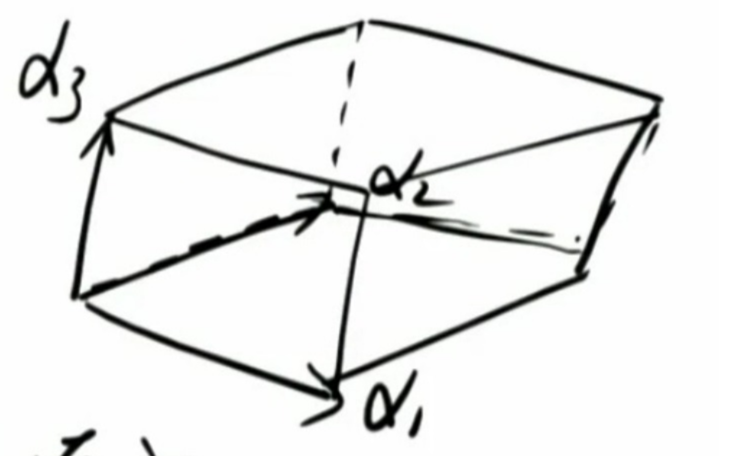
1. 几何法定义—杨西

1) $\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = S_{\square} = a_{11}a_{22} - a_{12}a_{21}$



$S_{\square} = l \cdot m \cdot \sin(\beta - \alpha)$
$= l \cdot m \cdot (\sin\beta\cos\alpha - \cos\beta\sin\alpha)$
$= a_{11}a_{22} - a_{12}a_{21}$

2) $\begin{array}{c} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{array} \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}_{3\times 3}$　棱



三阶行列式是以三个行向量为棱的平行六面体的体积.

3) $\begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix}_{n\times n}$

n阶行列式由n个n维向量组成. 其结果为n维图形体积.

4) 重要观点:

$D_n = |A_{n\times n}|$ $\begin{cases} \neq 0 \Rightarrow \text{(组成行列式的向量线性无关} \\ = 0 \Rightarrow \text{(组成行列式的向量线性相关.} \end{cases}$

$(1, 2)$
$(2, 4)$ →

5) 7大性质 (习惯上 $\alpha = \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{pmatrix}$ 列向量)

① $|A| = |A^T|$ (行、列地位等价)

② $|\alpha_1 \cdots \alpha_{i-1}\ 0\ \alpha_{i+1} \cdots \alpha_n| = 0$

(有0行或0列 $\Rightarrow |A| = 0$)

③ $|\alpha_1 \cdots \alpha_i, \cdots, k\alpha_i, \cdots \alpha_n| = 0$

④ 单列(行)可拆性

$|\alpha_1 \cdots \boxed{\alpha_i + \beta_i} \cdots \alpha_n| = |\alpha_1 \cdots \boxed{\alpha_i} \cdots \alpha_n| + |\alpha_1 \cdots \boxed{\beta_i} \cdots \alpha_n|$

⑤ $|\alpha_1 \cdots \alpha_i \cdots \alpha_j \cdots \alpha_n| = -|\alpha_1 \cdots \alpha_j \cdots \alpha_i \cdots \alpha_n|$

(互换)

⑥ $|\alpha_1 \cdots k\alpha_i \cdots \alpha_n| = k|\alpha_1 \cdots \alpha_i \cdots \alpha_n|$ (倍乘)

⑦ $|\alpha_1 \cdots \alpha_i \cdots \alpha_j \cdots \alpha_n|$　$|\alpha_1 \cdots \alpha_i \cdots \alpha_j + k\alpha_i \cdots \alpha_n|$