MA026IU
PROBABILITY, STATISTIC AND RANDOM PROCESS

Part 2C
Tests of Hypotheses for a Single Sample

Example of Hypothesis testing: Suppose that an engineer is designing an aircrew escape system that consists of an ejection seat and a rocket motor that powers the seat. The rocket motor contains a propellant, and for the ejection seat to function properly, the propellant should have a mean burning rate of 50 cm/sec. If the burning rate is too low, the ejection seat may not function properly,leading to an unsafe ejection and possible injury of the pilot.

So the practical engineering question that must be answered is: Does the mean burning rate of the propellant **equal 50 cm/sec, or is it some other value** (either higher or lower)? This type of question can be answered using a statistical technique called **hypothesis testing**.

Hypothesis Testing

Statistical Hypothesis

A **statistical hypothesis** is a statement about the parameters of one or more populations.

Two-sided Alternative Hypothesis

Let H_0: $\mu = 50$ centimeters per second and H_1: $\mu \neq 50$ centimeters per second

- The statement H_0: $\mu = 50$ is called the **null hypothesis.**
- The statement H_1: $\mu \neq 50$ is called the **alternative hypothesis.**

One-sided Alternative Hypothesis

- H_0: $\mu = 50$ centimeters per second $\qquad$ H_0: $\mu = 50$ centimeters per second

or

- H_1: $\mu < 50$ centimeters per second $\qquad$ H_1: $\mu > 50$ centimeters per second

Hypothesis Testing

Test of a Hypothesis

- A procedure leading to a decision about a particular hypothesis

- Hypothesis-testing procedures rely on using the information in a **random sample from the population of interest**

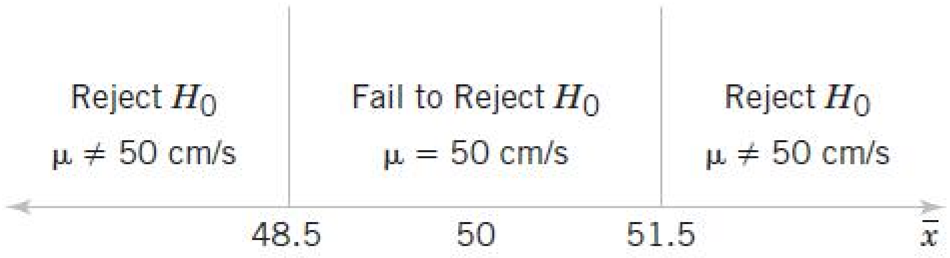
- If this information is *consistent* with the hypothesis, then we will conclude that the hypothesis is **true**; if this information is *inconsistent* with the hypothesis, we will conclude that the hypothesis is **false.**

Tests of Statistical Hypotheses

$$H_0\colon \mu = 50 \text{ centimeters per second}$$
$$H_1\colon \mu \neq 50 \text{ centimeters per second}$$

FIGURE 9.1

Decision criteria for testing H_0: $\mu = 50$ centimeters per second versus H_1: $\mu \neq 50$ centimeters per second.



Decisions in Hypothesis Testing

TABLE 9.1 Decisions in Hypothesis Testing

Decision	H_0 Is True	H_0 Is False
Fail to reject H_0	No error	Type II error
Reject H_0	Type I error	No error

Probability of Type I Error

$$\alpha = P(\text{type I error}) = P(\text{reject } H_0 \text{ when } H_0 \text{ is true}) \tag{9.3}$$

Probability of Type II Error

$$\beta = P(\text{type II error}) = P(\text{fail to reject } H_0 \text{ when } H_0 \text{ is false}) \tag{9.4}$$

Sometimes the type I error probability is called the **significance level,** or the α-**error,** or the **size** of the test.

