BRAC University (Department of Computer Science and Engineering)

CSE 330 (Numerical Methods)

Spring 2025

Quiz 3 [CO3]

Student ID: Set-A

Full Marks: 15

Section:

Name:

Duration: 20 minutes

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1. Consider the following dataset:

x	2.2	2.4	2.6	2.8
f(x)	20.2	24.3	26.5	32.6

a) Using the table above, compute $\mathbf{f^{(1)}(2.6)}$ upto **4 significant figures** using the **forward** difference method. [2 marks]

b) Compute $\mathbf{f^{(1)}(2.4)}$ upto **4 significant figures** using the **backward** difference method. [2 marks]

c) Compute the **truncation error** at x=4 using **central** difference method if the above data is generated by the function, $f(x) = 4x^2 + 3e^{-2x}$. [2 marks]

d) Compute the **upper bound** of truncation error at **x=5** using the **central** difference method if the above data is generated by the function, $f(x) = 4x^2 + 3e^{2x}$. [3 marks]

2. Consider a Runge function, $f(x) = 1/(1 + 16x^2)$ for the given interval [-4,4] and n = 3.

a) Calculate the equally angled points/θ_j [3 marks]

b) Calculate the value of Chebyshev nodes. [3 marks]

(2)(a) $\emptyset_j = \frac{(2j+1)\pi}{2(n+1)}$ $\theta_j = \frac{(2j+1)\pi}{2(3+1)}$ $j = 0, 1, 2, 3$

$\theta_0 = \frac{\pi}{8}$, $\theta_1 = \frac{3\pi}{8}$, $\theta_2 = \frac{5\pi}{8}$, $\theta_3 = \frac{7\pi}{8}$.

(b) $x_j = r\cos\phi_j + \text{center}$

$x_0 = 4\cos\left(\frac{\pi}{8}\right)$, $x_1 = 4\cos\left(\frac{3\pi}{8}\right)$, $x_2 = 4\cos\left(\frac{5\pi}{8}\right)$,

$x_3 = 4\cos\left(\frac{7\pi}{8}\right)$.

(a) $h = 2.4 - 2.2 = 0.2$

Using forward difference method,

$$f'(x) = \frac{F(x+h) - F(x)}{h}$$

$$f'(2.6) = \frac{F(2.6+0.2) - F(2.6)}{0.2}$$

$$= \frac{F(2.8) - F(2.6)}{0.2}$$

$$= \frac{32.6 - 26.5}{0.2}$$

$$= 30.50 \text{ (4sf)}$$

(b) Using backward difference method,

$$f'(x) = \frac{F(x) - F(x-h)}{h}$$

$$f'(2.4) = \frac{f(2.4) - f(2.4 - 0.2)}{0.2}$$

$$= \frac{f(2.4) - f(2.2)}{0.2}$$

$$= \frac{24.3 - 20.2}{0.2}$$

$$= 20.50 \text{ (4sf)}$$

(d) upper bound of truncation error at $x = 5$,

$$\left| \frac{f'''(\xi)}{3!} h^2 \right| \quad \xi \in [(x+h), (x-h)]$$

$$\xi \in [4.8, 5.2]$$

$$f'(x) = 8x + 6e^{2x}$$

$$f''(x) = 8 + 12e^{2x}$$

$$f'''(x) = 24e^{2x}$$

$24e^{2(4.8)} = 354354.7576$, $24e^{2(5.2)} = 788631.062$

$$\frac{788631.0162}{3!} * (0.2)^2 = 5257.54. \text{ (Answer)}$$

c

$x = 4,\ h = 0.2,\ f(x) = 4x^2 + 3e^{-2x}$

$f'(x) = 8x - 6e^{-2x}$

$f'(4) = 31.997987$

$$CD = \frac{f(x+h) - f(x-h)}{2h} = \frac{f(4.2) - f(3.8)}{2 \times 0.2}$$

$$= \frac{70.560675 - 57.7615}{2 \times 0.2}$$

$$= 31.99779375$$

Truncation error = |Actual - Central Difference|

$= |31.997987 - 31.99779375|$

$= 0.0000495.$