Regular Expression

Basics

Regular operations and Operators

- **Union: $A \cup B = \{x \mid x \in A \text{ or } x \in B\}$.**
- **Concatenation: $A \circ B = \{xy \mid x \in A \text{ and } y \in B\}$.**
- **Kleene Star : $A^* = \{x_1 x_2 \ldots x_k \mid k \geq 0 \text{ and each } x_i \in A\}$**

Positive Closure

- **Denoted by the + sign as superscript**
- **$A^+ = A^* \cup \{\varepsilon\}$**
- **If $\sum = \{0, 1\}$, $\sum^+ = \{0,00,01,10,11,001,....\}$**
- **All possible combinations of the alphabets WITHOUT empty string**

Union operation

- **Denoted by the ∪ operator, or | sign, or + sign**
- **Creates the set combining the two languages**
- **(0 ∪ 1) = {0, 1}**
- **(0 ∪ 1)* = { ε, 0, 1, 00,01,10,11….}**
- **(0 | 1) = { 0, 1}**
- **(ab + a) = {ab, a}**
- **$(ab + a)^+$ = {a, aa, ab, aab, aba…}**

If a alphabet set is {0,1} we can write it in shorthand notation

- **{0,1} = (0 U 1) = (0 | 1) = $\sum$**

Concatenation operation

- **Denoted by the ◦ sign**
- **(01) = 0 followed by 1, mandatory**
- **(01)* = {ε, 01, 0101, 010101….}**

Precedence

- The star operation is done first,
- Then concatenation,
- Finally union
- parentheses change the usual order. (parentheses comes first)

Things to remember

- $1 . \varepsilon = \{ 1 \}$
- $1 \cup \varepsilon = \{ 1, \varepsilon \}$
- $\varepsilon = \{ \varepsilon \}$; [
- $\emptyset = \{ \}$
- $1 \cup \emptyset = \{1\}$
- $1 . \emptyset = \emptyset$

EXAMPLE 1.53

In the following instances, we assume that the alphabet Σ is $\{0,1\}$.

1. $0^*10^* = \{w \mid w \text{ contains a single } 1\}$.
2. $\Sigma^*1\Sigma^* = \{w \mid w \text{ has at least one } 1\}$.
3. $\Sigma^*001\Sigma^* = \{w \mid w \text{ contains the string } 001 \text{ as a substring}\}$.
4. $1^*(01^+)^* = \{w \mid \text{every } 0 \text{ in } w \text{ is followed by at least one } 1\}$.
5. $(\Sigma\Sigma)^* = \{w \mid w \text{ is a string of even length}\}$.[5]
6. $(\Sigma\Sigma\Sigma)^* = \{w \mid \text{the length of } w \text{ is a multiple of } 3\}$.
7. $01 \cup 10 = \{01, 10\}$.
8. $0\Sigma^*0 \cup 1\Sigma^*1 \cup 0 \cup 1 = \{w \mid w \text{ starts and ends with the same symbol}\}$.
9. $(0 \cup \varepsilon)1^* = 01^* \cup 1^*$.
   The expression $0 \cup \varepsilon$ describes the language $\{0, \varepsilon\}$, so the concatenation operation adds either 0 or ε before every string in 1^*.
10. $(0 \cup \varepsilon)(1 \cup \varepsilon) = \{\varepsilon, 0, 1, 01\}$.
11. $1^*\emptyset = \emptyset$.
    Concatenating the empty set to any set yields the empty set.
12. $\emptyset^* = \{\varepsilon\}$.
    The star operation puts together any number of strings from the language to get a string in the result. If the language is empty, the star operation can put together 0 strings, giving only the empty string.