Computing the Probability of Type I Error

$$\alpha = P(\overline{X} < 48.5 \text{ when } \mu = 50) + P(\overline{X} > 51.5 \text{ when } \mu = 50)$$

The z-values that correspond to the critical values 48.5 and 51.5 are

$$z_1 = \frac{48.5 - 50}{0.79} = -1.90 \quad \text{and} \quad z_2 = \frac{51.5 - 50}{0.79} = 1.90$$

Therefore,

$$\alpha = P(z < -1.90) + P(z > 1.90) = 0.0287 + 0.0287 = 0.0574$$

This implies that 5.74% of all random samples would lead to rejection of the hypothesis $H_0: \mu = 50$.

Computing the Probability of Type II Error

$$\boxed{\beta = P(48.5 \le \overline{X} \le 51.5 \text{ when } \mu = 52)}$$

The z-values corresponding to 48.5 and 51.5 when $\mu = 52$ are

$$z_1 = \frac{48.5 - 52}{0.79} = -4.43 \qquad \text{and} \qquad z_2 = \frac{51.5 - 52}{0.79} = -0.63$$

Therefore,

$$\beta = P(-4.43 \le Z \le -0.63) = P(Z \le -0.63) - P(Z \le -4.43)$$
$$= 0.2643 - 0.0000 = 0.2643$$

This implies that the probability that we will fail to reject the false null hypothesis is 0.2643.

One-Sided and Two-Sided Hypotheses

Two-Sided Test

H_0: $\mu = \mu_0$
H_1: $\mu \neq \mu_0$

One-Sided Tests

H_0: $\mu = \mu_0$
H_1: $\mu > \mu_0$

or

H_0: $\mu = \mu_0$
H_1: $\mu < \mu_0$

In formulating one-sided alternative hypotheses, we should remember that rejecting H_0 is always a strong conclusion. Consequently, we should put the statement about which it is important to make a strong conclusion in the alternative hypothesis. In real-world problems, this will often depend on our point of view and experience with the situation.

P-value

P-Value

The ***P*-value** is the smallest level of significance that would lead to rejection of the null hypothesis H_0 with the given data.

P-value is the **observed significance level.**

P-values in Hypothesis Tests

- Consider the two-sided hypothesis test $H_0\colon \mu = 50$ against $H_1\colon \mu \neq 50$ with n 16 and $\sigma = 2.5$. Suppose that the observed sample mean is $\bar{x} = 51.3$ centimeters per second.

- The *P*-value of the test is the probability above 51.3 plus the probability below 48.7.

$$
\begin{aligned}
P\text{-value} &= 1 - P\left(48.7 < \overline{X} < 51.3\right) \\
&= 1 - P\left(\frac{48.7 - 50}{2.5/\sqrt{16}} < Z < \frac{51.3 - 50}{2.5/\sqrt{16}}\right) \\
&= 1 - P(-2.08 < Z < 2.08) \\
&= 1 - 0.962 = 0.038
\end{aligned}
$$

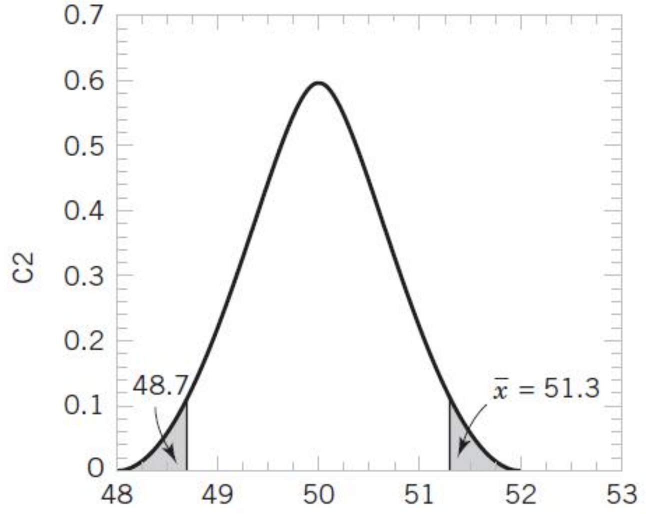


FIGURE 9.6

***P*-value is the area of the shaded region when $\bar{x} = 51.3$.**

Connection Between Hypothesis Tests and Confidence Intervals

- A close relationship exists between the test of a hypothesis for θ, and the confidence interval for θ.

