Electric Circuits I

Laboratory 6 – Storage Devices and *RC* Circuits

Objective:

- Learn the meaning of the time constant of a simple *s* circuit
- Measure the output of the same circuit in two different configurations.

I. Equipment:

- DC Power Supply (Keysight EDU36311A)
- Digital Multimeter (Keysight EDU34450A)
- Breadboard for connecting resistors

Waveform Generator (Keysight EDU33221A)	Digital Oscilloscope (Keysight EDUX1052G)
	

- Resistance Decade *R*.
- 5 capacitors in [1µF, 10µF].
- Three resistors (R_1, R_2, and R_3) are between 1kΩ and 3kΩ.

II. Background & Theory

A. Capacitor and Inductor:

Capacitor C and *inductor L* are nonlinear storage devices commonly used in analog and digital circuits. The voltage, current, and capacitance or inductance in a storage device are related, as shown in **Figure 1**. Capacitors and inductors have various sizes and shapes depending on the applications of circuits.

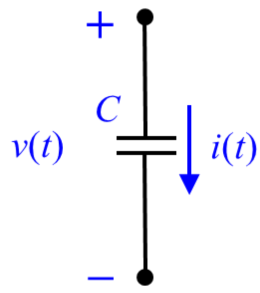
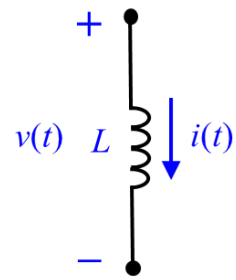
 $q(t) = Cv(t)$ $i(t) = C\dfrac{dv(t)}{dt}$ $w_C(t) = \dfrac{1}{2}Cv(t)^2$	 $v(t) = L\dfrac{di(t)}{dt}$ $w_L(t) = \dfrac{1}{2}Li(t)^2$

Figure 1 – Storage Element Capacitance *C* and inductance *L*

Equivalent capacitance and inductance of series and parallel connections are shown in **Figure 2**.

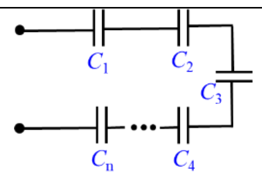
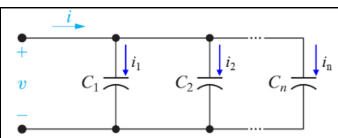
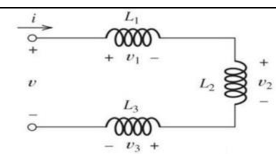
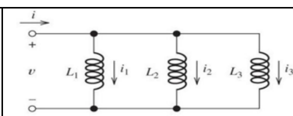
 $\dfrac{1}{C_{eq}} = \dfrac{1}{C_1} + \dfrac{1}{C_2} + \cdots + \dfrac{1}{C_n}$	 $C_{eq} = C_1 + C_2 + C_3 + \cdots C_n$	 $L_{eq} = L_1 + L_2 + \cdots + L_n$	 $\dfrac{1}{L_{eq}} = \dfrac{1}{L_1} + \dfrac{1}{L_2} + \cdots + \dfrac{1}{L_n}$

Figure 2 – Equivalent Capacitance and Inductance

Similar to the voltage divider of a resistive circuit, capacitors in series can divide the voltage of an input source according to their capacitance, as shown in **Figure 3**.

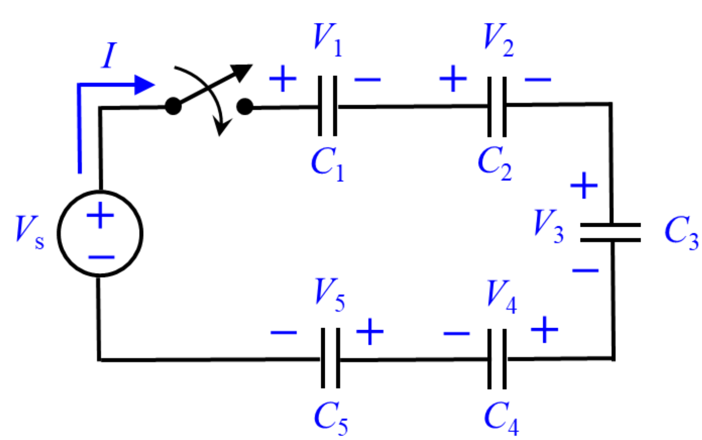


Figure 3 – Voltages across Capacitors in Series

Initially, all capacitors are empty. Since the capacitors are in series, they will experience the same current I from the voltage source V_s when the switch is closed. The capacitors will store the same amount of charge q until they are fully charged and become open circuits. This amount of charge is also the same if we replace all the capacitors with an equivalent capacitor C_{eq}. From this observation, we have:

$$q = C_{eq}V_s = C_1V_1 = C_2V_2 = C_3V_3 = C_4V_4 = C_5V_5 \qquad (1),$$

where

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \frac{1}{C_4} + \frac{1}{C_5}$$

