RESEARCH

Project-8

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Abstract

Goal of the project: This project uses functional maps to analyze shape differences and explore a collection of shapes through geometric data analysis. We compare shapes using inner products in their function spaces to understand variability. Specifically, by treating data points as geometric objects in the space of symmetric, positive-definite matrices, we aim to reveal patterns using methods like principal geodesic analysis (PGA).

Main results of the project: From the project, it is found that for the complex and non-linear dataset, PGA is more convenient to use for shape analysis as compared to PCA.

Personal key learning:

1 Sujan Darai: Learned point-to-point correspondence, ZoomOut algorithms
2 Samra Hamidovic: Learned to write Reports better and learned about integrating glyphs.
3 Matanat Mammadli: Learned to write results & discussion better and got a better understanding of functional maps and PGA with glyphs

Estimated working hours:

1 Sujan Darai: 8 hours
2 Samra Hamidovic: 8 hours
3 Matanat Mammadli: 8 hours

Project evaluation: 1

Number of words: 1639

Keywords: Geometric data analytics, Principal geodesic analysis (PGA), Statistical shape analysis, Functional map, Principal component analysis(PCA)

1 Introduction

Understanding and comparing shapes in computer graphics is important for design, search, and organization. Traditionally, a single similarity score was used to compare shapes, but this approach falls short when dealing with intricate shapes. This report presents a new method that dives more profound into the specifics of how shapes differ from one another. Our method enhances existing techniques that map one shape to another, with a particular focus on how this mapping process distorts or warps the shapes. This approach is beneficial because it accounts for the complexness of the mapping process, works independently of the shapes' 3D positioning, and aligns with traditional measures of distortion. Under certain conditions, it even allows us to reconstruct the original mapping. This method offers a more nuanced way to analyze shape differences by utilizing linear operators. This detailed analysis paves the way for more sophisticated applications in the field of computer graphics and shape analysis.[1]

Geometric analytics for functional shape descriptors is a cutting-edge approach in computer graphics and geometric modeling that focuses on understanding shapes by looking at their geometry and functionality, rather than just their appearance. This technique allows us to analyze how shapes perform and interact in different situations, providing a deeper insight into their properties. By using functional shape descriptors, we can measure and compare shapes more accurately and meaningfully. This approach also helps in creating better algorithms for recognizing, classifying, and manipulating shapes based on their unique geometric features. As a result, geometric analytics enhances our ability to design, search, and organize complex shapes in innovative ways, leading to advancements in areas like CAD, 3D modeling, and virtual reality.[1]

2 Goal of the project

The goal of this week's project is to use the functional maps framework to obtain shape differences and investigate a collection of shapes through geometric data analysis. By comparing shapes via the inner products of their corresponding function spaces, we aim to understand the variability within the shape collection.
Specifically, our task involves studying geometric variability by considering the data points as geometric objects in the space of symmetric, positive-definite (SPD) matrices. Since this space is a curved manifold where classical multivariate tools cannot be applied, we will use a generalized form of principal component analysis known as principal geodesic analysis (PGA) to analyze this variability. This project is structured around two main objectives: the functional characterization of shape differences and statistical shape analysis. Together, these methods will enhance our understanding of geometric differences and patterns within the shape collection, supporting various applications in fields such as computer graphics, medical imaging, and structural biology.

3 Data and preprocessings

Like in the last project, we worked with the FAUST shape dataset ($https : //nuage.lix.polytechnique.fr/index.php/s/LJFXrsTG22wYCXx/download?path = \%2F\&files = FAUST\_r.zip$). FAUST comes with high-quality correspondences that serve as ground truth. It contains 100 real human scans of 10 different subjects in 10 different poses, acquired with a high-accuracy 3D multi-stereo system. In this report, we have used two poses of the Faust dataset i.e. hand stretching upward and downward. The poses with the human shape with hand downward were labeled source while the shape with hand upward was labeled as target. Furthermore, we used data points, which are geometric objects that live in the space of symmetric, positive-definite (SPD) matrices.
The dataset was already cleaned and thus no further preprocessing was required. It was then loaded in the jupyter notebook or google collab and tasks were done.