- If $[l,\ u]$ is a $100(1-\alpha)\%$ confidence interval for the parameter θ, the test of size α of the hypothesis

$$H_0\text{: } \theta = \theta_0$$
$$H_1\text{: } \theta \neq \theta_0$$

will lead to rejection of H_0 if and only if θ_0 is **not** in the $100(1-\alpha)\%\ CI\ [l,\ u]$.

General Procedure for Hypothesis Tests

1. **Parameter of interest:** From the problem context, identify the parameter of interest.
2. **Null hypothesis, H_0:** State the null hypothesis, H_0.
3. **Alternative hypothesis, H_1:** Specify an appropriate alternative hypothesis, H_1.
4. **Test statistic:** Determine an appropriate test statistic.
5. **Reject H_0 if:** State the rejection criteria for the null hypothesis.
6. **Computations:** Compute any necessary sample quantities, substitute these into the equation for the test statistic, and compute that value.
7. **Draw conclusions:** Decide whether or not H_0 should be rejected and report that in the problem context.

Hypothesis Tests on the Mean

Suppose that we wish to test the hypotheses

$$H_0: \mu = \mu_0 \qquad H_1: \mu \neq \mu_0$$

Test Statistic

$$Z_0 = \frac{\overline{X} - \mu_0}{\sigma/\sqrt{n}} \tag{9.8}$$

Summary of Tests on the Mean, Variance Known

Testing Hypotheses on the Mean, Variance Known (Z-Tests)

Null hypothesis: $H_0: \mu = \mu_0$

Test statistic: $Z_0 = \dfrac{\overline{X} - \mu_0}{\sigma/\sqrt{n}}$

Alternative Hypotheses	*P*-Value	Rejection Criterion for Fixed-Level Tests
$H_1: \mu \neq \mu_0$	Probability above $\lvert z_0 \rvert$ and probability below $-\lvert z_0 \rvert$, $P = 2[1 - \Phi(\lvert z_0 \rvert)]$	$z_0 > z_{\alpha/2}$ or $z_0 < -z_{\alpha/2}$
$H_1: \mu > \mu_0$	Probability above z_0, $P = 1 - \Phi(z_0)$	$z_0 > z_\alpha$
$H_1: \mu < \mu_0$	Probability below z_0, $P = \Phi(z_0)$	$z_0 < -z_\alpha$

The *P*-values and critical regions for these situations are shown in Figures 9.10 and 9.11.

Example 2a | Propellant Burning Rate

- Air crew escape systems are powered by a solid propellant. The burning rate of this propellant is an important product characteristic. Specifications require that the mean burning rate must be 50 centimeters per second and the standard deviation is $\sigma = 2$ centimeters per second. The significance level of $\alpha = 0.05$ and a random sample of $n =$ 25 has a sample average burning rate of centimeters per second. Draw conclusions.
- The seven-step procedure is
  1. **Parameter of interest:** The parameter of interest is μ, the mean burning rate.
  2. **Null hypothesis:** H_0: $\mu = 50$ centimeters per second
  3. **Alternative hypothesis:** H_1: $\mu \neq 50$ centimeters per second

Example 2b | Propellant Burning Rate

4. **Test statistic:** The test statistic is $z_0 = \dfrac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$

5. **Reject H_0 if:** Reject H_0 if the P-value is less than 0.05. The boundaries of the critical region would be $z_{0.025} = 1.96\ and \quad z_{0.025} = -1.96$.

