Practice Problems Chapter 7: Numerical Integration

1. $\int_0^{0.8} (0.2 + 25x - 200x^2 + 675x^3 - 900x^4 + 400x^5)dx$
   (a) Find the exact value of the given integral.
   (b) Use the multi-segment Trapezoidal rule with m = 4 to approximate the integral.
   (c) Using the previous parts, calculate the relative error.

2. Compute the integration $\int_0^2 f(x)dx$ numerically by using Trapezoidal and Simpson's rules if the function f(x) is given as:
   (a) $f(x) = \sqrt{1 + x^2}$
   (b) $f(x) = sin(x)$
   (c) $f(x) = e^x$

3. Consider the following function: $f(x) = e^x - x$, which is continuous on the interval [1, 3]. Use this function to answer the following:
   (a) Find the actual integral value for this function.
   (b) Use the Composite Newton-Cotes formula to find the numerical integration for 4 segments.
   (c) Compute the error in percentage between the results obtained in the previous two parts. How can we decrease the error more?
   (d) Use the Simpson rule to find the numerical integration.

4. A function is given by $f(x) = 0.2 + 25x + 3x^2$.
Now answer the following based on this function:
   (a) Use the Trapezium rule to numerically integrate over the interval [0, 2]
   (b) Compute the exact integrated value of the given function.
   (c) Using the previous parts,calculate the relative error in percentage.

5. Consider a function $f(x) = \frac{1}{x(lnx)^2}$, which is continuous on the interval $[e, e + 1]$. Now answer the questions below based on this function:
   (a) Calculate the exact integrated value of the given function.
   (b) Find the numerical integration for m=4 using the Composite Newton-cotes formula.
   (c) Calculate the error in percentage from the above two parts.

6.A function is given by $f(x) = 1 + 4x - 2x^2$. Now answer the following based on this function:
   (a) Use the Trapezium rule to integrate over the interval [1, 3] numerically.
   (b) Compute the exact integrated value of the given function.
   (c) Using the previous parts, calculate the relative percentage error.

7. Compute the upper bound of error for the following example:

(a) $n = 1,\ f(x) = sin(x)\ ,\ [0, \frac{\pi}{2}]$

(b) $n = 2,\ f(x) = ln(1 + x)\ ,\ [0, 1]$

(c) $n = 2,\ f(x) = cos(x)\ ,\ [0, \frac{\pi}{4}]$

(d) $n = 1,\ f(x) = x^3\ ,\ [1, 2]$