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Group Number: 5

NAMES (FIRST AND LAST NAME):

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In-Class Assignment 1

ELEN 21/COEN 21 – Fall 2022
Instructor: Maria Kyrarini
Date: 10/4/2022
Time: 1 hour and 20 minutes
Number of Problems: 3

Important Notes:

- Be sure to read all the problems carefully and answer all questions.
- Be sure to answer all parts of each question.
- **Submit only one answer for each question.** Multiple solutions for one question will NOT be graded.
- **Clearly show all the steps of your work.**
- **Answers without detailed explanations will NOT be graded.**
- The Engineering School Honor Code applies.

Problem 1 (30 points)

Implement the following function using only 2-to-1 multiplexers and NOT gates:

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$$f = \overline{w}_2 w_3 + \overline{w}_1 w_2 \overline{w}_3 + w_2 \overline{w}_3 w_4 + w_1 \overline{w}_2 \overline{w}_4$$

Note: AND, NAND, OR, NOR, XOR gates are not available.

Problem 2 (35 points)

For the logic function $f(w,x,y,z)=\prod M(1,7,12,13,14,15) \cdot d(0,2,8,10)$ (**Note:** the d represents the don't cares):

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a) Show the truth table. [5 points]
b) Draw a completely labeled K-map. [5 points]
c) Write the algebraic expression for the minimized Sum of Products (SoP) and minimized Product of Sum (PoS) implementation of this function using the K-map. **Note:** Show the circles on the K-map. [10 points]
d) Write a Verilog program using structural code that implements the SoP. [5 points]

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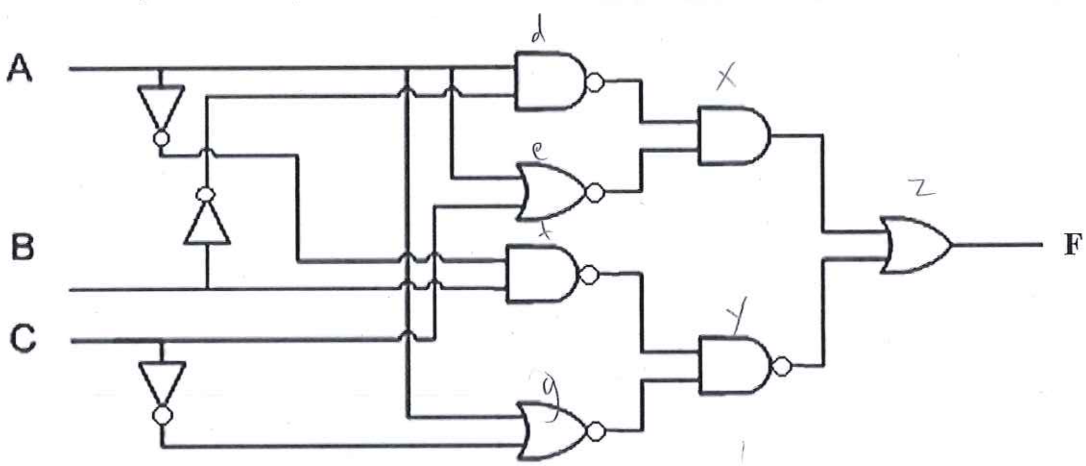
Problem 3 (35 points)

Simplify the following circuit by using Boolean Algebra properties. Assume that you have **only** 2-input and 3-input NAND gates. It is known that $\overline{x+y+z} = \bar{x} \cdot \bar{y} \cdot \bar{z}$. Draw the simplified circuit.

Note: NOT, AND, OR, NOR, XOR gates are not available.

Note: Simplification by K-maps will NOT be graded.