6. **Computations:** Since $\bar{x} = 51.3$ and $\sigma = 2$,

$$z_0 = \frac{51.3 - 50}{2/\sqrt{25}} = 3.25$$

7. **Conclusion:** Since $z_0 = 3.25$ and the p-value is $= 2[1 - \Phi(3.25)] = 0.0012$, we reject H_0: $\mu = 50$ at the 0.05 level of significance.

Practical Interpretation: The mean burning rate differs from 50 centimeters per second, based on a sample of 25 measurements.

Large-Sample Test

- A test procedure for the null hypothesis H_0: $\mu = \mu_0$ assuming that the population is normally distributed and that σ^2 known is developed. In most practical situations, σ^2 will be unknown. Even, we may not be certain that the population is normally distributed.
- In such cases, if n is large (say, $n > 40$) the sample standard deviation s can be substituted for σ in the test procedures. Thus, while we have given a test for the mean of a normal distribution with known σ^2, it can be easily converted into a *large-sample test procedure for unknown* σ^2 regardless of the form of the distribution of the population.
- Exact treatment of the case where the population is normal, σ^2 is unknown, and n is small involves use of the t distribution.

Hypothesis Tests on the Mean

Summary for the One-Sample *t*-test

Testing Hypotheses on the Mean of a Normal Distribution, Variance Unknown

Null hypothesis: $H_0: \mu = \mu_0$

Test statistic: $T_0 = \dfrac{\overline{X} - \mu_0}{S/\sqrt{n}}$

Alternative Hypotheses	*P*-Value	Rejection Criterion for Fixed-Level Tests
$H_1: \mu \neq \mu_0$	Probability above $\lvert t_0 \rvert$ and probability below $-\lvert t_0 \rvert$	$t_0 > t_{\alpha/2,n-1}$ or $t_0 < -t_{\alpha/2,n-1}$
$H_1: \mu > \mu_0$	Probability above t_0	$t_0 > t_{\alpha,n-1}$
$H_1: \mu < \mu_0$	Probability below t_0	$t_0 < -t_{\alpha,n-1}$

The calculations of the *P*-values and the locations of the critical regions for these situations are shown in Figures 9.13 and 9.15, respectively.

Example 3a | Golf Club Design

- An experiment was performed in which 15 drivers produced by a particular club maker were selected at random and their coefficients of restitution measured. It is of interest to determine if there is evidence (with $\alpha = 0.05$) to support a claim that the mean coefficient of restitution exceeds 0.82.
- The observations are:

0.8411	0.8191	0.8182	0.8125	0.8750
0.8580	0.8532	0.8483	0.8276	0.7983
0.8042	0.8730	0.8282	0.8359	0.8660

- The sample mean and sample standard deviation are $\bar{x} = 0.83725$ and s = 0.02456. The objective of the experimenter is to demonstrate that the mean coefficient of restitution exceeds 0.82, hence a one-sided alternative hypothesis is appropriate.

The seven-step procedure for hypothesis testing is as follows:

1. Parameter of interest: The parameter of interest is the mean coefficient of restitution, μ.

2. Null hypothesis: H_0: $\mu = 0.82$

3. Alternative hypothesis: H_1: $\mu > 0.82$

Example 3b | Golf Club Design

4. **Test Statistic:** The test statistic is

$$t_0 = \frac{\overline{x} - \mu_0}{s/\sqrt{n}}$$

5. **Reject H_0 if:** Reject H_0 if the P-value is less than 0.05.

6. **Computations:** Since $\overline{x} = 0.83725$, $s = 0.02456$, $\mu = 0.82$, and $n = 15$, we have

$$t_0 = \frac{0.83725 - 0.82}{0.02456/\sqrt{15}} = 2.72$$

7. **Conclusions:** From Appendix A Table II, for a t distribution with 14 degrees of freedom, $t_0 = 2.72$ falls between two values: 2.624, for which $\alpha = 0.01$, and 2.977, for which $\alpha = 0.005$. Since, this is a one-tailed test the P-value is between those two values, that is, $0.005 < P < 0.01$. Therefore, since $P < 0.05$, we reject H_0 and conclude that the mean coefficient of restitution exceeds 0.82.

