Section 1.1 HW

1. $X_1 + 5X_2 = 7$
$-2X_1 - 7X_2 = -5$

$$\left(\begin{array}{cc|c} 1 & 5 & 7 \\ -2 & -7 & -5 \end{array}\right)$$

$2R_1 + R_2 \to R_2$

$$\left(\begin{array}{cc|c} 1 & 5 & 7 \\ 0 & 3 & 9 \end{array}\right)$$

$\frac{1}{3}R_2 \to R_2$

$$\left(\begin{array}{cc|c} 1 & 5 & 7 \\ 0 & 1 & 3 \end{array}\right)$$

$R_1 - 5R_2 \to R_1$

$$\left(\begin{array}{cc|c} 1 & 0 & 7 \\ 0 & 1 & -8 \end{array}\right)$$

$X_1 = 7$
$X_2 = -8$ //

5. $$\left(\begin{array}{cccc|c} 1 & -4 & 5 & 0 & 7 \\ 0 & 1 & -3 & 0 & 6 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & -5 \end{array}\right)$$

$R_1 + 4R_2 \to R_1$

$$\left(\begin{array}{cccc|c} 1 & 0 & -7 & 0 & 31 \\ 0 & 1 & -3 & 0 & 6 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & -5 \end{array}\right)$$

$R_2 + 3R_3 \to R_2$

$$\left(\begin{array}{cccc|c} 1 & 0 & -7 & 0 & 31 \\ 0 & 1 & 0 & 0 & 12 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & -5 \end{array}\right)$$

Step #1:
$R_1 + 4R_2 \rightarrow R_1$:
Add Row 1 to 4 times Row 3

Step #2:
$R_2 + 3R_3 \rightarrow R_2$:
Add Row 2 to 3 times Row 3 //

9.
$$\left(\begin{array}{cccc|c} 1 & -1 & 0 & 0 & -4 \\ 0 & 1 & -3 & 0 & -7 \\ 0 & 0 & 1 & -3 & -1 \\ 0 & 0 & 0 & 0 & 4 \end{array}\right) \rightarrow 0 \neq 4$$

$$0x_1 + 0x_2 + 0x_3 + 0x_4 = 4$$
$$0 \neq 4$$

This system has no solution. //

11.
$$x_2 + 4x_3 = 4$$
$$x_1 + 3x_2 + 3x_3 = -2$$
$$3x_1 + 7x_2 + 5x_3 = 6$$

$$\left(\begin{array}{ccc|c} 0 & 1 & 4 & 4 \\ 1 & 3 & 3 & -2 \\ 3 & 7 & 5 & 6 \end{array}\right)$$

$R_3 - 3R_2 \rightarrow R_3$

$$\left(\begin{array}{ccc|c} 0 & 1 & 4 & 4 \\ 1 & 3 & 3 & -2 \\ 0 & -2 & -4 & 12 \end{array}\right)$$

$R_2 - 3R_1 \rightarrow R_2$

$$\left(\begin{array}{ccc|c} 0 & 1 & 4 & 4 \\ 1 & 0 & -9 & -14 \\ 0 & -2 & -4 & 12 \end{array}\right)$$

$R_1 \leftrightarrow R_2$

$$\left(\begin{array}{ccc|c} 1 & 0 & -9 & -14 \\ 0 & 1 & 4 & 4 \\ 0 & -2 & -4 & 12 \end{array}\right)$$

$R_2 + R_3 \longrightarrow R_2$

$$\left(\begin{array}{ccc|c} 1 & 0 & -9 & -14 \\ 0 & -1 & 0 & 16 \\ 0 & -2 & -4 & 12 \end{array}\right)$$

$-R_2 \longrightarrow R_2$

$$\left(\begin{array}{ccc|c} 1 & 0 & -9 & -14 \\ 0 & 1 & 0 & -16 \\ 0 & -2 & -4 & 12 \end{array}\right)$$

$2R_2 + R_3 \longrightarrow R_3$

$$\left(\begin{array}{ccc|c} 1 & 0 & -9 & -14 \\ 0 & 1 & 0 & -16 \\ 0 & 0 & -4 & -20 \end{array}\right)$$

$R_1 - \frac{9}{4} R_3 \longrightarrow R_1$

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 31 \\ 0 & 1 & 0 & -16 \\ 0 & 0 & -4 & -20 \end{array}\right)$$

$-\frac{1}{4} R_3 \longrightarrow R_3$

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 31 \\ 0 & 1 & 0 & -16 \\ 0 & 0 & 1 & 5 \end{array}\right)$$

$X_1 = 31$
$X_2 = -16$
$X_3 = 5$

2.

$$\left(\begin{array}{cc|c} 1 & -4 & 1 \\ 2 & -1 & -3 \\ -1 & -3 & 4 \end{array}\right)$$

$R_1 + R_3 \to R_3$

$$\left(\begin{array}{cc|c} 1 & -4 & 1 \\ 2 & -1 & -3 \\ 0 & -7 & 5 \end{array}\right)$$

$R_2 - 2R_1 \to R_2$

$$\left(\begin{array}{cc|c} 1 & -4 & 1 \\ 0 & 7 & -5 \\ 0 & -7 & 5 \end{array}\right)$$

$R_2 + R_3 \to R_3$

$$\left(\begin{array}{cc|c} 1 & -4 & 1 \\ 0 & 7 & -5 \\ 0 & 0 & 0 \end{array}\right)$$

$R_1 + \frac{4}{7}R_2 \to R_1$

$$\left(\begin{array}{cc|c} 1 & 0 & -\frac{13}{7} \\ 0 & 7 & -5 \\ 0 & 0 & 0 \end{array}\right)$$

$\frac{1}{7}R_2 \to R_2$

$$\left(\begin{array}{cc|c} 1 & 0 & -\frac{13}{7} \\ 0 & 1 & -\frac{5}{7} \\ 0 & 0 & 0 \end{array}\right)$$

They have a common point of intersection:

$$\left(-\frac{13}{7}, -\frac{5}{7}\right)$$

23.

$$\left(\begin{array}{cc|c} 1 & h & 4 \\ 3 & 6 & 8 \end{array}\right) \xrightarrow{3R_1 - R_2 \to R_2} \left(\begin{array}{cc|c} 1 & h & 4 \\ 0 & 3h-6 & 4 \end{array}\right)$$

Matrix like this below is inconsistent:

$$\left(\begin{array}{cc|c} 1 & h & 4 \\ 0 & 0 & 4 \end{array}\right)$$

$3h - 6 = 0$

$3h = 6$

$h = 2 \rightarrow$ inconsistent

To be consistent: $\boxed{h \neq 2}$

27. True.
For instance, addition, multiplication, and row switch are reversible.

29. False.
Matrix dimensions: $m \times n$
rows columns

33. True.
Questions: (1.) Do solutions exist?
(2.) Are solutions unique if they exist?

34. True.
Equivalent: Have same solution set.

39. 1st into 2nd: Swap Row 1 and Row 2

2nd into 1st: Swap Row 1 and Row 2