练　习　十　七

1．设 $f_1(t)=\begin{cases}0, & t<0\\ 1, & t\geq 0\end{cases}$，$f_2(t)=\begin{cases}0, & t<0\\ e^{-t}, & t\geq 0\end{cases}$ 求 $f_1(t)*f_2(t)$。

解：$f_1(t)*f_2(t)=\int_{-\infty}^{+\infty}f_1(\tau)f_2(t-\tau)d\tau=\int_0^t e^{\tau-t}d\tau=1-e^{-t}$

$\therefore f_1(t)*f_2(t)=\begin{cases}0, & t<0\\ 1-e^{-t}, & t\geq 0\end{cases}$

2．求下列函数的傅氏变换。

（1）$f(t)=\sin\omega_0 t\cdot u(t)$

解：$\mathscr{F}[f(t)]=\dfrac{1}{2\pi}\mathscr{F}[\sin\omega_0 t]*\mathscr{F}[u(t)]$

$$=\frac{1}{2\pi}\int_{-\infty}^{+\infty}j\pi[\delta(\tau+\omega_0)-\delta(\tau-\omega_0)]\cdot(\frac{1}{j(u-\tau)}+\pi\delta(u-\tau))d\tau$$

$$=\frac{\omega_0}{\omega_0^2-\omega^2}-\frac{\pi}{2j}[\delta(\omega-\omega_0)-\delta(\omega+\omega_0)]$$

（2）$f(t)=e^{j\omega_0 t}\cdot t\cdot u(t)$

解：$\mathscr{F}[te^{j\omega_0 t}]=2\pi j\delta'(\omega-\omega_0)=F_1(\omega)$

$\mathcal{F}[u(t)]=\dfrac{1}{j\omega}+\pi\delta(\omega)=F_2(\omega)$

$\therefore\mathcal{F}[f(t)]=\dfrac{1}{2\pi}\mathcal{F}[(te^{j\omega_0 t})\,\mathcal{F}(u(t))=\dfrac{1}{2\pi}\int_{-\infty}^{\infty}F_1(\tau)F_1(\omega-\tau)d\tau$

$$=\frac{-1}{(\omega-\omega_0)^2}+\pi j\delta'(\omega-\omega_0)$$

3．证明：

$a[f_1(t)*f_2(t)]=[af_1(t)]*F_2(t)$　　　　（a 为常数）

证：$a\int_{-\infty}^{+\infty}f_1(\tau)f_2(t-\tau)d\tau=\int_{-\infty}^{+\infty}af_1(\tau)f_2(t-\tau)d\tau=(af_1(t))*f_2(t)$

4．若 $F_1(\omega)=\mathscr{F}[f_1\cdot(t)]$，$F_1(\omega)=\mathscr{F}[f_2(t)]$，证明：

$\mathscr{F}[f_1(t)\cdot f_2(t)]=\dfrac{1}{2\pi}F_1(\omega)*F_2(\omega)$

证： $\frac{1}{2\pi}F_1(\omega)*F_2(\omega)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}(\int_{-\infty}^{+\infty}f_1(t)e^{-j\tau t}dt)\cdot F_2(\omega-\tau)d\tau$

$$=\frac{1}{2\pi}\int_{-\infty}^{+\infty}f_1(t)(\int_{-\infty}^{+\infty}F_2(\omega-\tau)e^{-j\tau t}d\tau)dt$$

$$=\frac{1}{2\pi}\int_{-\infty}^{+\infty}f_1(t)[\int_{-\infty}^{+\infty}F_2(\omega-\tau)e^{-j(\omega-\tau)t}d(\omega-\tau)]e^{-j\omega t}dt$$

$$=\int_{-\infty}^{+\infty}f_1(t)f_1(t)e^{-j\omega t}dt$$

$$=\mathscr{F}[f_1(t)\cdot f_2(t)]$$