Practical Interpretation: There is strong evidence to conclude that the mean coefficient of restitution exceeds 0.82.

Hypothesis Tests on the Variance

Tests on the Variance of a Normal Distribution

Null hypothesis: $H_0\colon \sigma^2 = \sigma_0^2$

Test statistic: $$\chi_0^2 = \frac{(n-1)S^2}{\sigma_0^2}$$

Alternative Hypothesis	Rejection Criteria
$H_1\colon \sigma^2 \neq \sigma_0^2$	$\chi_0^2 > \chi^2_{\alpha/2,n-1}$ or $\chi_0^2 < \chi^2_{1-\alpha/2,n-1}$
$H_1\colon \sigma^2 > \sigma_0^2$	$\chi_0^2 > \chi^2_{\alpha,n-1}$
$H_1\colon \sigma^2 < \sigma_0^2$	$\chi_0^2 < \chi^2_{1-\alpha,n-1}$

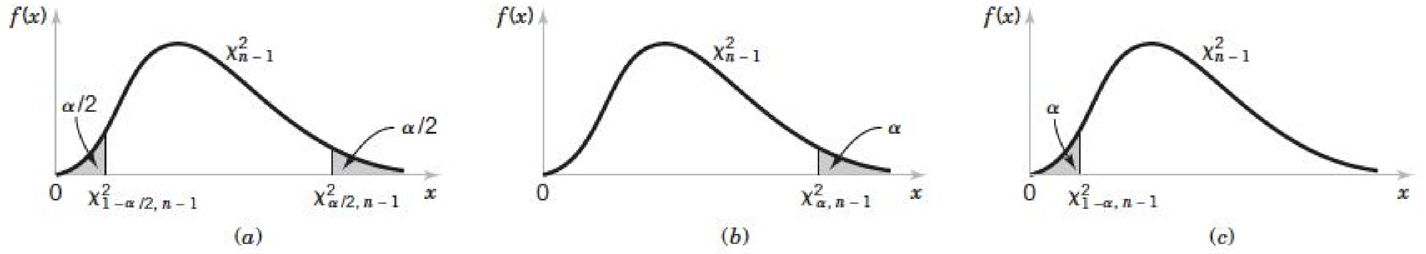


FIGURE 9.17

Reference distribution for the test of $H_0\colon \sigma^2 = \sigma_0^2$ with critical region values for (a) $H_1\colon \sigma^2 \neq \sigma_0^2$, (b) $H_1\colon \sigma^2 > \sigma_0^2$, and (c) $H_1\colon \sigma^2 < \sigma_0^2$.

Example 4 | Automated Filling

- An automated filling machine is used to fill bottles with liquid detergent. A random sample of 20 bottles results in a sample variance of fill volume of $s^2 = 0.0153$ (fluid ounces)^2. If the variance of fill volume exceeds 0.01 (fluid ounces)^2, an unacceptable proportion of bottles will be underfilled or overfilled. Is there evidence in the sample data to suggest that the manufacturer has a problem with underfilled or overfilled bottles? Use $\alpha = 0.05$, and assume that fill volume has a normal distribution.
- Using the seven-step procedure results in the following:

1. **Parameter of interest:** The parameter of interest is the population variance σ^2.
2. **Null hypothesis:** $H_0: \sigma^2 = 0.01$
3. **Alternative hypothesis:** $H_0: \sigma^2 > 0.01$
4. **Test statistic:** The test statistic is $\chi_0^2 = \dfrac{(n-1)s^2}{\sigma_0^2}$
5. **Reject H_0 if:** Use $\alpha = 0.05$, and reject H_0 if $\chi_0^2 > \chi_{0.05,19}^2 = 30.14$
6. **Computations:** $\chi_0^2 = \dfrac{19(0.0153)}{0.01} = 29.07$
7. **Conclusions:** Because $\chi_0^2 = 29.07 < \chi_{0.05,19}^2 = 30.14$, we conclude that there is no strong evidence that the variance of fill volume exceeds 0.01 (fluid ounces)^2. So there is no strong evidence of a problem with incorrectly filled bottles.