Boolean Algebra Properties

5*a*. $x \cdot 0 = 0$

5*b*. $x + 1 = 1$

6*a*. $x \cdot 1 = x$

6*b*. $x + 0 = x$

7*a*. $x \cdot x = x$

7*b*. $x + x = x$

8*a*. $x \cdot \overline{x} = 0$

8*b*. $x + \overline{x} = 1$

9. $\overline{\overline{x}} = x$

10*a*.	$x \cdot y = y \cdot x$	*Commutative*
10*b*.	$x + y = y + x$	
11*a*.	$x \cdot (y \cdot z) = (x \cdot y) \cdot z$	*Associative*
11*b*.	$x + (y + z) = (x + y) + z$	
12*a*.	$x \cdot (y + z) = x \cdot y + x \cdot z$	*Distributive*
12*b*.	$x + y \cdot z = (x + y) \cdot (x + z)$	
13*a*.	$x + x \cdot y = x$	*Absorption*
13*b*.	$x \cdot (x + y) = x$	
14*a*.	$x \cdot y + x \cdot \overline{y} = x$	*Combining*
14*b*.	$(x + y) \cdot (x + \overline{y}) = x$	
15*a*.	$\overline{x \cdot y} = \overline{x} + \overline{y}$	*DeMorgan's theorem*
15*b*.	$\overline{x + y} = \overline{x} \cdot \overline{y}$	
16*a*.	$x + \overline{x} \cdot y = x + y$	
16*b*.	$x \cdot (\overline{x} + y) = x \cdot y$	
17*a*.	$x \cdot y + y \cdot z + \overline{x} \cdot z = x \cdot y + \overline{x} \cdot z$	*Consensus*
17*b*.	$(x + y) \cdot (y + z) \cdot (\overline{x} + z) = (x + y) \cdot (\overline{x} + z)$	

Problem 1

$f = \bar{w}_2 w_3 + \bar{w}_1 w_2 \bar{w}_3 + w_2 \bar{w}_3 w_4 + w_1 \bar{w}_2 \bar{w}_4$

$\bar{w}_2(w_3 + w_1 \bar{w}_4) + w_2(\bar{w}_1 \bar{w}_3 + \bar{w}_3 w_4)$

$\bar{w}_2(w_3 + w_1 \bar{w}_4) + w_2(\bar{w}_3(\bar{w}_1 + w_4))$

$\bar{w}_2(w_3 + w_1 \bar{w}_4) + w_2(\bar{w}_3(\bar{w}_1 + w_4) + w_3(0))$

$\bar{w}_2(w_3 + w_1 \bar{w}_4) + w_2(\bar{w}_3(\bar{w}_1(1) + w_1(w_4)) + w_3(0))$

$\bar{w}_2(w_3(1) + \bar{w}_3(w_1 \bar{w}_4)) + w_2(\bar{w}_3(\bar{w}_1(1) + w_1(w_4)) + w_3(0))$

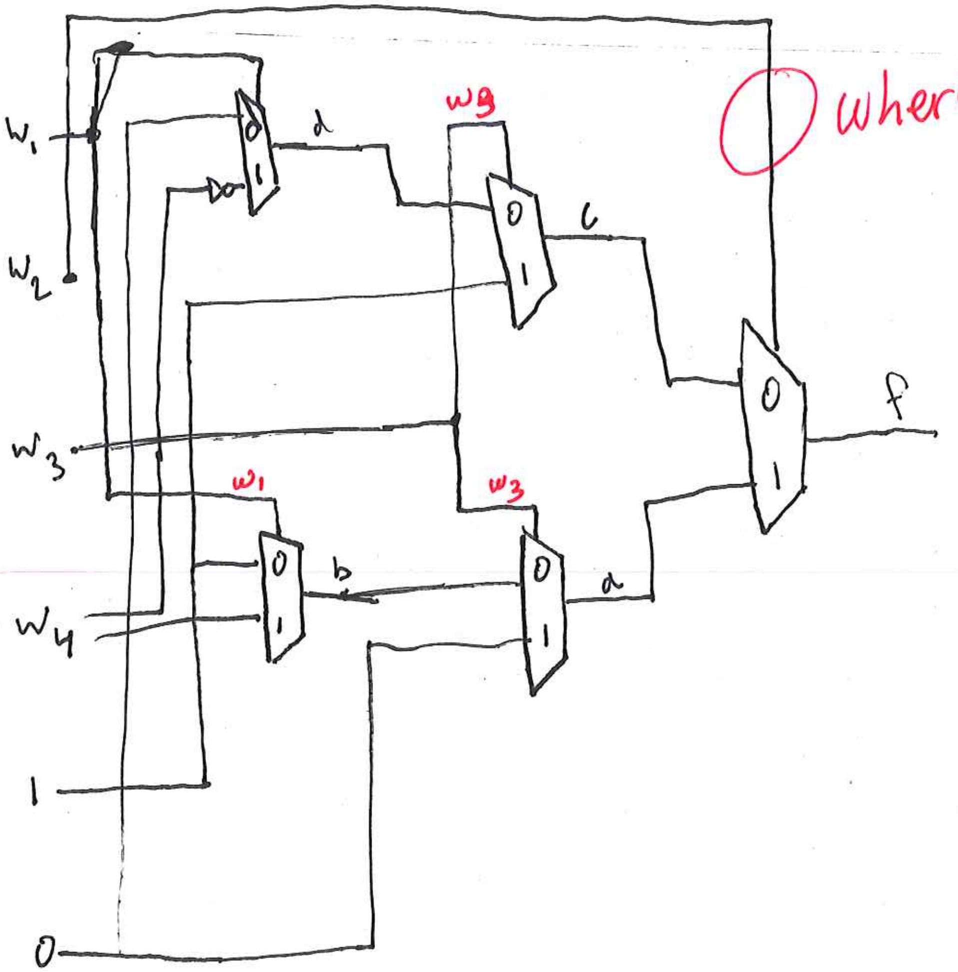
$\bar{w}_2(w_3(1) + \bar{w}_3(\bar{w}_1(0) + w_1(\bar{w}_4))) + w_2(\bar{w}_3(\bar{w}_1(1) + w_1(w_4)) + w_3(0))$

