CSE330 Numerical Methods
Quiz 5 - SET[B]

Name:	ID:	Section:

Question 1: A student decided to set up a stall at the university fair to sell handmade items. He creates custom t-shirts (T), mugs (M), and tote bags (B). His friends came to support his small side business, and the following purchases were made:

- Friend 1 bought 3 t-shirts, 2 mugs, and 4 tote bags for a total of 450 tk. ... (i)
- Friend 2 bought 4 t-shirts, 3 mugs, and 2 tote bags for a total of 500 tk. ... (ii)
- Friend 3 bought 5 t-shirts, 2 mugs, and 3 tote bags for a total of 550 tk. ... (iii)

In total, the student earned 2500 tk by selling 50 t-shirts, 20 mugs, and 30 tote bags. ... (iv)

a. **[1.5 mark]** Write down the linear equations that relate the variables T, M & B.
b. **[1.5 mark]** Identify the matrices A, x, and b so that the equations in the previous question can be expressed in the standard matrix equation form Ax = b.
c. **[7 marks]** From matrix A in the previous question, compute the matrices Q and R such that A = QR, where the symbols have their usual meanings.
d. **[5 marks]** Evaluate QTb, and finally solve the system by evaluating x (that is, evaluate T, M & B).

a

$$3T+2M+4B=450$$
$$4T+3M+2B=500$$
$$5T+2M+3B=550$$
$$50T+20M+30B=2500$$

b

$$\begin{bmatrix} 3 & 2 & 4 \\ 4 & 3 & 2 \\ 5 & 2 & 3 \\ 50 & 20 & 30 \end{bmatrix} \begin{bmatrix} T \\ M \\ B \end{bmatrix} = \begin{bmatrix} 450 \\ 500 \\ 550 \\ 2500 \end{bmatrix}$$

A x b

$Ax = b$

c

$$\begin{bmatrix} 3 & 2 & 4 \\ 4 & 3 & 2 \\ 5 & 2 & 3 \\ 50 & 20 & 30 \end{bmatrix}$$
$u_1 \quad u_2 \quad u_3$

$$p_1 = u_1 = \begin{bmatrix} 3 \\ 4 \\ 5 \\ 50 \end{bmatrix} ; \quad |p_1| = 5\sqrt{102}$$

$$q_1 = \frac{p_1}{|p_1|} = \frac{1}{5\sqrt{102}} \begin{bmatrix} 3 \\ 4 \\ 5 \\ 50 \end{bmatrix}$$

$$P_2 = U_2 - (U_2^T q_1) q_1$$

$$= \begin{bmatrix} 2 \\ 3 \\ 2 \\ 20 \end{bmatrix} - 20.35743 \begin{bmatrix} \frac{3}{5\sqrt{102}} \\ \frac{4}{5\sqrt{102}} \\ \frac{5}{5\sqrt{102}} \\ \frac{50}{5\sqrt{102}} \end{bmatrix}$$

$$U_2^T q_1 = \begin{bmatrix} 2 & 3 & 2 & 20 \end{bmatrix} \times \frac{1}{5\sqrt{102}} \begin{bmatrix} 3 \\ 4 \\ 5 \\ 50 \end{bmatrix}$$

$$= 20.35743$$

$$= \begin{bmatrix} 0.79058 \\ 1.38745 \\ -0.015685 \\ -0.15685 \end{bmatrix}$$

; $|P_2| = \sqrt{(0.79058)^2 + (1.38745)^2 + (-0.015685)^2 + (-0.1585)^2}$

$$= 1.60464$$

$$q_2 = \frac{P_2}{|P_2|} = \begin{bmatrix} 0.49268 \\ 0.86464 \\ -0.009774 \\ -0.09774 \end{bmatrix}$$