Large-Sample Tests on a Proportion

Summary of Approximate Tests on a Binomial Proportion

Testing Hypotheses on a Binomial Proportion

Null hypotheses: $H_0: p = p_0$

Test statistic: $$Z_0 = \frac{X - np_0}{\sqrt{np_0(1 - p_0)}}$$

Alternative Hypotheses	*P*-Value	Rejection Criterion for Fixed-Level Tests
$H_1: p \neq p_0$	Probability above $\lvert z_0 \rvert$ and probability below $-\lvert z_0 \rvert$, $P = 2[1 - \Phi(z_0)]$	$z_0 > z_{\alpha/2}$ or $z_0 < -z_{\alpha/2}$
$H_1: p > p_0$	Probability above z_0, $P = 1 - \Phi(z_0)$	$z_0 > z_\alpha$
$H_1: p < p_0$	Probability below z_0, $P = \Phi(z_0)$	$z_0 < -z_\alpha$

Example 5 | Automobile Engine Controller

A semiconductor manufacturer produces controllers used in automobile engine applications. The customer requires that the process fallout or fraction defective at a critical manufacturing step not exceed 0.05 and that the manufacturer demonstrate process capability at this level of quality using $\alpha = 0.05$. The semiconductor manufacturer takes a random sample of 200 devices and finds that four of them are defective. Can the manufacturer demonstrate process capability for the customer?

1. **Parameter of Interest:** The parameter of interest is the process fraction defective *p.*
2. **Null hypothesis:** H_0:$p = 0.05$
3. **Alternative hypothesis:** H_1: $p < 0.05$
4. **Test Statistic:** $z_0 = \dfrac{x - np_0}{\sqrt{np_0(1 - p_0)}}$ where $x = 4$, $n = 200$, and $p_0 = 0.05$.
5. **Reject H_0 if:** Reject H_0: $p = 0.05$ if the p-value is less than 0.05.
6. **Computations:** The test statistic is $z_0 = \dfrac{4 - 200(0.05)}{\sqrt{200(0.05)(0.95)}} = -1.95$
7. **Conclusions:** Since $z_0 = -1.95$, the *P*-value is $\Phi(-1.95) = 0.0256$, so we reject H_0 and conclude that the process fraction defective *p* is less than 0.05.

Large-Sample Tests on a Proportion

Another form of the test statistic Z_0 is

$$Z_0 = \frac{X/n - p_0}{\sqrt{p_0(1-p_0)/n}} \quad \text{or} \quad Z_0 = \frac{\widehat{P} - p_0}{\sqrt{p_0(1-p_0)/n}}$$

Testing for Goodness of Fit

- Based on chi-square distribution
- Requires a random sample of size *n* from the population whose probability distribution is unknown
- Let O_i be the observed frequency in the ith class interval.
- Let E_i be the expected frequency in the ith class interval.

Goodness-of-Fit Test Statistic

$$\chi_0^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i} \tag{9.47}$$

Example 6 | Printed Circuit Board Defects-Poisson Distribution

- The number of defects in printed circuit boards is hypothesized to follow a Poisson distribution. A random sample of *n* = 60 printed boards has been collected, and the following number of defects observed.