$a = \bar{w}_3(\bar{w}_1 + w_4)$
if $\bar{w}_3 = 0$; $a = \bar{w}_1 + w_4$
if $w_3 = 1$; $a = 0$

$b = \bar{w}_1 + w_4$
if $\bar{w}_1 = 0$; $b = 1$
if $w_1 = 1$; $b = w_4$

$c = w_3 + w_1 \bar{w}_4$
if $\bar{w}_3 = 0$; $c = w_1 \bar{w}_4$
if $w_3 = 1$; $c = 1$

$d = w_1 \bar{w}_4$
if $\bar{w}_1 = 0$; $d = 0$
if $w_1 = 1$; $d = \bar{w}_4$



① where is it connected? w_2 or w_1?

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complicated explanation

Problem 2

$$f(w, x, y, z) = \prod M(1, 7, 12, 13, 14, 15) \cdot d(0, 2, 8, 10)$$

a.

w	x	y	z	f	
0	0	0	0	d	0
0	0	0	1	0	1
0	0	1	0	d	2
0	0	1	1	1	3
0	1	0	0	1	4
0	1	0	1	1	5
0	1	1	0	1	6
0	1	1	1	0	7
1	0	0	0	d	8
1	0	0	1	1	9
1	0	1	0	d	10
1	0	1	1	1	11
1	1	0	0	0	12
1	1	0	1	0	13
1	1	1	0	0	14
1	1	1	1	0	15

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b. 5

yz \ wx	00	01	11	10
00	d	1	0	d
01	0	1	0	1
11	1	0	0	1
10	d	1	0	d

Problem 2 h i j k

c. SOP: $f = w \cdot \bar{x} + \bar{x} \cdot y + \bar{w} \cdot \bar{z} + \bar{w} \cdot x \cdot \bar{y}$ 5

POS: $f = \cancel{\bar{w} \cdot \bar{x} \cdot \bar{y} + x \cdot y \cdot z + w \cdot x}$

$= (w + x + y) \cdot (\bar{x} + \bar{y} + \bar{z}) \cdot (\bar{w} + \bar{x})$

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3 $\bar{y}\cdot x\cdot\bar{w} + \bar{z}\cdot\bar{w} + y\cdot\bar{x} + \bar{x}\cdot w$

$\cancel{x\cdot w + z\cdot y\cdot x + \bar{y}\cdot\bar{x}\cdot\bar{w}}$

$(\bar{x} + \bar{w})\cdot(\bar{z} + \bar{y} + \bar{x})\cdot(y + x + w)$

2

d. SOP structural Verilog:

```
module problem2d (w, x, y, z, f);
    input w, x, y, z;
    output f;

    not (a, w);        // a = w̄
    not (b, x);        // b = x̄
    not (c, y);        // c = ȳ
    not (d, z);        // d = z̄

    and (h, w, b);
    and (i, b, y);
    and (j, a, d);
    and (k, a, x, c);
    or (f, h, i, j, k);
endmodule
```

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+10

Problem 3:

$$F = \left(\overline{A\overline{B}} \cdot \overline{A+C}\right) + \overline{\overline{\overline{A}B} \cdot \overline{A+\overline{C}}}$$

Properties used:

