

Homework #31. a. $|011100111|$ Left-most bit: 1 \rightarrow negativeFlip: $|011100111| \rightarrow 0|00011000| + 1 = 0|0001100|$

$$0 \cdot 2^9 + 1 \cdot 2^8 + 0 \cdot 2^7 + 0 \cdot 2^6 + 0 \cdot 2^5 + 1 \cdot 2^4 + 1 \cdot 2^3 + 0 \cdot 2^2 + 0 \cdot 2^1 + 1 \cdot 2^0$$

$$= (281)_{10}$$

Due to negative: $(-281)_{10}$ b. $|111111110|$ Left-most bit: 1 \rightarrow negativeFlip: $|111111110| \rightarrow 000000000| + 1 = 00000000|0$

$$0 \cdot 2^9 + 0 \cdot 2^8 + 0 \cdot 2^7 + 0 \cdot 2^6 + 0 \cdot 2^5 + 0 \cdot 2^4 + 0 \cdot 2^3 + 0 \cdot 2^2 + 1 \cdot 2^1 + 0 \cdot 2^0 = 2$$

(cont.)

1. b. Due to negative: $(-2)_{10}$

c. 011101110

Left-most bit: positive \rightarrow Flipping is NOT needed

$$0 \cdot 2^9 + 1 \cdot 2^8 + 1 \cdot 2^7 + 1 \cdot 2^6 + 0 \cdot 2^5 + 1 \cdot 2^4 \\ + 1 \cdot 2^3 + 1 \cdot 2^2 + 1 \cdot 2^1 + 0 \cdot 2^0$$

$$= \underline{(478)_{10}}$$

2. Left Part:

$$X \oplus (Y \oplus Z) = \overline{X} (Y \oplus Z) + X \overline{(Y \oplus Z)}$$

$$= \overline{X} (\overline{Y} Z + Y \overline{Z}) + X (\overline{\overline{Y} Z + Y \overline{Z}})$$

$$= \overline{X} (\overline{Y} Z + Y \overline{Z}) + X \overline{\overline{Y} Z} \overline{Y \overline{Z}}$$

$$= \overline{X} (\overline{Y} Z + Y \overline{Z}) + X (\overline{\overline{Y}} + \overline{Z}) \overline{Y \cdot Z}$$

$$= \overline{X} (\overline{Y} Z + Y \overline{Z}) + X (Y + \overline{Z}) (\overline{Y} + \overline{\overline{Z}})$$

$$= \overline{X} (\overline{Y} Z + Y \overline{Z}) + X (Y + \overline{Z}) (\overline{Y} + Z)$$

$$= X Y Z + X \overline{Y} \overline{Z} + X Z \overline{\overline{Z}} + X \overline{Y} \overline{Z} \\ + \overline{X} Y \overline{Z} + \overline{X} \overline{Y} Z$$

$$= X Y Z + X \overline{Y} \overline{Z} + \overline{X} Y \overline{Z} + \overline{X} \overline{Y} Z$$

(cont.)

2.

Right Part:

$$(X \oplus Y) \oplus Z$$

$$= \overline{(X \oplus Y)} Z + (X \oplus Y) \overline{Z}$$

$$= (\overline{XY} + X\overline{Y}) Z + (\overline{XY} + X\overline{Y}) \overline{Z}$$

$$= (\overline{XY})(\overline{X\overline{Y}}) Z + (\overline{XY} + X\overline{Y}) \overline{Z}$$

$$= (\overline{X} + \overline{Y})(\overline{X} + \overline{Y}) Z + (\overline{XY} + X\overline{Y}) \overline{Z}$$

$$= (X + \overline{Y})(\overline{X} + Y) Z + (\overline{XY} + X\overline{Y}) \overline{Z}$$

$$= XY Z + X\overline{X} Z + X\overline{Y} Z + Y\overline{Y} Z + \overline{X} Y \overline{Z} + \overline{X} \overline{Y} \overline{Z}$$

$$= XY Z + X\overline{Y} Z + \overline{X} Y \overline{Z} + \overline{X} \overline{Y} \overline{Z} \Rightarrow \text{(Proven associative)}$$

3.

$$\begin{array}{r} 00110110 \\ + 000001 \\ \hline 0111011 \end{array}$$

$$\begin{array}{r} (+54) \\ + (+69) \\ \hline 123 \end{array}$$

$$\begin{array}{r} 111111 \\ 01110101 \\ + 1101110 \\ \hline 101010011 \end{array}$$

$$\begin{array}{r} (+117) \\ + (-34) \\ \hline 83 \end{array}$$

↓
ignore

$$\begin{array}{r} 111111 \\ 11011111 \\ + 10111000 \\ \hline 110010111 \end{array}$$

$$\begin{array}{r} (-33) \\ + (-72) \\ \hline -105 \end{array}$$

↓
ignore