1 Mathematical Preparations

$$\begin{aligned} f(x) &= e^{-x^2} \\ g(x) &= -0.6x^2 + 1 \\ x &\epsilon [0, 1] \end{aligned}$$

integrate $g(x)$

$$\begin{aligned} I_g &= \int_0^1 \mathrm{dx} g(x) \\ &= \int_0^1 \mathrm{dx}(-0.6x^2 + 1) \\ &= \left[-0.2x^3 + x\right]_0^1 \\ &= -0.2 + 1 - 0 \\ &= 0.8 \end{aligned}$$

integral of interest:

$$\begin{aligned} I &= \int_0^1 \mathrm{dx} f(x) \\ &= \int_0^1 \mathrm{dx}(f(x) - g(x)) + I_g \\ &\approx \frac{1}{N} \sum_i (f(x_i) - g(x_i)) + I_g \end{aligned}$$

2 Computational Setup

This exercise was solved by implementing only one non-main method. This method computes the integral by control variates (cv). Therefore, I_g and I were calculated as shown above. In the program, implementation was very similar to that of the simple-sampling-method of exercise 1 (with altered $f(x)$ and correction value of I_g).
For comparing cv with other methods, results of exercise 2 where copied.
The main method is used for organizing calls of control-variates-method, increasing number of samples and writing into file.

3 Observations

For comparing the cv method, only importance sampling was used. This is because the variance of cv is more likely to that of importance sampling and not that of simple sampling.
While number of samples increases, the integrals calculated by both methods

(as in the previous exercises) approach the exact value of 0.746824. For me, it was surprising, that the variance of the cv method goes down more quickly than the one of the importance sampling. I didn't expect that because the amount of mathematical preparations for cv is small compared to that one of importance sampling.
This is probably a wrong conclusion because the actual amount of preparation for cv is "hidden" in finding $g(x)$ which was given for this exercise.

While the digits are fixed by importance sampling as in the previous exercise (1st >1, 2nd >10, 3rd >1000, 4th >10.000, all >100.000) cv fixes them as follows: 1st and 2nd > 1, 3rd > 100, 4th > 10.000. Thus, cv fixes the first digits faster and with less variance than importance sampling but has less accuracy (fixed numbers of digits) at higher number of samples.