From **Eq. 1**, the voltages across capacitors are:

$V_1 = \left(\frac{C_{eq}}{C_1}\right)V_s,$	$V_4 = \left(\frac{C_{eq}}{C_4}\right)V_s,$
$V_2 = \left(\frac{C_{eq}}{C_2}\right)V_s,$	$V_5 = \left(\frac{C_{eq}}{C_5}\right)V_s$
$V_3 = \left(\frac{C_{eq}}{C_3}\right)V_s,$	

B. Response of *RC* Circuit

When resistor R and capacitor C are connected in series, they form a first-order resistor-capacitor circuit (*RC* circuit), as shown in **Figure 4**.

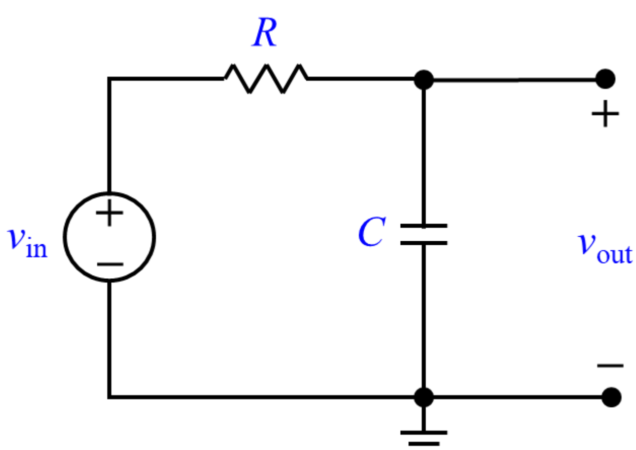


Figure 4 – *RC* Circuit

The circuit will respond to a step function, for example, from 0V to V_m or from V_m to 0V, with a time constant: $\tau = RC$. The time constant will determine how fast the *RC* circuit will respond to the change of the voltage source V_{in}. **Figure 5** shows the response of the voltage across the capacitor C to a square function of v_{in}. The capacitor will charge and discharge each cycle of a square function.

