RESEARCH

Project-7

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Abstract

Goal of the project: This project aims to improve point-to-point map recovery between shapes with Manifold Harmonics and Functional maps. By developing a robust framework for accurate correspondence estimation and exploring spectral upsampling for enhanced functional map quality, it advances shape analysis and deformation modeling. This contributes to efficient solutions for shape matching and manipulation in computational geometry and computer graphics, showcasing innovative techniques in non-rigid shape matching.

Main results of the project: Comparative analysis was conducted using the Faust dataset, visualizing two human body poses labeled as source (S) and target (T), which revealed how their manifold harmonics varied in spatial distribution and strength, revealing deformations across different poses. By defining a functional map C, we successfully aligned the spectral embeddings of S and T. The aligned spectral embedding of S closely resembled T, indicating successful orientation alignment despite initial pose disparities. However, using point-to-point shape correspondence and Zoomout algorithms, we found that while S closely matched T in point coverage, T exhibited fewer corresponding points on S, indicating potential geometric mismatches and alignment complexities in mesh comparisons.

Personal key learning:

1 Sujan Darai: learned to use Google Collab, got inside of functional map, manifold harmonics

2 Samra Hamidovic: learned to use Google Collab

3 Matanat Mammadli: learned to use Google Collab, learned about manifold harmonics and functional maps.

Estimated working hours:

1 Sujan Darai: 8 hours

2 Samra Hamidovic: 8 hours

3 Matanat Mammadli: 8 hours

Project evaluation: 1

Number of words: 1963

Keywords: Functional maps, Manifold Harmonics, point-to-point maps, Spectral upsampling

1 Introduction

Shape analysis and processing play a crucial role in various fields such as computer graphics, computer vision, and computational anatomy. One fundamental aspect of shape analysis is the representation of maps between pairs of shapes, which enables efficient inference and manipulation. In this context, the concepts of Basis representation, Manifold Harmonics, Functional Map Representation, and Laplace-Beltrami operators have emerged as key components in shaping the landscape of non-rigid

shape matching and deformation analysis.
The notion of Basis representation, particularly utilizing multi-scale bases such as the eigenfunctions of the Laplace-Beltrami operator, provides a compact and effective way to represent maps between shapes. By leveraging the intrinsic geometry of shapes through Manifold Harmonics, these basis functions offer a powerful tool for capturing shape deformations and preserving important shape characteristics during mapping processes.
Functional Map Representation introduces a novel approach to map shapes by putting in correspondence real-valued functions defined on the shapes, rather than relying solely on point-to-point correspondences. This innovative representation not only simplifies the mapping process but also enables the incorporation of global constraints such as descriptor preservation, landmark correspondences, and part preservation in a linear framework.
Central to the success of Functional Maps is the Laplace-Beltrami operator, which plays a pivotal role in computing functional constraints and capturing the intrinsic geometry of shapes. The eigenfunctions of the Laplace-Beltrami operator serve as a natural basis for functional representations, offering a multi-scale and geometry-aware framework for shape analysis. By exploiting the properties of Laplace-Beltrami eigenfunctions, researchers have developed state-of-the-art algorithms for isometric shape matching and deformation analysis, leading to significant advancements in the field of shape processing.[2]
In this report, we delve into the intricate interplay between Basis representation, Manifold Harmonics, Functional Map Representation, and Laplace-Beltrami operators, exploring their collective impact on shape analysis and deformation modeling.

2 Goal of the project

The primary goal of this project is to leverage advanced techniques such as Manifold Harmonics and Functional maps to enhance the process of recovering point-to-point maps between shapes. By exploring the capabilities of Functional maps and their relationship with Manifold Harmonics, we aim to develop a robust framework for efficiently estimating point-to-point correspondences between shapes.
Furthermore, a key objective of this project is to investigate the application of spectral upsampling techniques in the context of functional map estimation. By utilizing spectral upsampling methods, we seek to improve the accuracy and reliability of functional map estimation processes, ultimately enhancing the quality of recovered point-to-point maps between shapes.
Through the integration of Manifold Harmonics, Functional maps, and spectral upsampling techniques, this project aims to advance the state-of-the-art in shape analysis and deformation modeling. By addressing the challenges associated with recovering point-to-point maps and exploring innovative approaches for functional map estimation, we strive to contribute to the development of efficient and effective solutions for shape matching and manipulation in diverse applications within the field of computational geometry and computer graphics.

