CSE331: Practice sheet-2 (CFG)

1. Give context-free grammars that generate the following languages

a) L= {w | w contains at least three 1's}.
= $S \rightarrow R1R1R1R$
$R \rightarrow 0R|1R|\varepsilon$

b) L={w | w starts and ends with the same symbol}.
= $S \rightarrow 0E0|1E1|\varepsilon$
$E \rightarrow 1E|0E|\varepsilon$

c) L= {w | the length of w is odd}
= $S \rightarrow X|0Z|1Z$
$Z \rightarrow 0XZ|1XZ|\varepsilon$
$X \rightarrow 0|1$

d) L={w | the length of w is odd and its middle is 0}
$S \rightarrow ZSZ|0$
$Z \rightarrow 0|1$

or

$S \rightarrow 0S0|0S1|1S0|1S1|0$

e) L={w | w = w^R , w is a palindrome}
= $S \rightarrow 0S0|1S1|\varepsilon|0|1$

f) L={w | w = w^R AND |w| is even, w is a palindrome}
= $S \rightarrow 0S0|1S1|\varepsilon$

g) The empty set
= $S \rightarrow \varepsilon$

h) L={w | w contains twice as many 1s as 0s} [for each 0s, two 1s]
= $S \rightarrow SS|\varepsilon|S011|0S11|01S1|011S$

i) L={w | w contains more as than bs}
= $S \rightarrow TaT$
$T \rightarrow TT|aTb|bTa|a|\varepsilon$

2. Give a context-free grammar for each of the following languages.

a) $L(G) = \{0^n1^m0^m \mid n, m \geq 0\}$ over the terminals {0,1}

$S \rightarrow AB$
$A \rightarrow 0A \mid \varepsilon$
$B \rightarrow 1B0 \mid \varepsilon$

b) $L(G) = \{0^n1^m0^m1^n \mid n, m \geq 0\}$ over the terminals {0,1}

$S \rightarrow 0S1 \mid A \mid \varepsilon$
$A \rightarrow 1A0 \mid \varepsilon$

c) $L(G) = \{ a^nb^mc^k \mid n, m, k \geq 0 \text{ and } n=2m+3k\}$ over $\Sigma=\{a,b,c\}$

here, $a^nb^mc^k = a^{2m+3k}b^mc^k = a^{3k}a^{2m}b^mc^k$

$S \rightarrow aaaSc \mid B \mid \varepsilon$
$B \rightarrow aaBb \mid \varepsilon$

d) $L(G) = \{ a^nb^m \mid 0 < n < m < 3n\}$. $\Sigma=\{a,b\}$

$S \rightarrow aZbb$
$Z \rightarrow aZb \mid aZbb \mid aZbbb \mid \varepsilon$

e) $L(G) = \{ a^ib^jc^k \mid i, j, k \geq 0 \text{ and } i=j \text{ or } i=k\}$. $\Sigma=\{a,b,c\}$

$S \rightarrow AC \mid S'$
$A \rightarrow aAb \mid \varepsilon$
$C \rightarrow cC \mid \varepsilon$
$S' \rightarrow aBc \mid B$
$A' \rightarrow aA' \mid \varepsilon$
$B \rightarrow bB \mid \varepsilon$

f) $L(G) = \{ a^ib^jc^k \mid i, j, k \geq 0 \text{ and } i=j \text{ or } j=k\}$. $\Sigma=\{a,b,c\}$

$S \rightarrow AC \mid S'$
$A \rightarrow aAb \mid C$
$C \rightarrow cC \mid \varepsilon$
$S' \rightarrow A'B$
$A' \rightarrow aA' \mid \varepsilon$
$B \rightarrow bBb \mid A'$

g) $L(G) = \{ a^nb^mc^md^{2n} \mid n \geq 0, m > 0\}$.

$S \rightarrow aSdd \mid B$
$B \rightarrow bBc \mid \varepsilon$

h) L(G) = set of all strings w over {a, b} such that w is not palindrome.