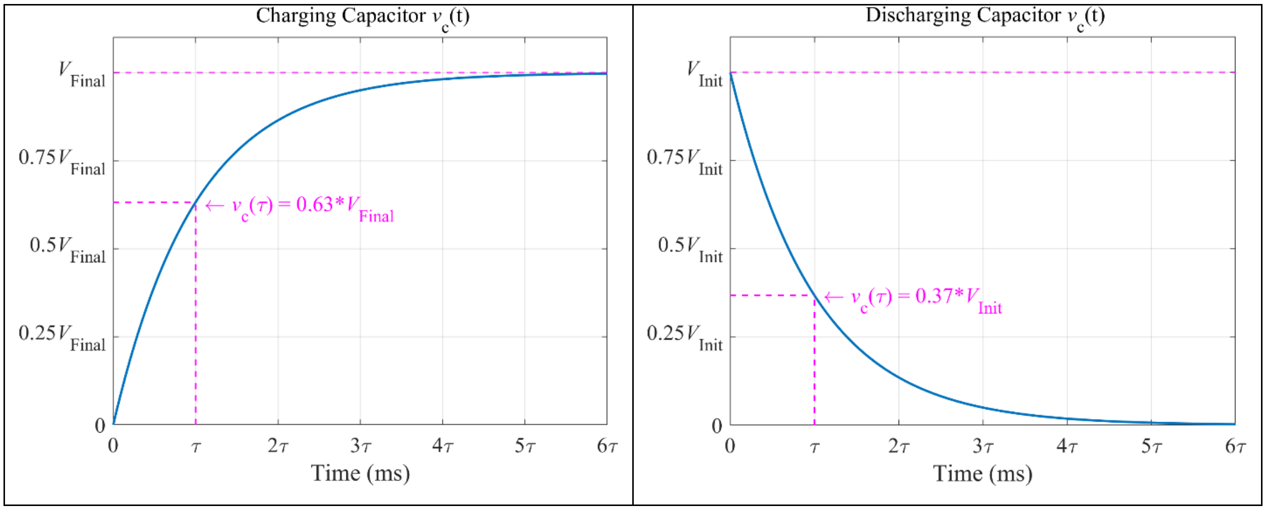


Figure 5 – Values of $v_C(t)$ at the Time Constant τ

In the charging phase, the capacitor will charge from 0V to V_{Final}. The time where the capacitor is charged up to 0.63x of V_{Final} is the time constant τ. In the discharging phase, the time where the capacitor discharges to 0.37x of $V_{Initial}$ is the time constant τ. The time points of 63% or 37% are also how the time constant τ of an *RC* circuit is measured.

C. Step Response of An *RC* Circuit

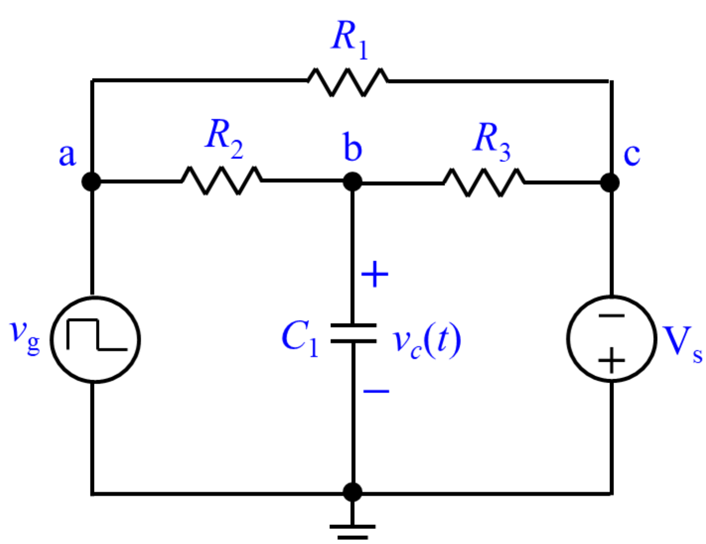


Figure 6 – Step Response of an *RC* Circuit

In **Figure 6**, v_g functions as a software switch: when $t < 0$, $v_g = 0V$; when $t \geq 0$, $v_g = V_p$. The step response of the voltage $v_c(t)$ is:

$$v_c(t) = V_t + (V_o - V_t)e^{-t/\tau};\ \tau = R_{eq}C,$$

or

$$v_c(t) = V_{th} + [v_c(0) - V_{th}]e^{-t/\tau}; \tau = R_{th}C,$$

where V_t is the final voltage across the capacitor when $t \rightarrow \infty$ or the Thévenin voltage V_{th} of the Thévenin equivalent circuit across the capacitor C, V_o or $v_c(0)$ is the initial voltage across the capacitor when $t < 0$, R_{eq} is the equivalent resistance across the capacitor, and R_{th} is the Thévenin equivalent resistance.

Table 1 – Standard Capacitance Values

Standard Capacitor Values (±10%)						
10pF	100pF	1000pF	.010μF	.10μF	1.0μF	10μF
12pF	120pF	1200pF	.012μF	.12μF	1.2μF	
15pF	150pF	1500pF	.015μF	.15μF	1.5μF	
18pF	180pF	1800pF	.018μF	.18μF	1.8μF	
22pF	220pF	2200pF	.022μF	.22μF	2.2μF	22μF
27pF	270pF	2700pF	.027μF	.27μF	2.7μF	
33pF	330pF	3300pF	.033μF	.33μF	3.3μF	33μF
39pF	390pF	3900pF	.039μF	.39μF	3.9μF	
47pF	470pF	4700pF	.047μF	.47μF	4.7μF	47uF
56pF	560pF	5600pF	.056μF	.56μF	5.6μF	
68pF	680pF	6800pF	.068μF	.68μF	6.8μF	
82pF	820pF	8200pF	.082μF	.82μF	8.2μF	

III. Prelab Assignment:

The prelab assignments should be completed and submitted to Camino before the lab.

