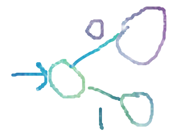
Pumping Lemma

Wednesday, September 25, 2024 5:58 PM

$C = \{w \mid w$ has an equal number of 0s and 1s$\}$, and NRL

$D = \{w \mid w$ has an equal number of occurrences of 01 and 10 as substrings$\}$. RL



0011

0101

L = {length at least 4}

1010 = w

$1\ 0\ 1\ 0\ 1\ 0\ 1\ 0 = w'$

x y y z

$xy^2z \in R$

10,1 X

1001 ✓

Our technique for proving nonregularity stems from a theorem about regular languages, traditionally called the ***pumping lemma***. This theorem states that all regular languages have a special property. If we can show that a language does not have this property, we are guaranteed that it is not regular. The property states that all strings in the language can be "pumped" if they are at least as long as a certain special value, called the ***pumping length***. That means each string contains a section that can be repeated any number of times with the resulting string remaining in the language.

$\notin R$

$S = 1\ 0\ 1\ 0\ 0\ 0\ 0\ 1$

p

p

$1\ 0\ 1\ 0\ 0 \mid 0\ 1$

y z

$|x| = 0$

$|y| = p$

Pumping lemma If A is a regular language, then there is a number p (the pumping length) where if s is any string in A of length at least p, then s may be divided into three pieces, $s = xyz$, satisfying the following conditions:

1. for each $i \geq 0$, $xy^iz \in A$,
2. $|y| > 0$, and
3. $|xy| \leq p$.

p

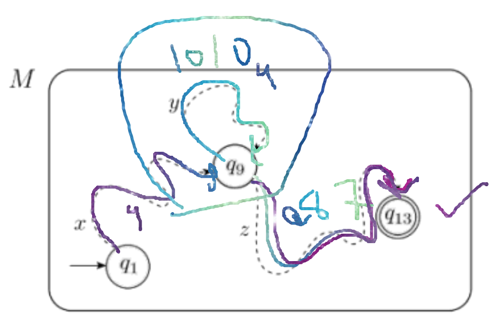
S1 = 000110101 1010 1.....0 [100 length]

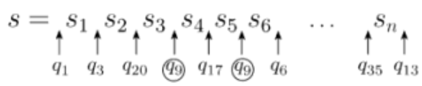
x = 9 y =4 z = 87

S2 = 000110101 1010 1010 1......0 [104 length]

9 i=1 i=2 87

Dfa with 13 states





for $\forall$ long strings ω (length at least P), $\exists xyz$, $\omega = xyz$, such that $\forall i$, $xy^iz \in L$

0^P

S'

We need to negate this statement where there exists one string w with all regular language properties where exists an "i" which in xy^iz doesn't belong to language

To use the pumping lemma to prove that a language B is not regular, first assume that B is regular in order to obtain a contradiction. Then use the pumping lemma to guarantee the existence of a pumping length p such that all strings of length p or greater in B can be pumped. Next, find a string s in B that has length p or greater but that cannot be pumped. Finally, demonstrate that s cannot be pumped by considering all ways of dividing s into x, y, and z (taking condition 3 of the pumping lemma into account if convenient) and, for each such division, finding a value i where $xy^iz \notin B$. This final step often involves grouping the various ways of dividing s into several cases and analyzing them individually. The existence of s contradicts the pumping lemma if B were regular. Hence B cannot be regular.

Example 1 : L = { $0^n 1^n$; n>=0 }

Let L is a regular language. It has three special properties.
pumping length = p

Now we are going to select such a string which is at least length p.
w = $0^{p/2} 1^{p/2}$ == length p
/// $0^p 1^p$ == length 2p

$|xy| \leq P$

$0 \,..\, 0^P \mid 1\,1\,1 \,..\, 1^P$

$x = 0^{P-2}$

$i=1$ $\quad \underbrace{0^{P-2}}_{x}\ \underbrace{0^2}_{y} \mid \underbrace{1^P}_{z} \Rightarrow i = 2$

$xyyz \Rightarrow 0^{P-2}\, 0^2\, 0^2\, 1^P$

$w' \Rightarrow 0^{P+2}\, 1^P$

$x = 0 \quad \underbrace{0^p}_{y} \mid \underbrace{1^p}_{z}$ $\qquad w' \Rightarrow 0^{p+2} 1^p$

$|xy| = \frac{p}{2} + 1$

$w_2 = 0^{p/2} \; 1^{p/2}$

$0 0 \text{---} 0^{p/2} \; 1 \mid \underbrace{1 \text{---} 1^{p/2}}_{2 = p/2 - 1}$

$\underbrace{00 \cdots 0}_{x} \; \underbrace{0 1}_{y} \mid 1 \text{---} 1$

$\underbrace{0 \text{---} 0}_{x} \; \underbrace{01}_{y} \; \underbrace{01}_{y} \; 1 \text{---} 1$

Example 2 : L = {ww : w e {0,1}*}

$|xy| \leq p$

$w = 0^p 1$

$s = ww = 0^p 1 0^p 1$

① $i = 1 \quad \underbrace{0 \text{---} 0^p}_{|y| = p} \mid \underbrace{1 \; 0 \text{---} 0^p \, 1}_{z}$

$|x| = 0$

$i = 2 \quad 0^p 0^p 1 0^p 1$

$\Rightarrow 0^{2p} 1 0^p 1$

② $w = 0^p$

$s = 0^p 0^p \qquad \overbrace{0 \text{---} 0^p}^{p} \mid \underbrace{0 \text{---} 0^p}_{|z| = p}$

$|x| = p-2 \quad |y| = 2 \quad |z| = p$

$i = 2 \qquad \underbrace{0^{p-2}}_{x}\ \underbrace{00}_{y}\ \underbrace{00}_{y}\ \underbrace{0^{p}}_{z}$

$0^{2+p} = 0^{p} \quad p+2 \notin L$

$= \underbrace{0^{p+1}}_{w}\ \underbrace{0^{p+1}}_{w} \in L$