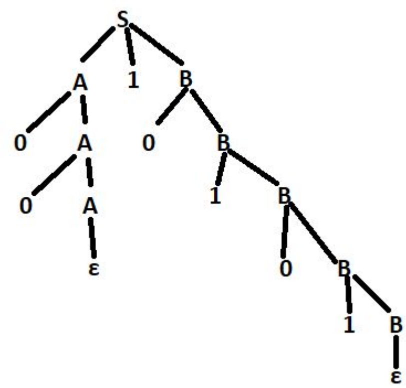
$Y \rightarrow aYa \mid bYb \mid aZb \mid bZa$
$Z \rightarrow aZ \mid bZ \mid \varepsilon$

3. Consider the following context-free grammar Σ={0,1}.

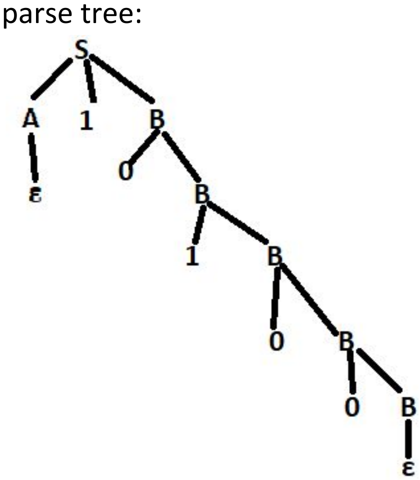
S → A 1 B
A → 0A | ε
B → 0B |1B| ε

Give a)leftmost and b)rightmost derivations and c)parse tree for the following strings.

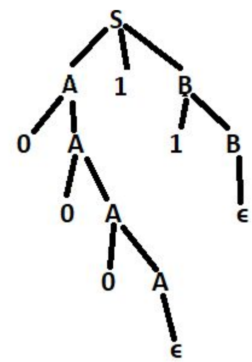
a) 0010101

leftmost derivation:	rightmost derivation:	Parse tree:
S→ A1B → 0A1B → 00A1B → 00 ε 1B → 001B → 0010B → 00101B → 001010B → 0010101B → 0010101 ε → 0010101	S→ A1B → A10B → A101B → A101 ε → 0A101 → 00A101 → 001A101 → 0010 ε 101 → 0010101	

b) 10100

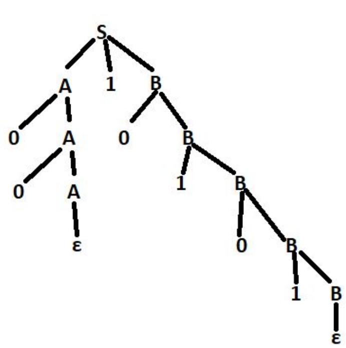
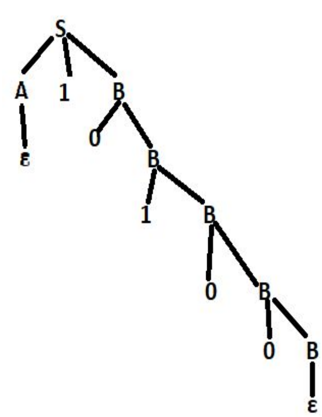
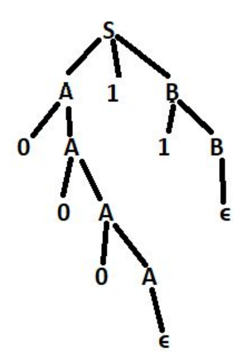
leftmost derivation:	rightmost derivation:	parse tree:
S → A1B → ϵ1B → 10B → 101B → 1010B → 10100ϵ → 10100	S→ A1B → A10B → A101B → A1010B → A10100 → ϵ10100 → 10100	

c) 00011

leftmost derivation:	rightmost derivation:	parse tree:
S→ A1B → 0A1B → 00A1B → 000A1B → 000ϵ1B → 0001B → 00011B → 00011ϵ → 00011	S→ A1B → A11B → A11ϵ → A11 → 0A11 → 00A11 → 000A11 → 000ϵ11 → 00011	

d) Explain/Prove why the grammar below is unambiguous.

