$$C = KM \pmod n$$

$$M = K^{-1} C \pmod n$$

4) $AA^{-1} = E,\ \Delta A \neq 0$

$$K = \begin{pmatrix} 8 & 4 & 3 \\ 11 & 7 & 15 \\ 5 & 23 & 13 \end{pmatrix} \quad M = \begin{pmatrix} 7 \\ 3 \\ 8 \end{pmatrix}$$

$n = 29$ n-простое число

$$C = \begin{pmatrix} 8 & 4 & 3 \\ 11 & 7 & 15 \\ 5 & 23 & 13 \end{pmatrix} \begin{pmatrix} 7 \\ 3 \\ 8 \end{pmatrix} \pmod{29}$$

$$C = \begin{pmatrix} 8\cdot 7 + 4\cdot 3 + 3\cdot 8 \\ 11\cdot 7 + 7\cdot 3 + 15\cdot 8 \\ 5\cdot 7 + 23\cdot 3 + 13\cdot 8 \end{pmatrix}$$

$$\begin{pmatrix} 92 \\ 218 \\ 208 \end{pmatrix} \pmod{29} = \begin{pmatrix} 5 \\ 15 \\ 5 \end{pmatrix} = C$$

$$\Delta K = a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13}$$

$$A_{ij} = (-1)^{i+j} \cdot \Delta M_{ij}$$

$$A_{11} = (-1)^{1+1}(7\cdot 13 - 23\cdot 15)$$

$$A_{12} = (-1)^{1+2}(11\cdot 13 - 5\cdot 15)$$

$$A_{13} = (-1)^{1+3}(11\cdot 23 - 5\cdot 7)$$

$A_{11} = -254 \quad A_{12} = -68 \quad A_{13} =$

$$\Delta K = 8\cdot(-254) + 4\cdot(-68) + 3\cdot 218 = -1650 \equiv -1650 \pmod{29} = 3$$

$$A^{-1} = \frac{1}{\Delta A} A^*$$

$$A^* = \begin{pmatrix} A_{11} & A_{21} & \dots & A_{n1} \\ A_{12} & A_{22} & \dots & A_{n2} \\ \dots & \dots & \dots & \dots \\ A_{1n} & A_{2n} & \dots & A_{nn} \end{pmatrix}$$

$$A_{21} = (-1)^{2+1}(4\cdot 13 - 23\cdot 3)$$

$$A_{22} = (-1)^{2+2}(8\cdot 13 - 5\cdot 3)$$

$$A_{23} = (-1)^{2+3}(8\cdot 23 - 5\cdot 4)$$

$A_{21} = 17 \quad A_{22} = 89 \quad A_{23} = -164$

$$A_{31} = (-1)^{3+1}\cdot(4\cdot 15 - 7\cdot 3)$$

$$A_{32} = (-1)^{3+2}\cdot(8\cdot 15 - 11\cdot 3)$$

$$A_{33} = (-1)^{3+3}\cdot(8\cdot 7 - 11\cdot 4)$$

$A_{31} = 39 \quad A_{32} = -87 \quad A_{33} = 12$

$$A^{-1} = \frac{1}{\det A} A^*$$

$$A^* = \begin{pmatrix} -254 & 17 & 39 \\ -68 & 89 & -87 \\ 218 & -164 & 12 \end{pmatrix}$$

$$\begin{pmatrix} 7 & 17 & 10 \\ 19 & 2 & 0 \\ 15 & 10 & 12 \end{pmatrix} \pmod{29}$$

$$K = \begin{pmatrix} 8 & 4 & 3 \\ 11 & 7 & 15 \\ 5 & 23 & 13 \end{pmatrix} \quad 7\cdot 13 - 23\cdot 15$$

$$\det A \cdot \det A^{-1} \equiv 1 \pmod n$$

$3 \cdot x \equiv 1 \pmod{29}$

$3 \cdot 10 \equiv 30 \pmod{29}$

$30 - 29 = 1$

3 ** (29-2) % 29 = 10

$$A^{-1} \equiv \det A^{-1} \cdot A^* \pmod n$$

$$A^{-1} = 10 \cdot \begin{pmatrix} 7 & 17 & 10 \\ 19 & 2 & 0 \\ 15 & 10 & 12 \end{pmatrix} =$$

$$= \begin{pmatrix} 70 & 170 & 100 \\ 190 & 20 & 0 \\ 150 & 100 & 120 \end{pmatrix} \equiv \begin{pmatrix} 12 & 25 & 13 \\ 16 & 20 & 0 \\ 5 & 13 & 4 \end{pmatrix} \pmod{29}$$

$$M = K^{-1} C$$

$$\begin{pmatrix} 12 & 25 & 13 \\ 16 & 20 & 0 \\ 5 & 13 & 4 \end{pmatrix} \begin{pmatrix} 5 \\ 15 \\ 5 \end{pmatrix} =$$

$$= \begin{pmatrix} 12\cdot 5 + 25\cdot 15 + 13\cdot 5 \\ 16\cdot 5 + 20\cdot 15 + 0\cdot 5 \\ 5\cdot 5 + 13\cdot 15 + 4\cdot 5 \end{pmatrix} = \begin{pmatrix} 500 \\ 380 \\ 240 \end{pmatrix}$$

$$\begin{pmatrix} 500 \\ 380 \\ 240 \end{pmatrix} \bmod 29 = \begin{pmatrix} 7 \\ 3 \\ 8 \end{pmatrix} = M$$