A. Part 1 – Test Equipment

1. Quickly read through the manual of Digital Multimeter (Multimeter_EDU34450-90002.pdf from page 16 onward) to understand how to measure capacitance.
2. Quickly read through the manual of Waveform Generator (WaveformGenerator_EDU332102A_UsersGuide.pdf from page 16 onward) to understand how to set up different waveforms such as square or sine waves.
3. Quickly read through the manual of Digital Oscilloscope (Oscilloscope_EDUX1052G_97014.pdf from page 16 onward) to understand how to set up input channels to capture and measure a waveform.

B. Part 2 – *RC* Circuit

1. Select 5 capacitors (C_1 to C_5) between 1μF to 10μF from the typical capacitance values in **Table 1**.

2. Create a MATLAB script with V_s = 1V:
    a. Determine the voltages across the capacitors in **Figure 3** when V_s = 1V.
    b. Calculate the time constant τ of an *RC* circuit (*C* is one of the chosen capacitors) with different values of *R*: 200Ω, 400Ω, 600Ω, 800Ω, 1kΩ, 1.2kΩ.

C. Part 3 – Step Response of An *RC* Circuit

1. Create a MATLAB script with *C* = 1µF, V_p = 2V and V_s = 0.5V:
    a. Derive the step response $v_c(t)$ of the *RC* circuit in **Figure 6**.
    b. Calculate the time constant τ and $v_c(\tau)$.
    c. Plot the function $v_c(t)$ from 0 to 6τ and to save the plot of $v_c(t)$.

IV. Laboratory

Verify the results of MATLAB scripts of your prelab assignment with the TA before working on the lab procedures.

A. Capacitors in Series

1. Use the digital multimeter to measure the resistor and capacitors chosen in the prelab assignments. To measure capacitance, set the impedance of the multimeter to AUTO. Record their measured values in **Table 2** below.

Table 2 – Capacitance Values

	C_1 (µF)	C_2 (µF)	C_3 (µF)	C_4 (µF)	C_5 (µF)
Nominal Values					
Measured Values					

2. With the measured values of *C's*, use the MATLAB script to recalculate the voltages across capacitors in **Figure 3**. Record the theoretical results in **Table 3**.
3. Build the circuit in **Figure 3** with V_s = 12V. Measure the voltage across each capacitor and record all measurements in **Table 3**.

Table 3 – Capacitance Voltage Measurements

	Theoretical	Measured	Percent Error
V_1			
V_2			
V_3			
V_4			
V_5			

4. If there are any discrepancies, explain them.

B. *RC* Circuit

1. Build the *RC* circuit in **Figure 4** with the resistance decade *R*.
2. Set v_{in} to a square wave: amplitude of $1V_{pp}$ (peak-to-peak), 1V offset, frequency of 10Hz, and 50% duty cycle.
3. Connect one channel of the oscilloscope to measure v_{out}.
4. Measure the time constant τ by determining the time where the voltage across the capacitor V_{out} increases to about 63% of its maximum value. Record the time measurement for different values of *R*.

Table 4 – Time Constant Measurements

R	Theoretical	Measured	Percent Error
200Ω			
400Ω			
600Ω			
800Ω			
1kΩ			
1.2kΩ			

5. If there are any discrepancies, explain them.
6. Derive comments or conclusions from the results.

C. Step Response of An *RC* Circuit

1. Use the digital multimeter to measure the resistors chosen in the prelab assignments. Record their measured values in **Table 5** below.

Table 5 – Resistance and Capacitance Values

	R_1 (kΩ)	R_2 (kΩ)	R_3 (kΩ)	C_1 (μF)
Nominal Values				
Measured Values				

2. Build the *RC* circuit in **Figure 6** with R_1, R_2, R_3, and $V_s = 0.5V$.
3. Use the MATLAB script to recalculate the time constant τ and $v_c(\tau)$ with the measured values of *R's* and C_1.
4. Set v_g to a square wave: amplitude of $1V_{pp}$ (peak-to-peak), 1V offset, frequency of 5Hz,

and 50% duty cycle.

5. Connect one channel of the oscilloscope to measure $v_c(t)$.
6. Measure the time constant τ by determining the time when the voltage across the capacitor $v_c(t) = v_c(\tau)$.

Table 6 – Time Constant Measurements

	Theoretical	Measured	Percent Error
$v_c(\tau)$ (V)			
τ (ms)			

7. If there are any discrepancies, explain them.
8. Derive comments or conclusions from the results.

Make sure to check off with the TA before leaving the lab section.

V. Laboratory Report:

Include the measurements, computations, and answers to questions from the laboratory procedure. Clearly label all steps.