= the grammar is unambiguous since only one parse tree is possible for every string.

parse tree for: 0010101	parse tree for: 10100	parse tree for: 00011
Here, we can draw only one parse tree for the string 0010101. If we try to draw something else other than this, we will get stuck at some point and we will not be able to complete the given string. The grammar is unambiguous since only one parse tree is possible for every string. 	Here, we can draw only one parse tree for the string 10100. If we try to draw something else other than this, we will get stuck at some point and we will not be able to complete the given string. The grammar is unambiguous since only one parse tree is possible for every string. 	Here, we can draw only one parse tree for the string 00011. If we try to draw something else other than this, we will get stuck at some point and we will not be able to complete the given string. The grammar is unambiguous since only one parse tree is possible for every string. 

4. Which language generates the grammar G given by the productions.

S→aSa | aBa

B→bB | b

= L(G)

={w|w over {a,b}* AND w has equal number of ‘a’ at start AND at end, AND at least 1 ‘b’ in the middle}

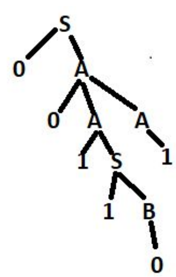
= L(G) = { $a^ib^ja^i$ | i, j ≥ 0}, Σ={a,b,c}

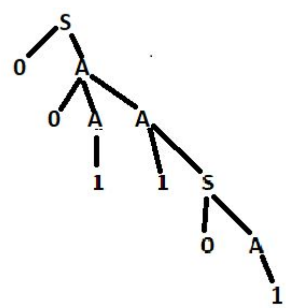
5. Explain/Prove why the grammar below is ambiguous.

S → 0A | 1B
A → 0AA | 1S | 1
B → 1BB | 0S | 0

<u>Solution:</u>

let’s assume, a string w= 001101





Since, two different trees are generated, the grammar is ambiguous.

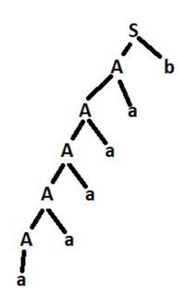
6. Given the following ambiguous context free grammar.

S → Ab | aaB
A → a | Aa
B → b

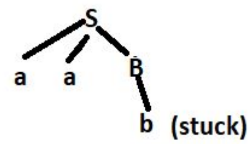
(a) Find leftmost and rightmost derivations for aaaaab, aabb, ab.

<u>Solution:</u>

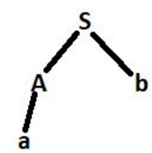
i) aaaaab

leftmost derivation:	rightmost derivation:	parse tree:
S→ Ab → Aab → Aaab → Aaaab → Aaaaab → aaaaab	S→ Ab → Aab → Aaab → Aaaab → Aaaaab → aaaaab	

ii) aabb

Leftmost derivation: S→Ab → Aab → aab(stuck)	rightmost derivation: S→ Ab → stuck	parse tree:  aabb does not belong to L(G)

iii) ab

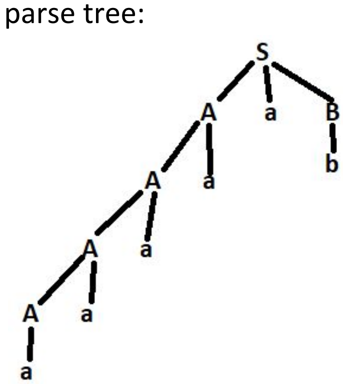
leftmost derivation: S→ Ab → ab	rightmost derivation: S→ Ab → ab	parse tree: 

(c) Is the above grammar ambiguous, give an example?
=the grammar is unambiguous since only one parse tree is possible for every string.

