BRAC University (Department of Computer Science and Engineering)

CSE 330 (Numerical Methods)

Spring 2025

Quiz 3 [CO3]

Student ID: Set-B **Full Marks: 15**

Section:

Name: **Duration: 20 minutes**

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1. Consider the following dataset:

x	2.1	2.3	2.5	2.7
f(x)	18.2	22.3	24.5	30.6

a) Compute $\mathbf{f^{(1)}(2.5)}$ upto 4 significant figures using the **central** difference method. [2 marks]

b) Compute $\mathbf{f^{(1)}(2.3)}$ upto 4 significant figures using the **backward** difference method. [2 marks]

c) Compute the **truncation error** at x=4 using **backward** difference method if the above data is generated by the function, $f(x) = 4x^2 + 3e^{-2x}$. [2 marks]

d) Compute the **upper bound** of truncation error at x=4 using **central** difference method, if the above data is generated by the function, $f(x) = 4x^2 + 3e^{-2x}$. [3 marks]

2. Consider a Runge function, $f(x) = 3/(4 + 9x^2)$ for the given interval [-4,4] and n = 3.

a) Calculate the equally angled points/θ_j [3 marks]

b) Calculate the value of Chebyshev nodes. [3 marks]

(2) (a) $\phi_j = \frac{(2j+1)\pi}{2(n+1)}$ $\theta_j = \frac{(2j+1)\pi}{2(3+1)}$ $j = 0, 1, 2, 3$

$\theta_0 = \frac{\pi}{8}$, $\theta_1 = \frac{3\pi}{8}$, $\theta_2 = \frac{5\pi}{8}$, $\theta_3 = \frac{7\pi}{8}$.

(b) $x_j = r\cos\phi_j + \text{center}$

$x_0 = 4\cos\left(\frac{\pi}{8}\right)$, $x_1 = 4\cos\left(\frac{3\pi}{8}\right)$, $x_2 = 4\cos\left(\frac{5\pi}{8}\right)$,

$x_3 = 4\cos\left(\frac{7\pi}{8}\right)$.

(1)(a) $h = 2.3 - 2.1 = 0.2$

Using central difference method,

$$f'(x) = \frac{f(x+h) - f(x-h)}{2h}$$

$$f'(2.5) = \frac{f(2.5+0.2) - f(2.5-0.2)}{2 * 0.2}$$

$$f'(2.5) = \frac{f(2.7) - f(2.3)}{0.4}$$

$$f'(2.5) = \frac{30.6 - 22.3}{0.4}$$

$$f'(2.5) = 20.75$$

(b) Using backward difference method,

$$f'(x) = \frac{f(x) - f(x-h)}{h}$$

$$f'(2.3) = \frac{f(2.3) - f(2.3-0.2)}{0.2}$$

$$f'(2.3) = \frac{f(2.3) - f(2.1)}{0.2}$$

$$f'(2.3) = \frac{22.3 - 18.2}{0.2}$$

$$f'(2.3) = 20.50$$

(d) upper bound of truncation error at $x = 4$,

$$\left| \frac{f'''(\xi)}{3!} h^2 \right| \qquad \xi \in [(x+h), (x-h)]$$

$$\xi \in [4.2, 3.8]$$

$$f'(x) = 8x - 6e^{-2x}$$

$$f''(x) = 8 + 12e^{-2x}$$

$$f'''(x) = -24e^{-2x}$$

$$|-24e^{-2(4.2)}| = |-5.3968 * 10^{-3}|$$

$$|-24e^{-2(3.8)}| = |-0.01201|$$

$$\frac{0.01201}{3!} * (0.2)^2 = 8.0067 * 10^{-5}.$$

c

$x=4,\ h=0.2,\ f(x) = 4x^2 + 3e^{-2x}$

$f'(x) = 8x - 6e^{-2x}$

$f'(4) = 31.997987.$

$$BD = \frac{f(x) - f(x-h)}{h} = \frac{f(4) - f(3.8)}{0.2}$$

$$= \frac{64.001006 - 57.7615}{0.2}$$

$$= 31.19753$$

Truncation error = | Actual − Backward Difference |

$$= |31.997987 - 31.19753|$$

$$= 0.800457.$$