4 Methods

4.1 Functional characterization of shape differences

For the functional characterization of shape differences, we used manifold harmonics and functional maps. Each shape's mesh was represented by computing the Laplace-Beltrami operator and deriving its mass and stiffness matrices. Eigenvalues

and eigenvectors from spectral decomposition formed the manifold harmonics.
A ZoomOut algorithm refined initial functional maps by iteratively computing point-to-point maps and applying singular value decomposition (SVD) for orthonormality. Shape differences were captured using an area-based shape difference operator, $D = (H^M)^{-1} F^T H^N F$, where H^M and H^N are mass matrices, and F is the functional map.
The spectral analysis of this operator, through its eigenvalues and eigenvectors, identified regions with significant shape variations. This framework effectively quantifies and visualizes intrinsic shape differences, useful in various fields. [1]

4.2 Principal geodesic analysis (PGA)

Principal Geodesic Analysis (PGA) generalizes Principal Component Analysis (PCA) for data on non-Euclidean spaces, such as Riemannian manifolds. Traditional PCA is inadequate for complex geometric representations like medial representations (m-reps) that lie on these manifolds. In this study, PGA was employed to analyze shape variability using m-reps, where each medial atom, defined by a tuple $m = (x, r, n_0, n_1)$, represents a geometric object in a Riemannian symmetric space $M(1) = \mathbb{R}^3 \times \mathbb{R}^+ \times S^2 \times S^2$. For objects with multiple medial atoms, the space $M(n)$ is the direct product of individual atom spaces. The intrinsic mean µ of a set of points $x_1, x_2, ..., x_N \in M$ is computed as the point minimizing the sum-of-squared Riemannian distances, using a gradient descent algorithm. Principal geodesic directions, which maximize projected variance, are then computed in the tangent space T_M at μ. These directions form the basis for geodesic submanifolds, where data points are projected using a linear approximation in the tangent space.
In a schizophrenia study with hippocampus m-rep models, the PGA method aligned models, and computed their mean and principal geodesics, revealing significant shape variations linked to schizophrenia-related anatomical changes.

5 Results and discussion

First, we investigated the functional map between two shapes from the FAUST dataset using manifold harmonics and Principal Geodesic Analysis (PGA). The analysis focused on computing and visualizing the dominant eigenfunction difference between the two shapes. We loaded two shapes (shape1 and shape2) from the FAUST dataset in .off format and computed manifold harmonics to represent their intrinsic geometry. The functional map (C) was computed using zoomout strategies, aligning the harmonic representations of shape1 to shape2. We also calculated manifold harmonics for both shapes to capture their intrinsic geometry represented by eigenvalues (evals) and eigenvectors (evecs). The functional map C was computed to establish point-to-point correspondences between shape1 and shape2, facilitating comparison based on their geometric features. We derived the area-based shape difference area_diff using the formula $D = (H^M)^{-1} F^T H^N F$, where H^M and H^N are diagonal matrices derived from the harmonic spectra of shape1 and shape2, respectively., and F is the functional map. The eigenvalues and eigenvectors of area_diff were computed to understand the spectrum of shape differences between shape1 and shape2. The most dominant eigenfunction difference was visualized on shape2 (see Figure 1) by interpolating it back to the original point space. This allowed us to observe how shape1 and shape2 differ in their intrinsic geometry based on the most significant eigenfunction.

The plot of the most dominant eigenfunction on shape2 (Figure 1) shows localized variations in the shape that contribute most significantly to the overall shape difference. The areas of highest and lowest values in the plot indicate regions where shape1 and shape2 exhibit the most substantial geometric disparities.

Through the analysis, we quantified and visualized the intrinsic shape differences between two shapes from the FAUST dataset using advanced geometric analysis techniques.

Later we performed Principal Geodesic Analysis (PGA) on a computed area difference matrix (area_diff) from shape data. It utilizes geometric and statistical libraries to analyze and visualize shape differences.

We prepared the data by creating a list containing the computed area difference matrix (area_diff) and converting area_diff to Sparse Positive Definite (SPD) matrices using sparse.csr_matrix(diff @ diff.T). We also initialized a manifold for SPD matrices and Converted SPD matrices to dense arrays (data) for compatibility with JAX. We computed PGA (pga) using the manifold and data prepared. PGA identifies principal geodesics in the data, capturing variations in shape. We created a 3D scatter plot (scatter plot) of PGA coefficients (coeffs) using Pyvista (see Figure 2). Each point in the scatter plot represents a coefficient vector colored by subject and labeled by pose. PGA computes the principal geodesics in the shape data, revealing significant directions of shape variation. The manifold (mfd) and data are crucial inputs for PGA, ensuring the analysis captures the intrinsic geometry of the shapes. The scatterplot displays the first three principal geodesic coefficients. Different subjects are represented by distinct colors, aiding in distinguishing shape variations across subjects. Poses are represented by glyphs (text labels) at each data point, showing pose-specific variations. The distribution and spread of points in the scatter plot reveal the dominant directions of shape variation captured by PGA. For the PCA, data was cleaned by removing the infinite values, and null values were dropped because it caused an error in PCA visualization. Only two components were taken and it was visualized which is as shown in figure 3. The data points were spread inhomogeneously all over the graph in standard PCA visualization and this might be due to the presence of non-linearity in data and PCA is applicable for simple and linear data. It can be concluded that for the complex shape analysis, PGA is more convenient to use as compared to PCA.