3 Data and preprocessings

We worked with the FAUST shape dataset (https://nuage.lix.polytechnique.fr/index .php/s/LJFXrsTG22wYCXx/download?path=%2F&files=FAUST_r.zip). FAUST comes with high-quality correspondences that serve as ground truth. FAUST contains 300 real human scans of 10 different subjects in 30 different poses, acquired with a high-accuracy 3D multi-stereo system. In this report, we have used only two poses of the Faust dataset i.e. hand stretching upward and downward. The poses with the human shape with hand downward were labeled source while the shape with hand upward was labeled as target.

To recover correspondence for remeshed versions of the original data, we went through the steps according to the paper [2]. The dataset was already cleaned and thus no further preprocessing was required. It was then loaded in the jupyter notebook or google collab and tasks were done.

4 Methods

4.1 Shape matching and Functional map

Shape correspondence or shape matching is a major issue in computer graphics, geometric processing, computer vision, and many other areas. It involves matching features between different shapes. Traditional methods use handcrafted features or deformation models, but these often fall short in robustness and generalization. Recent machine learning approaches have improved accuracy but usually require large datasets and still rely on pre-computed descriptors, which can limit their effectiveness[1].

A functional map is an innovative computer vision and graphics method for matching features between different shapes. Instead of matching points, it checks functions defined over the shapes' surfaces. This is done by using a set of simple functions that catch the nature of each shape. The functional map then finds a way to translate these functions from one shape to another shape. This approach is more robust to noise and small changes, offers a more compact representation, and can handle intricate shape deformations. Functional maps are widely used in tasks like shape matching, texture transfer, and shape interpolation, making them versatile and powerful tools for analyzing and comparing shapes[1, 2].

4.2 Manifold harmonics

Manifold Harmonics is a mathematical framework used in shape analysis and geometry processing. It involves decomposing functions defined on a manifold (such as a surface) into a set of basis functions that capture the intrinsic geometry of the shape. These basis functions are derived from the eigenfunctions of the Laplace-Beltrami operator, which encodes important geometric information about the shape. By representing functions on the manifold in terms of these basis functions, Manifold Harmonics provides a compact and efficient way to analyze and manipulate shapes, enabling tasks such as shape matching, correspondence estimation, and deformation modeling.[2]

4.3 Spectral upsampling

Spectral upsampling is a technique used in signal processing and geometry processing to increase the resolution or detail of a signal or a geometric shape. It involves leveraging the spectral domain, particularly the eigenfunctions of operators like the

Laplace-Beltrami operator, to interpolate or extrapolate information in a higher-dimensional space. By utilizing the spectral properties of the data, spectral upsampling can enhance the quality of signals or shapes, enabling more accurate analysis, reconstruction, and manipulation. In the context of shape analysis, spectral upsampling can be applied to improve the resolution of functional maps between shapes, leading to better correspondence estimation and shape-matching results.[2]

5 Results and discussion

At first, we visualized the Faust dataset with two poses of human body scan i.e. hands downward labeled as source (S) and hands upward labeled as a target (T) which are as shown in figures 1, 2 respectively. For both of the poses, five figures in each pose with different colors denote the manifold harmonics i.e. generalization of Fourier basis, i.e. eigenfunction of Laplace (Beltrami) operator (which is an analogue of the classical Laplace equation in the context of curved surfaces. These functions describe the behavior of quantities (e.g., temperature, pressure, or potentials) that satisfy certain boundary conditions on the mesh). Each subplot in the visualization shows the mesh colored according to the scalar values of the corresponding harmonic. By viewing all 5 harmonics side by side, one can compare their spatial distributions and relative strengths, which helps in understanding how different harmonic frequencies contribute to the overall shape and deformation of the mesh.

The colors on the mesh indicate the intensity or amplitude of each harmonic mode. Higher values are typically represented by warmer colors (reds, yellows), while lower values are cooler colors (blues, greens). Differences in these two meshes lead to different distributions of harmonic values. This results in varying color patterns and intensity on the meshes.

