Practice Problems Chapter 7: Numerical Integration

1.
$$\int_{0}^{0.8} (0.2 + 25x - 200x^2 + 675x^3 - 900x^4 + 400x^5) dx$$

- (a) Find the exact value of the given integral.
- (b) Use the multi-segment Trapezoidal rule with m = 4 to approximate the integral.
- (c) Using the previous parts, calculate the relative error.
- 2. Compute the integration $\int_{0}^{2} f(x)dx$ numerically by using Trapezoidal and Simpson's rules if the function f(x) is given as:

(a)
$$f(x) = \sqrt{1 + x^2}$$

(b)
$$f(x) = \sin(x)$$

(c)
$$f(x) = e^x$$

- 3. Consider the following function: $f(x) = e^x x$, which is continuous on the interval [1, 3]. Use this function to answer the following:
 - (a) Find the actual integral value for this function.
 - (b) Use the Composite Newton-Cotes formula to find the numerical integration for 4 segments.
 - (c) Compute the error in percentage between the results obtained in the previous two parts. How can we decrease the error more?
 - (d) Use the Simpson rule to find the numerical integration.
- 4. A function is given by $f(x) = 0.2 + 25x + 3x^{2}$.

Now answer the following based on this function:

- (a) Use the Trapezium rule to numerically integrate over the interval [0, 2]
- (b) Compute the exact integrated value of the given function.
- (c) Using the previous parts, calculate the relative error in percentage.
- 5. Consider a function $f(x) = \frac{1}{x(\ln x)^2}$, which is continuous on the interval [e, e+1]. Now answer the questions below based on this function:
 - (a) Calculate the exact integrated value of the given function.
 - (b) Find the numerical integration for m=4 using the Composite Newton-cotes formula.
 - (c) Calculate the error in percentage from the above two parts.
- 6.A function is given by $f(x) = 1 + 4x 2x^2$. Now answer the following based on this function:
 - (a) Use the Trapezium rule to integrate over the interval [1, 3] numerically.
 - (b) Compute the exact integrated value of the given function.
 - (c) Using the previous parts, calculate the relative percentage error.

7. Compute the upper bound of error for the following example:

(a)
$$n = 1$$
, $f(x) = \sin(x)$, $[0, \frac{\pi}{2}]$

(b)
$$n = 2$$
, $f(x) = ln(1 + x)$, [0, 1]

(b)
$$n = 2$$
, $f(x) = ln(1 + x)$, $[0, 1]$
(c) $n = 2$, $f(x) = cos(x)$, $[0, \frac{\pi}{4}]$

(d)
$$n = 1$$
, $f(x) = x^3$, [1, 2]