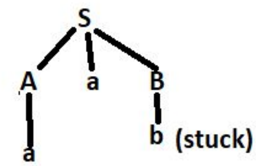
7. **Given the following ambiguous context free grammar.**

S → Ab | AaB
A → a | Aa
B → b

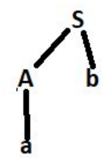
(a) Find leftmost and rightmost derivations & parse tree for aaaaab, aabb, ab.
i) aaaaab

leftmost derivation: S→ AaB → AaaB → AaaaB → AaaaaB → aaaaaB → aaaaab	rightmost derivation: S→ AaB → Aab → Aaab → Aaaab → Aaaaab → aaaaab	parse tree: 

ii) aabb

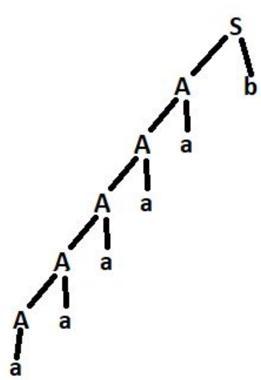
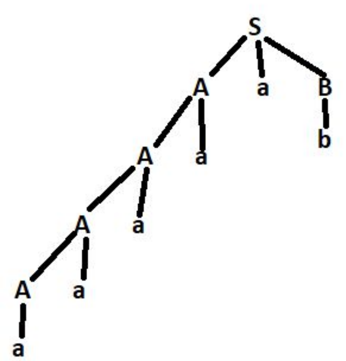
leftmost derivation: S→ AaB → aaB → aab(stuck)	rightmost derivation: S→ AaB → Aab → aab(stuck)	parse tree:  aabb does not belong to L(G)

iii)ab

leftmost derivation: S→ Ab → ab	rightmost derivation: S→ Ab → ab	parse tree: 

(c) Is the above grammar ambiguous, give an example? Find an equivalent <u>unambiguous context-free grammar.</u>

<u>Solution:</u>
For the first string aaaaab, we can find two parse trees.

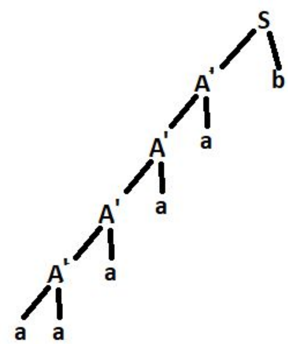
	

So the grammar is ambiguous.

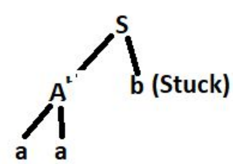
<u>Equivalent unambiguous CFG:</u>
S→ A’b|ab
A’ → A’a|aa

(d) Give the **unique** leftmost derivation and parse tree for the above strings generated from the unambiguous grammar above.

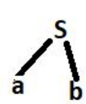
i) aaaaab

<table>
<tr>
<td>leftmost derivation:
S→ A’b
→ A’ab
→ A’aab
→ A’aaab
→ aaaaab</td>
<td>parse tree:
</td>
</tr>
</table>

ii)aabb

<table>
<tr>
<td>leftmost derivation:
S→ A’b
→ aab(stuck)</td>
<td>parse tree:

aabb does not belong to L(G)</td>
</tr>
</table>

iii)ab

<table>
<tr>
<td>leftmost derivation:
S→ ab</td>
<td>parse tree:
</td>
</tr>
</table>