After loading meshes and computing multiscale finite difference harmonics, we defined manually functional map C, that is a 4x4 matrix, for alignment. This matrix defines a mapping between the harmonics of mesh S and mesh T. Then we plotted spectral embeddings of source mesh and target mesh using the first three eigenvectors, and also the spectral embedding of mesh S aligned to mesh T using the functional map C (after alignment). The results of this can be seen in Figure 3. For Figure 4, we did those same steps, but additionally, after computing mfd harmonics, we computed spectral representations of the vertex points of meshes S and T using their respective eigenvectors (phi_S and phi_T) and mass matrices (M_S and M_T), and we also computed the functional map C using least-squares regression. In both figures, the colors represent scalar values associated with the mesh points. When we compare the spectral embeddings of the source and target meshes directly, we can see differences in orientation: arms, legs and head are standing in different directions. However, when we compare the original source mesh spectral embedding with its aligned version using C, we see similarities with aligned target (light blue in this case) mesh, when it comes to directions of arms and legs.

On the next task, we used the point-to-point shape correspondence that maps the spectral embeddings of Φ_M C of one shape M for source to the spectral embedding of another shape Φ_N. Φ_M and Φ_N are the eigenfunction matrices (basis functions) representations of source and target respectively. The term C is a linear transformation taking functions on M to functions on N; in matrix notation. For this we

used the NearestNeighbors package from the scikit learn library. In this task value of C was adjusted using the least square technique for the shape correspondence between the source and target which is as shown in figure 5. Also, we implemented the Zoomout algorithms between the source and target for shape correspondence using 4 steps of stepsize 100 and different C depending upon the value of steps which is as shown in figure 6. However, it is surprising that both figures 5 and 6 look similar.
In both figures, the source mesh is colored in light blue with blue points, indicating its vertices, and the target mesh is colored in light green with red points, indicating the corresponding vertices from S. As we can observe, the source mesh S is perfectly covered with blue points, indicating that every point on S has found a corresponding point on T, however, the target mesh T has very few red points compared to the blue points on S. This suggests that not every point on T has a corresponding point on S, indicating potential mismatches or gaps in the correspondence (the reason could be geometry and sampling differences or most likely, alignment issues).

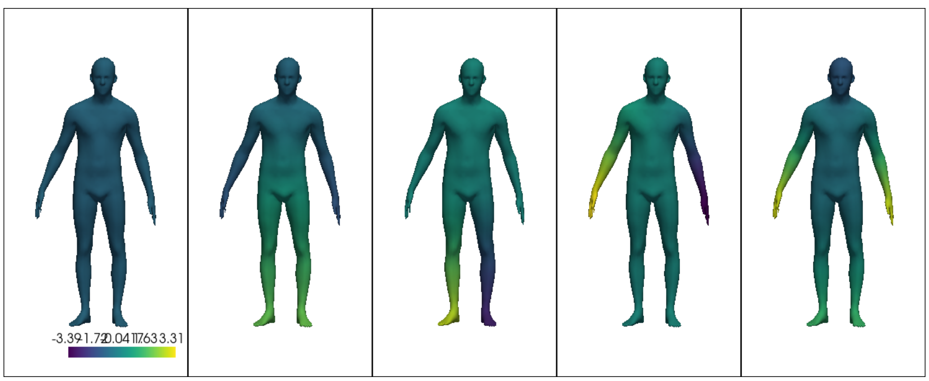

Figure 1: Faust dataset of human body scan with the hands downward (labeled as source 'S')

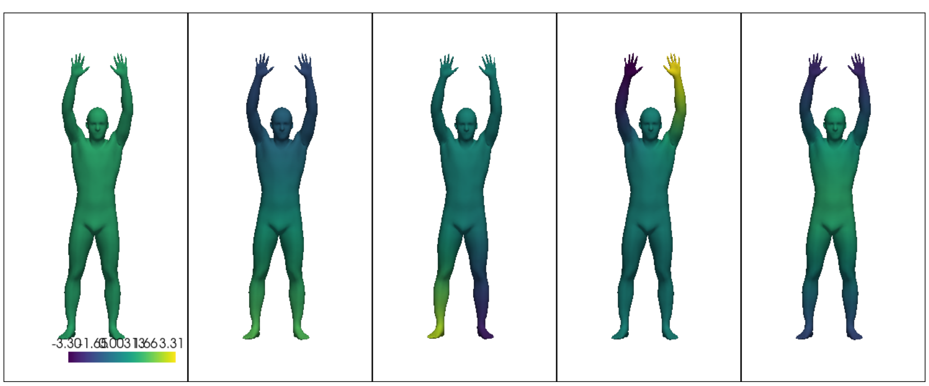

Figure 2: Faust dataset of human body scan with the hands upward (labeled as target 'T')