Number of Defects	Observed Frequency
0	32
1	15
2	9
3	4

- The estimate of the mean number of defects per board is the sample average, (32·0 + 15·1 + 9·2 + 4·3)/60 = 0.75. From the Poisson distribution with parameter 0.75, we may compute p_i, the theoretical, hypothesized probability associated with the *i*th class interval. We may find the p_i as follows:

$$p_1 = P(X = 0) = \frac{e^{-0.75}(0.75)^0}{0!} = 0.472$$

$$p_2 = P(X = 1) = \frac{e^{-0.75}(0.75)^1}{1!} = 0.354$$

$$p_3 = P(X = 2) = \frac{e^{-0.75}(0.75)^2}{2!} = 0.133$$

$$p_4 = P(X \geq 3) = 1 - (p_1 + p_2 + p_3) = 0.041$$

Example 6 | Printed Circuit Board Defects-Poisson Distribution

- The expected frequencies are computed by multiplying the sample size $n = 60$ times the probabilities p_i. That is, $E_i = np_i$. The expected frequencies follow:

Number of Defects	Probability	Expected Frequency
0	0.472	28.32
1	0.354	21.24
2	0.133	7.98
3 (or more)	0.041	2.46

- Because the expected frequency in the last cell is less than 3, we combine the last two cells.

Number of Defects	Observed Frequency	Expected Frequency
0	32	28.32
1	15	21.24
2 (or more)	13	10.44

Example 8 | Printed Circuit Board Defects-Poisson Distribution

The seven-step hypothesis-testing procedure may now be applied, using α = 0.05, as follows:

1. **Parameter of interest:** The variable of interest is the form of the distribution of defects in printed circuit boards.
2. **Null hypothesis:** H_0: The form of the distribution of defects is Poisson.
3. **Alternative hypothesis:** H_1: The form of the distribution of defects is not Poisson.
4. **Test statistic:** The test statistic is $\chi_0^2 = \sum_{i=1}^{k} \frac{(o_i - E_i)^2}{E_i}$
5. **Reject H_0 if:** Reject H_0 if the *P*-value is less than 0.05.
6. **Computations:** $$\chi_0^2 = \frac{(32 - 28.32)^2}{28.32} + \frac{(15 - 21.24)^2}{21.24} + \frac{(13 - 10.44)^2}{10.44} = 2.94$$
7. **Conclusions:** We find from Appendix Table III that $\chi^2_{0.10,1} = 2.71$ and $\chi^2_{0.05,1} = 3.84$ Because $\chi_0^2 = 2.94$ lies between these values, we conclude that the *P*-value is between 0.05 and 0.10. Therefore, since the *P*-value exceeds 0.05 we are unable to reject the null hypothesis that the distribution of defects in printed circuit boards is Poisson. The exact *P*-value computed from Minitab is 0.0864.

Contingency Table Tests

TABLE 9.2 An $r \times c$ Contingency Table

		Columns			
		1	2	$\cdots$	c
	1	O_{11}	O_{12}	$\cdots$	O_{1c}
Rows	2	O_{21}	O_{22}	$\cdots$	O_{2c}
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
	r	O_{r1}	O_{r2}	$\cdots$	O_{rc}

Assuming independence, the estimators of u_i and v_j are:

$$\hat{u}_i = \frac{1}{n} \sum_{j=1}^{c} O_{ij}$$

$$\hat{v}_j = \frac{1}{n} \sum_{i=1}^{r} O_{ij}$$

Contingency Table Tests

Therefore, the expected frequency of each cell is

$$E_{ij} = n\hat{u}_i \hat{v}_j = \frac{1}{n} \sum_{j=1}^{c} O_{ij} \sum_{i=1}^{r} O_{ij}$$

Then, for large *n*, the statistic

$$\chi_0^2 = \sum_{i=1}^{r} \sum_{j=1}^{c} \frac{(O_{ij} - E_{ij})^2}{E_{ij}}$$

has an approximate chi-square distribution with $(r - 1)(c - 1)$ degrees of freedom if the null hypothesis is true. We should reject the null hypothesis if the value of the test statistic χ_0^2 is too large. The *P*-value would be calculated as the probability beyond χ_0^2 on the $\chi^2_{(r-1)(c-1)}$ distribution, or $P = p\left(\chi^2_{(r-1)(c-1)} > \chi_0^2\right)$. For a fixed-level test, we would reject the hypothesis of independence if the observed value of the test statistic χ_0^2 exceeded $\chi^2_{\alpha,(r-1)(c-1)}$.

