CSE330- Numerical Methods
Quiz 05; Summer'24

Name: Sakit ID: 1930 Section: ----

Marks: 15 points **Time: 25 minutes**

Instructions: **Answer all questions on the space provided below for each.**

Question 1: CO3 (4+4+2+5 points): A linear system is described by the following equations:

$$x_1 + 6x_2 + 2x_3 = 10$$
$$3x_1 + 2x_2 + x_3 = 6$$
$$2x_1 + 5x_2 + 2x_3 = 9$$

Based on these equations, answer the questions below.

(a) From the given linear equations, identify the matrices A, x and b such that the linear system can be expressed as a matrix equation. Find the value of det(A).
(b) Construct the Frobenius matrices $F^{(1)}$ and $F^{(2)}$ from this system.
(c) Compute the unit lower triangular matrix L.
(d) Now find the solution of the linear system using the LU decomposition method. Use the unit lower triangular matrix found in the previous question.

a

$$A = \begin{bmatrix} 1 & 6 & 2 \\ 3 & 2 & 1 \\ 2 & 5 & 2 \end{bmatrix}; \quad x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}; \quad b = \begin{bmatrix} 10 \\ 6 \\ 9 \end{bmatrix}$$

$$\det(A) = 1\{(2\times2)-(5\times1)\} - 6\{(3\times2)-(1\times2)\} + 2\{(3\times5)-(2\times2)\}$$
$$= -3$$

b

$$A^1 = \begin{bmatrix} 1 & 6 & 2 \\ 3 & 2 & 1 \\ 2 & 5 & 2 \end{bmatrix}$$

$$m_{21} = \frac{3}{1} = 3$$
$$m_{31} = \frac{a_{31}}{a_{11}} = \frac{2}{1} = 2$$

$$F^1 = \begin{vmatrix} 1 & 0 & 0 \\ -m_{21} & 1 & 0 \\ -m_{31} & 0 & 1 \end{vmatrix}$$
$$= \begin{vmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ -2 & 0 & 1 \end{vmatrix}$$

$$A^2 = F^1 \times A^1 = \begin{vmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ -2 & 0 & 1 \end{vmatrix} \times \begin{vmatrix} 1 & 6 & 2 \\ 3 & 2 & 1 \\ 2 & 5 & 2 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 6 & 2 \\ 0 & -16 & -5 \\ 0 & -7 & -2 \end{vmatrix} \quad \text{now,} \quad m_{32} = \frac{a_{32}}{a_{22}} = \frac{-7}{-16} = \frac{7}{16}$$

$$F^2 = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -m_{32} & 1 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -\frac{7}{16} & 1 \end{vmatrix}$$

$$A^3 = \underbrace{\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -\frac{7}{16} & 1 \end{vmatrix}}_{F^2} \times \underbrace{\begin{vmatrix} 1 & 6 & 2 \\ 0 & -16 & -5 \\ 0 & -7 & -2 \end{vmatrix}}_{A^2} = \underbrace{\begin{vmatrix} 1 & 6 & 2 \\ 0 & -16 & -5 \\ 0 & 0 & \frac{3}{16} \end{vmatrix}}_{U}$$

$\underset{\sim}{e}$

$$L = \begin{vmatrix} 1 & 0 & 0 \\ m_{21} & 1 & 0 \\ m_{31} & m_{32} & 1 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 2 & \frac{7}{16} & 1 \end{vmatrix}$$

$\underset{\sim}{d}$

$Ly = b$

$$\begin{vmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 2 & \frac{7}{16} & 1 \end{vmatrix} \begin{vmatrix} y_1 \\ y_2 \\ y_3 \end{vmatrix} = \begin{vmatrix} 10 \\ 6 \\ 9 \end{vmatrix}$$

$\rightarrow y_1 = 10$

$\rightarrow 3y_1 + y_2 = 6$

$\Rightarrow 3 \times 10 + y_2 = 6$

$\Rightarrow y_2 = -24$

$\rightarrow 2y_1 + \frac{7}{16} y_2 + y_3 = 9$

$\Rightarrow y_3 = 9 - (2 \times 10) - (\frac{7}{16} \times -24)$

$= -0.5$

$Ux = y$

$$\begin{vmatrix} 1 & 6 & 2 \\ 0 & -16 & -5 \\ 0 & 0 & \frac{3}{16} \end{vmatrix} \cdot \begin{vmatrix} x_1 \\ x_2 \\ x_3 \end{vmatrix} = \begin{vmatrix} 10 \\ -24 \\ -0.5 \end{vmatrix}$$

$\rightarrow \frac{3}{16} x_3 = -0.5$

$x_3 = -\frac{8}{3}$

$\rightarrow -16x_2 - 5x_3 = -24$

$\Rightarrow x_2 = \frac{7}{3}$

$\rightarrow x_1 + 6x_2 + 2x_3 = 10$

$\Rightarrow x_1 = 10 - (6 \times \frac{7}{3}) - (2 \times -\frac{8}{3})$

$= \frac{4}{3}.$

$$\begin{vmatrix} x_1 \\ x_2 \\ x_3 \end{vmatrix} = \begin{vmatrix} \frac{4}{3} \\ \frac{7}{3} \\ -\frac{8}{3} \end{vmatrix}$$

(Ans).