1 Preparations

$$\begin{aligned} f(x) &= e^{-x^2} \\ g(x) &= c \cdot e^{-x} \\ x &\epsilon [0,1] \end{aligned}$$

normalization:

$$\begin{aligned} 1 &= \int_0^1 \mathrm{dx} g(x) \\ &= c \cdot \int_0^1 \mathrm{dx} e^{-x} \\ &= c \cdot \left[-e^{-x}\right]_0^1 \\ &= c\left(-\frac{1}{e}+1\right) \\ c &= \frac{1}{1-e^{-1}} \end{aligned}$$

integration:

$$\begin{aligned} G(x) &= \int_0^x \mathrm{dx}' g(x') \\ &= c\int_0^x \mathrm{dx}' e^{-x'} \\ &= \frac{1}{1-e^{-1}} \cdot \left[-e^{-x'}\right]_0^x \\ y &= \frac{1-e^{-x}}{1-e^{-1}} \end{aligned}$$

boundaries of $G(x)$:

$$\begin{aligned} G(0) &= 0 \\ G(1) &= 1 \end{aligned}$$

getting x again:

$$\begin{aligned} y(1-e^{-1}) &= 1-e^{-x} \\ 1-y(1-e^{-1}) &= e^{-x} \\ \ln(1-y(1-e^{-1})) &= -x \\ x &= -\ln(1-y(1-e^{-1})) \end{aligned}$$