Example 8 | Health Insurance Plan Preference

A company has to choose among three health insurance plans. Management wishes to know whether the preference for plans is independent of job classification and wants to use α= 0.05. The opinions of a random sample of 500 employees are shown in Table 9.3.

TABLE 9.3 **Observed Data for Example 9.14**

	Health Insurance Plan			
Job Classification	1	2	3	Totals
Salaried workers	160	140	40	340
Hourly workers	40	60	60	160
Totals	200	200	100	500

To find the expected frequencies, we must first compute

$$\hat{u}_2 = (160/500) = 0.32, \quad \hat{v}_1 = (200/500) = 0.40, \quad \hat{v}_2 = (200/500) = 0.40, \quad \text{and}$$

$$\hat{u}_1 = (340/500) = 0.68, \quad \hat{v}_3 = (100/500) = 0.20$$

Example 9 | Health Insurance Plan Preference

- The expected frequencies may now be computed from Equation 9.49.
- For example, the expected number of salaried workers favoring health insurance plan 1 is

$$E_{11} = n\hat{u}_1\hat{v}_1 = 500\left(0.68\right)\left(0.40\right) = 136$$

- The expected frequencies are shown in the table below

TABLE 9.4 **Expected Frequencies for Example 9.14**

	Health Insurance Plan			
Job Classification	1	2	3	Totals
Salaried workers	136	136	68	340
Hourly workers	64	64	32	160
Totals	200	200	100	500

Example 10 | Health Insurance Plan Preference

The seven-step hypothesis-testing procedure may now be applied to this problem.

1. Parameter of Interest: The variable of interest is employee preference among health insurance plans.

2. Null hypothesis: H_0: Preference is independent of salaried versus hourly job classification.

3. Alternative hypothesis: H_1: Preference is not independent of salaried versus hourly job classification.

4. Test statistic: The test statistic is $\chi_0^2 = \sum_{i=1}^{r} \sum_{j=1}^{c} \frac{(o_{ij} - E_{ij})^2}{E_{ij}}$

5. Reject H_0 if: We will use a fixed-significance level test with a = 0.05. Therefore, since $r = 2$ and $c = 3$, the degrees of freedom for chi-square are $(r – 1)(c – 1) = (1)(2) = 2$, and we would reject H_0 if $\chi_0^2 = 49.63 > \chi_{0.05,2}^2 = 5.99$

Example 11 | Health Insurance Plan Preference

5. **Reject H_0 if:** We will use a fixed-significance level test with a = 0.05. Therefore, since r = 2 and c = 3, the degrees of freedom for chi-square are (r – 1)(c – 1) = (1)(2) = 2, and we would reject H_0 if
6. **Computations:**

$$\chi_0^2 = \sum_{i=1}^{2}\sum_{j=1}^{3}\frac{(o_{ij} - E_{ij})^2}{E_{ij}}$$

$$= \frac{(160 - 136)^2}{136} + \frac{(140 - 136)^2}{136} + \frac{(40 - 68)^2}{68} + \frac{(40 - 64)^2}{64} + \frac{(60 - 64)^2}{64} + \frac{(60 - 32)^2}{32}$$

$$= 49.63$$

7. **Conclusions:** Since $\chi_0^2 = 49.63 > \chi_{0.05,2}^2 = 5.99$, we reject the hypothesis of independence and conclude that the preference for health insurance plans is not independent of job classification. The P-value for $\chi_0^2 = 49.63$ $P = 1.671 \times 10^{-11}$ (This value was computed from computer software).