6 Contributions

Sujan Darai: Writing Introduction, results and discussion

Matanat Mammadli: Running and implementing Task 1 & 2, Writing Results & Discussion

Samra Hamidovic: Running Task 1 & 2, Writing Goal of the project, Data & preprocessing, Methods

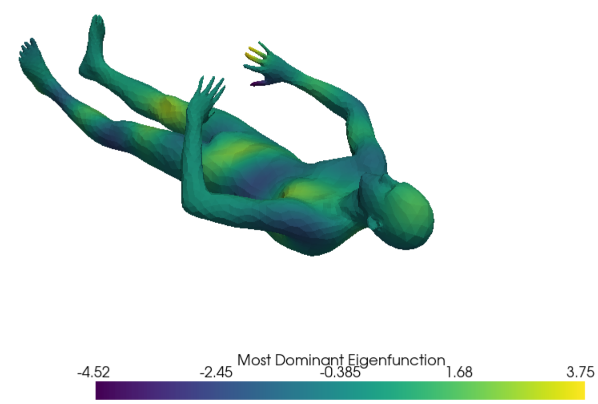

Figure 1: The most dominant eigenfunction on shape2

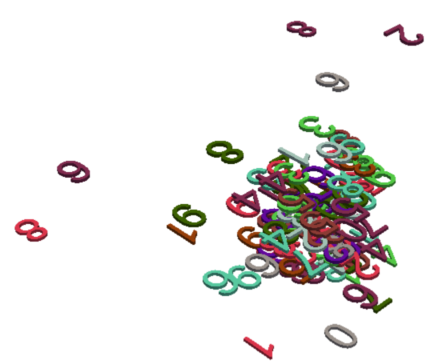

Figure 2: The scatter plot of PGA coefficients

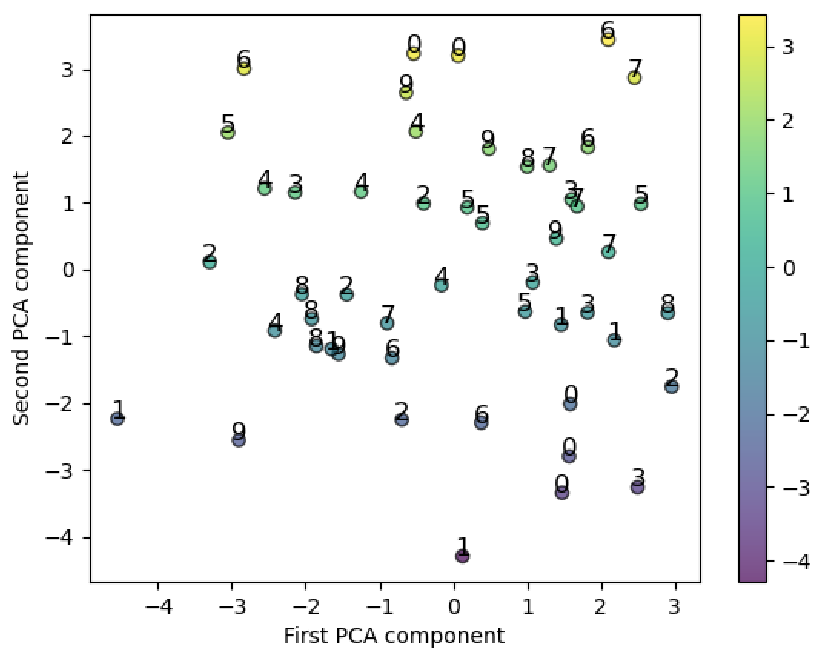

Figure 3: Scatter plot of Standard 2D-PCA components of Faust dataser

7 Appendix

References

1. Raif M. Rustamov, Maks Ovsjanikov, Omri Azencot, Mirela Ben-Chen, Frédéric Chazal, and Leonidas Guibas. Map-based exploration of intrinsic shape differences and variability. *ACM Transactions on Graphics*, 32(4):72:1–72:12, July 2013.