CSE330- Numerical Methods

Quiz 02; Spring'25

Set A

Name: Sahil _____ **ID:** 1930... _____ **Section:** _____

Marks: 15 points **Time: 20 minutes**

<u>Instructions:</u> Answer all questions on the space provided below for each.

Question 1: CO2 (6+1 points): The velocity of a test object measured at different times has been given below:

Time (seconds)	Velocity (m/s)
2	20
4	32
8	50

- Using Lagrange basis, construct a polynomial that goes through the above nodes.
- Use the polynomial to find the approximate velocity at Time = 21 seconds.

Question 2: CO3 (3+4+1 points): The following nodes come from the function f(x) = xln(5):

x	f(x)
0.5	0.804
1	1.609

- Using Newton's divided difference method, find the equation of a first degree polynomial which fits the above data points.
- Add another node '1.5' to the above nodes and find out the interpolating polynomial of appropriate degree.
- Find out the relative error at x = 4.

Q-1

$$P_2(x) = l_0(x)f(x_0) + l_1(x)f(x_1) + l_2(x)f(x_2)$$

$$l_0(x) = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} = \frac{(x-4)(x-8)}{(2-4)(2-8)} = \frac{(x-4)(x-8)}{12}$$

$$l_1(x) = \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} = \frac{(x-2)(x-8)}{(4-2)(4-8)} = -\frac{(x-2)(x-8)}{8}$$

$$l_2(x) = \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} = \frac{(x-2)(x-4)}{(8-2)(8-4)} = \frac{(x-2)(x-4)}{24}$$

$$P_2(x) = \frac{(x-4)(x-8)}{12} \times 20 - \frac{(x-2)(x-8)}{8} \times 32 + \frac{(x-2)(x-4)}{24} \times 50$$

at $x/t = 21s$, the value of $P_2(x)$

$$P_2(21) = \frac{(21-4)(21-8)}{12} \times 20 - \frac{(21-2)(21-8)}{8} \times 32 + \frac{(21-2)(21-4)}{24} \times 50$$

$$= 53.25. \quad \text{(Ans)}.$$

Q-2

$x_0 = 0.5 \quad f[x_0] = 0.804$

$x_1 = 1 \quad f[x_1] = 1.609$

$x_2 = 1.5 \quad f[x_2] = 2.414$

$f[x_0, x_1] = \dfrac{1.609 - 0.804}{1 - 0.5} = 1.61$

$f[x_1, x_2] = \dfrac{2.414 - 1.609}{1.5 - 1} = 1.61$

$f[x_0, x_1, x_2] = \dfrac{1.61 - 1.61}{1.5 - 0.5} = 0$

a) $P_1(x) = f[x_0] + f[x_0, x_1](x - x_0) = 0.804 + 1.61(x - 0.5)$

b) $P_2(x) = f[x_0] + f[x_0, x_1](x - x_0) + f[x_0, x_1, x_2](x - x_0)(x - x_1)$

$= 0.804 + 1.61(x - 0.5) + 0$

c) $P_2(4) = 0.804 + 1.61(4 - 0.5) = 6.439$

$f(4) = 4\ln(5) = 6.43775$

$\text{relative error} = \dfrac{|6.43775 - 6.439|}{|6.43775|} = 1.939 \times 10^{-4}$