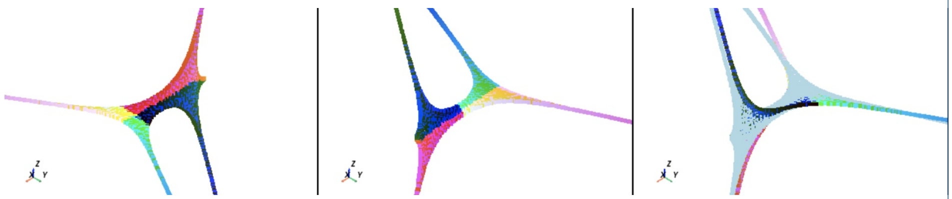

Figure 3: Spectral embedding of source mesh, target mesh and aligned source to target mesh using the first three eigenvectors

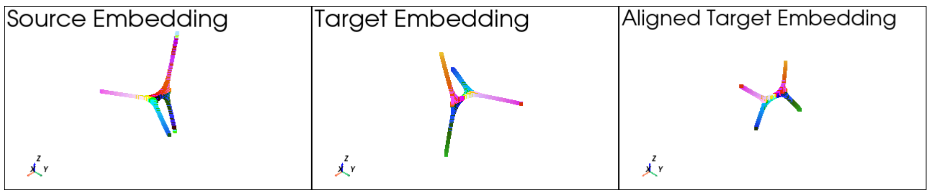

Figure 4: Spectral embedding of source mesh, target mesh and aligned target mesh using functional map C (after least-squares regression)

6 Contributions

Sujan Darai: Running and Implementing Task 1, 2, 3 and 4, Writing Methods, Results & Discussion
Matanat Mammadli: Running and Implementing Task 1, 2, 3 and 4, Writing Abstract, Results & Discussion
Samra Hamidovic: Running and Implementing Task 1, 2, 3 and 4, Writing Abstract, Introduction, Goal of the project, Data & preprocessings, Methods

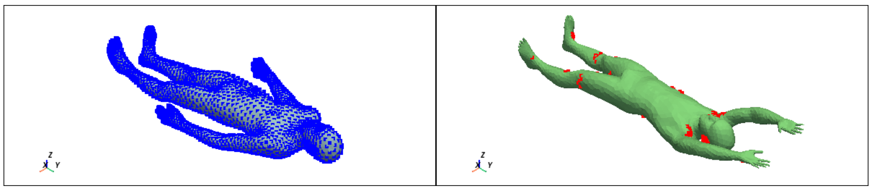

Figure 5: Point-to-point shape correspondence between the source on left and target on right

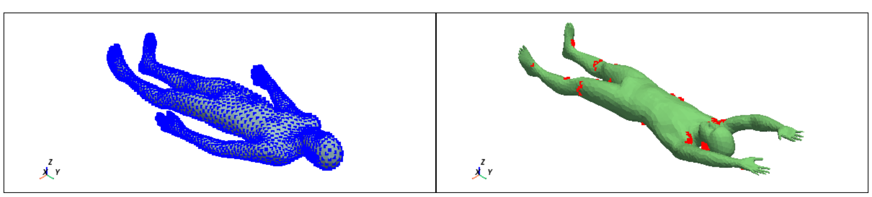

Figure 6: Functional map shape correspondence between the source on left and target on right using the Zoomout algorithms

7 Appendix

References

1. Nicolas Donati, Abhishek Sharma, and Maks Ovsjanikov. Deep geometric functional maps: Robust feature learning for shape correspondence. In *Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition*, pages 8592–8601, 2020.
2. Maks Ovsjanikov, Mirela Ben-Chen, Justin Solomon, Adrian Butscher, and Leonidas Guibas. Functional maps: a flexible representation of maps between shapes. *ACM Transactions on Graphics*, 31(4):30:1–30:11, July 2012.