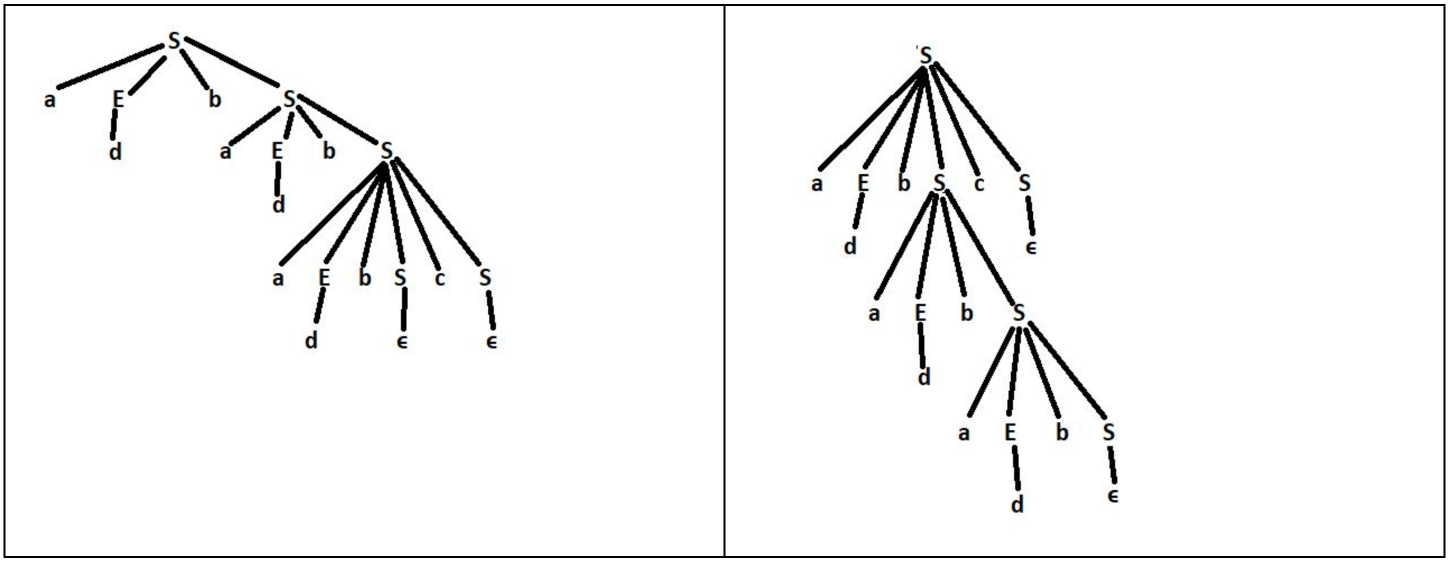
8. Show that the following grammar is ambiguous.

S → aEbS
S → aEbScS | ϵ
E → d

Solution:

let’s assume a string w= adbadbadbc

we can find two parse trees for this grammar:



so the grammar is ambiguous.

9. Consider the grammar with start symbol D, Σ = {c, a, b, . , 0, 1}

D → TL
T → c | Tc
L → L.V | V
V → a | b | 0 | 1| Va | Vb | V0 | V1

i) Derive ccab.01 using leftmost derivation	ii) Derive cabb0011.ab1 using rightmost derivation
D→ TL	D→ TL
→ TcL	→ TL.V
→ ccL	→ TL.V1
→ ccLV	→ TL.Vb1
→ ccV.V	→ TL.ab1
→ ccVb.V	→ TV.ab1
→ ccab.V	→ TV1.ab1
→ ccab.V1	→ TV11.ab1
→ ccab.01	→ TV011.ab1
	→ TV0011.ab1
	→ TVb0011.ab1
	→ TVbb0011.ab1
	→ Tabb0011.ab1
	→ cabb0011.ab1

10. Show that the following grammar is ambiguous. [hint use string aab]

S → aS | aSbS | ϵ

Give a)leftmost and b)rightmost derivations and c)parse tree for the following strings.

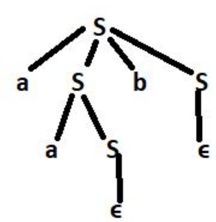
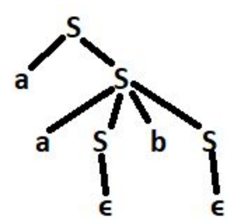
a) leftmost derivations for aab:

S→ aS	S→ aSbS
→ aaSbS	→ aaSbS
→ aaϵbS	→ aaϵbS
→ aabS	→ aabS
→ aabϵ	→ aabϵ
→ aab	→ aab

b) rightmost derivations for aab:

S→ aS	S→ aSbS
→ aaSbS	→ aSbϵ
→ aaSbϵ	→ aSb
→ aaSb	→ aaSb
→ aaϵb	→ aaϵb
→ aab	→ aab

c) parse trees:

Since, the grammar has more than one parse tree, <u>it is ambiguous.</u>

d)Remove ambiguity of the above grammar.

After removing ambiguity:

S→ aS|aTbS|ϵ
T→ aTbT|ϵ