练 习 十 六

1．求下列函数的傅氏变换。

（1）$\operatorname{sgn} t = \begin{cases} -1 & t < 0 \\ 1 & t > 0 \end{cases}$

解：设$\operatorname{sgn} t = 2u(t) - 1$，则

$$\mathscr{F}[\operatorname{sgn} t] = \mathscr{F}[2u(t) - 1] = 2\mathcal{F}[u(t)] - \mathcal{F}[1]$$

$$= 2(\frac{1}{j\omega} + \pi\delta(\omega)) - 2\pi\delta(\omega) = \frac{2}{j\omega}$$

（2）$f(t) = \cos t \sin t$

解：$\mathscr{F}(\omega) = \int_{-\infty}^{+\infty} f(t) e^{-j\omega t} dt = \int_{-\infty}^{+\infty} \cos t \sin t e^{-j\omega t} dt$

$$= \frac{1}{2}\int_{-\infty}^{+\infty} \sin 2t e^{-j\omega t} dt$$

$$= \frac{\pi}{2} j[(\delta(\omega + 2) - \delta(\omega - 2))]$$

（3）$f(t) = \sin^3 t$

解：因$\sin^3 t = (\dfrac{e^{jt} - e^{-jt}}{2j})^3$

故有$\mathscr{F}[f(t)] = \int_{-\infty}^{+\infty} \sin^3 t e^{-j\omega t} dt = \int_{-\infty}^{+\infty} \dfrac{e^{3jt} - 3e^{jt} + 3e^{-jt} - e^{-j3t}}{-8j} dt$

$$= \frac{\pi}{4} j[\delta(\omega - 3) - 3\delta(\omega - 1) + 3\delta(\omega + 1) - \delta(\omega + 3)]$$

2．已知$F(\omega) = \pi[\delta(\omega + \omega_0) + \delta(\omega - \omega_0)]$为函数$f(t)$的傅氏变换，求$f(t)$。

解：$F(\omega) = \pi[\delta(\omega + \omega_0) + \delta(\omega - \omega_0)] = \dfrac{1}{2}\int_{-\infty}^{\infty} [e^{-j(\omega - \omega_0)} + e^{-jt(c_0 + \omega_0)}] dt$

$$= \frac{1}{2}\int_{-\infty}^{+\infty} e^{-j\omega t}(e^{-j\omega t} + e^{jt\omega_0}) dt = \int_{-\infty}^{+\infty} \cos\omega_0 t e^{-j\omega t} dt = \mathcal{F}[F(t)]$$

所以$f(t) = \cos\omega_0 t$

3．求函数$f(t) = \dfrac{1}{2}[\delta(t + a) + \delta(t - a) + \delta(t + \dfrac{a}{2}) + \delta(t - \dfrac{a}{2})]$的傅氏变换。

解：$F(\omega) = \mathscr{F}[f(t)] = \dfrac{1}{2}\int_{-\infty}^{\infty} [\delta(t + a) + \delta(t - a) + \delta(t + \dfrac{a}{2}) + \delta(t - \dfrac{a}{2})] e^{-j\omega t} dt$

$$=\frac{1}{2}\cdot\left[e^{-j\omega t}\Big|_{t=-a}+e^{-j\omega t}\Big|_{t=a}+e^{-j\omega t}\Big|_{t=-\frac{a}{2}}+e^{-j\omega t}\Big|_{t=\frac{a}{2}}\right]$$

$$=\frac{1}{2}[e^{+j\omega a}+e^{-j\omega a}]+\frac{1}{2}[e^{+j\omega\cdot\frac{a}{2}}+e^{-j\omega\cdot\frac{a}{2}}]$$

$$=\cos a\omega+\cos\frac{a\omega}{2}$$

4．证明：若$\mathscr{F}[e^{j\varphi(t)}]=F(\omega)$, 其中$\varphi(t)$为一实函数，则

$$\mathscr{F}[\cos\varphi(t)]=\frac{1}{2}[F(\omega)+\overline{F(-\omega)}]$$

$$\mathscr{F}[\sin\varphi(t)]=\frac{1}{2j}[F(\omega)-\overline{F(-\omega)}]$$

其中$\overline{F(-\omega)}$为$F(\omega)$的共轭函数。

证明：$\mathscr{F}[e^{j\varphi(t)}]=F(\omega)=\int_{-\infty}^{+\infty}e^{j\varphi(t)}e^{-j\omega t}dt$

$$=\int_{-\infty}^{+\infty}[\cos\varphi(t)+j\sin\varphi(t)]e^{-j\omega t}dt$$

$$\overline{F(\omega)}=\int_{-\infty}^{+\infty}[(\cos\varphi(t)-j\sin\varphi(t))]e^{j\omega t}dt$$

$$\overline{F(-\omega)}=\int_{-\infty}^{+\infty}[(\cos\varphi(t)-j\sin\varphi(t))]e^{-j\omega t}dt$$

所以$\frac{1}{2}[F(\omega)+\overline{F(\omega)}]=\int_{-\infty}^{+\infty}\cos\varphi(t)e^{-j\omega t}dt=\mathscr{F}[\cos\varphi(t)]$

同理：$\frac{1}{2j}[F(\omega)+\overline{F(\omega)}]=\mathscr{F}[\sin\varphi(t)]$