$= \overline{\overline{A\overline{B}} + (A+C)} + \overline{\overline{\overline{A}B + A + \overline{C}}}$	DeMorgan's Theorem: $\overline{X} \cdot \overline{Y} = \overline{X+Y}$ Note: $X \rightarrow A\overline{B}$ $Y \rightarrow A+C$
$= \overline{A\overline{B} + A + C} + \overline{A}B + A + \overline{C}$	(1.) Associative property (2.) $\overline{\overline{X}} = X$
$\downarrow \overline{(A\overline{B})} \cdot \overline{A} \cdot \overline{C} = (\overline{A}+B) \cdot \overline{A} \cdot \overline{C} \leftarrow$ This is the correct This is wrong ??? $= \overline{A} \cdot \overline{\overline{B}} \cdot \overline{A} \cdot \overline{C} + \overline{A}B + A + \overline{C}$	DeMorgan's Theorem: $\overline{X+Y} = \overline{X} \cdot \overline{Y}$ $\overline{X+Y+Z} = \overline{X} \cdot \overline{Y} \cdot Z$
$= (\overline{A} + B) \cdot \overline{A} \cdot \overline{C} + \overline{A}B + A + \overline{C}$	$\overline{\overline{X}} = X$
$= \overline{C} + (\overline{A} + B)\overline{A} \cdot \overline{C} + \overline{A}B + A$	Commutative property
$= \overline{C}\left(1 + (\overline{A} + B) \cdot \overline{A}\right) + \overline{A}B + A$	Factoring/ Distributive Property
$= \overline{C} \cdot 1 + \overline{A}B + A$	Disable Property: $X + 1 = 1$
$= \overline{C} + \overline{A}B + A$	Enable Property: $X \cdot 1 = X$
$= \overline{C} + A + \overline{A}B$	Commutative Property
$= \overline{C} + A + B$	$X + \overline{X} \cdot Y = X + Y$
$= A + B + \overline{C}$	Commutative Property
$= \overline{\overline{A + B + \overline{C}}}$	$X = \overline{\overline{X}}$
$= \overline{\overline{A} \cdot \overline{B} \cdot C}$	(1.) DeMorgan Theorem: $\overline{X+Y+Z} = \overline{X} \cdot \overline{Y} \cdot \overline{Z}$ (2.) $\overline{\overline{C}} = C$

Continued for problem 3:

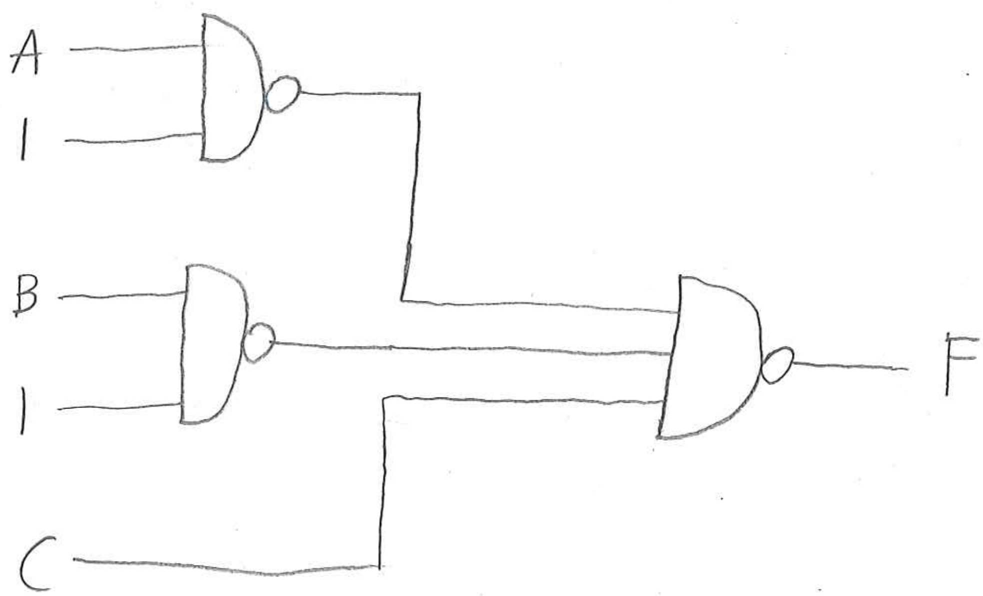
$$F = \overline{\bar{A} \cdot \bar{B} \cdot C}$$

$$= \overline{\overline{A \cdot 1} \cdot \overline{B \cdot 1} \cdot C}$$

Reverse Enable property:

$X = X \cdot 1$

Simplified Circuit:



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