$$P_3 = U_3 - (U_3^T q_2) q_2 - (U_3^T q_1) q_1$$

$$= \begin{bmatrix} 4 \\ 2 \\ 3 \\ 30 \end{bmatrix} - 0.738478 \begin{bmatrix} 0.49268 \\ 0.8646 \\ -0.009774 \\ -0.09774 \end{bmatrix} - 30.39752 \begin{bmatrix} \frac{3}{5\sqrt{102}} \\ \frac{4}{5\sqrt{102}} \\ \frac{5}{5\sqrt{102}} \\ \frac{50}{5\sqrt{102}} \end{bmatrix}$$

$$U_3^T q_2 = \begin{bmatrix} 4 & 2 & 3 & 30 \end{bmatrix} \begin{bmatrix} 0.49268 \\ 0.8646 \\ -0.009774 \\ -0.09774 \end{bmatrix}$$

$$= 0.738478$$

$$U_3^T q_1 = \begin{bmatrix} 4 & 2 & 3 & 30 \end{bmatrix} \begin{bmatrix} \frac{3}{5\sqrt{102}} \\ \frac{4}{5\sqrt{102}} \\ \frac{5}{5\sqrt{102}} \\ \frac{50}{5\sqrt{102}} \end{bmatrix}$$

$$= 30.39752$$

$$P_3 = \begin{bmatrix} 1.83028 \\ -1.04635 \\ -0.0025850 \\ -0.02585 \end{bmatrix} \quad ; \quad |P_3| = 2.108423$$

$$q_3 = \frac{P_3}{|P_3|} = \begin{bmatrix} 0.86808 \\ -0.49627 \\ -0.001226 \\ -0.01226 \end{bmatrix}$$

$$Q = \begin{bmatrix} \frac{3}{5\sqrt{102}} & 0.49268 & 0.86808 \\ \frac{4}{5\sqrt{102}} & 0.86464 & -0.49627 \\ \frac{5}{5\sqrt{102}} & -0.009774 & -0.001226 \\ \frac{50}{5\sqrt{102}} & -0.09774 & -0.01226 \end{bmatrix}$$

$$R = \begin{bmatrix} U_1^T q_1 & U_2^T q_1 & U_3^T q_1 \\ 0 & U_2^T q_2 & U_3^T q_2 \\ 0 & 0 & U_3^T q_3 \end{bmatrix} = \begin{bmatrix} 50.4579 & 20.3574 & 30.39752 \\ 0 & 1.60493 & 0.738478 \\ 0 & 0 & 2.1083 \end{bmatrix}$$

$$Q^T b = \begin{bmatrix} \frac{3}{5\sqrt{102}} & \frac{4}{5\sqrt{102}} & \frac{5}{5\sqrt{102}} & \frac{50}{5\sqrt{102}} \\ 0.49268 & 0.8044 & -0.009774 & -0.09774 \\ 0.86808 & -0.49627 & -0.001226 & -0.01226 \end{bmatrix} \times \begin{bmatrix} 450 \\ 500 \\ 550 \\ 2500 \end{bmatrix}$$

$$= \begin{bmatrix} 2595.7905 \\ 409.3003 \\ 111.1767 \end{bmatrix}$$

$$Rx = Q^T b$$

$$\begin{bmatrix} 50.4579 & 20.3574 & 30.39752 \\ 0 & 1.60493 & 0.738478 \\ 0 & 0 & 2.1083 \end{bmatrix} \begin{bmatrix} T \\ M \\ B \end{bmatrix} = \begin{bmatrix} 2595.7905 \\ 409.3003 \\ 111.1767 \end{bmatrix}$$

$$B = \frac{111.1764}{2.1083} = 52.7327$$

$$M = \frac{409.3003 - 0.738478 \times 52.7327}{1.60493} = 227.6475$$

$$T = \frac{2595.7905 - 20.3574 \times 227.6475 - 30.39752 \times 52.7327}{50.2579}$$

$= -72.16836$

$$\begin{bmatrix} T \\ M \\ B \end{bmatrix} = \begin{bmatrix} -72.16836 \\ 227.6475 \\ 52.7327 \end{bmatrix}$$

(Ans).