# Dividimos y No Conquistamos (D&!C)

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}

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### 1 Template

### 1.1 C++ Template

```
#include <bits/stdc++.h>
   using namespace std;
   #define LOCAL
   #define L(i, j, n) for (int i = (j); i < (int)n; i ++)
   #define LI(i, j, n) for (int i = (j); i \le (int)n; i \leftrightarrow (int)n
   #define R(i, j, n) for (int i = (j); i > (int)n; i --)
   #define RI(i, j, n) for (int i = (j); i \ge (int)n; i --)
   #define SZ(x) int((x).size())
   #define ALL(x) begin(x),end(x)
   #define IS_IN(x, v) ((x).find(v) != (x).end())
   #define vec vector
   #define pb push_back
14
   using ll = long long;
   using ld = long double;
   using pii = pair<int, int>;
   using pil = pair<int, ll>;
   using pli = pair<ll, int>;
   using pll = pair<ll, 11>;
21
22
   const int N = (int)2e5+5;
   const int MOD = (int)1e9 + 7;
   const int oo = (int)1e9;
26
   void solve()
27
28
29
30
31
   int main()
32
33
       ios::sync_with_stdio(0);cin.tie(0);
34
       int TT = 1:
35
       //cin >> TT:
36
       while (TT--)
37
       {
38
           solve();
39
```

```
}
40
41 }
                           1.2 Fast Python
import os, sys, io
finput = io.BytesIO(os.read(0, os.fstat(0).st_size)).readline
3 fprint = sys.stdout.write
                           1.3 Policy Based
 #include <ext/pb_ds/assoc_container.hpp>
   using namespace __gnu_pbds;
   template<typename Key, typename Val=null_type>
   using indexed_set = tree<Key, Val, less<Key>, rb_tree_tag,
                            tree_order_statistics_node_update>;
   // indexed_set<char> s;
   // char val = *s.find_by_order(0); // access por indice
   // int idx = s.order_of_key('a'); // busca indice del valor
   template<class Key, class Val=null_type>using htable=gp_hash_table<Key,
       Val>:
10 // como unordered_map (o unordered_set si Val es vacio), pero sin metodo
                                    Search
                              2.1 Ternary
1 // Minimo de 'f' en '(l,r)'.
   template<class Fun>11 ternary(Fun f, 11 1, 11 r) {
     for (11 d = r-1; d > 2; d = r-1) {
       11 a = 1 + d/3, b = r - d/3;
       if (f(a) > f(b)) l = a; else r = b;
5
6
     return 1 + 1;
8
   // para error < EPS, usar iters=log((r-1)/EPS)/log(1.618)</pre>
   template<class Fun>double golden(Fun f, double 1, double r, int iters){
     double const ratio = (3-sqrt(5))/2;
11
     double x1=1+(r-1)*ratio, x2=r-(r-1)*ratio, f1=f(x1), f2=f(x2);
12
     while (iters--) {
       if (f1 > f2) 1=x1, x1=x2, f1=f2, x2=r-(r-1)*ratio, f2=f(x2);
14
                    r=x2, x2=x1, f2=f1, x1=1+(r-1)*ratio, f1=f(x1);
15
```

```
return (1+r)/2;
18 }
                              Data structures
                                  3.1 BIT
   #define LSOne(S) (S & -S)
2
   struct BIT {
3
       vector<int> B;
4
       int n;
5
       BIT(int n = 1): B(n + 1), n(n+1){}
6
       BIT(vector<int> &v): B(v.size()+1), n(v.size()+1) {
           for (int i = 1; i <= n; i ++){
8
               B[i] += v[i-1];
9
               if (i + LSOne(i) <= n){</pre>
10
                    B[i + LSOne(i)] += B[i];
11
               }
12
           }
13
       }
14
       void update(int i, int x){
15
           while (i \le n){
16
               B[i] += x;
17
               i += LSOne(i);
18
           }
19
       }
20
       int sum(int i){
21
           int res = 0;
^{22}
           while (i > 0){
23
               res += B[i];
24
               i -= LSOne(i);
25
26
           return res;
27
       }
28
       int range_sum(int 1, int r){
29
           return sum(r) - sum(1 - 1);
30
       }
31
32 };
                                 3.2 DSU
1 | struct DSU {
```

```
vector<int> par, sz;
2
3
       int n;
       DSU(int n = 1): par(n), sz(n, 1), n(n) {
4
           for (int i = 0;i < n; i ++) par[i] = i;
5
       }
6
       int find(int a){
           return a == par[a] ? a : par[a] = find(par[a]);
8
9
       void join(int a, int b){
10
           a=find(a);
           b=find(b);
12
           if (a != b){
               if (sz[b] > sz[a]) swap(a,b);
               par[b] = a;
               sz[a] += sz[b];
16
           }
17
18
19 };
                        3.3 Index Compession
1 template<class T>
   struct Index{ // If only 1 use Don't need to copy T type
       vec<T> d;
       int sz;
4
       Index(vec<T> &a): d(ALL(a)){
5
           sort(ALL(d)); // Sort
6
           d.erase(unique(ALL(d)), end(d)); // Erase continuous duplicates
           sz = SZ(d); }
       int of(T e){return lower_bound(ALL(d), e) - begin(d);} // get index
       T at(int i){return d[i];} // get value of index
11 };
                           3.4 Sparse Table
int log2_floor(unsigned long long i) {
       return i ? __builtin_clzll(1) - __builtin_clzll(i) : -1;
2
3
   }
4
   const int MAXN = 10;
  int K = log2_floor(MAXN);
   int st[K + 1][MAXN];
   // Load Array to st[0][i]
```

```
build(RC(v), m + 1, R);
std::copy(array.begin(), array.end(), st[0]);
                                                                                   27
                                                                                                   st[v] = merge(st[LC(v)], st[RC(v)]);
                                                                                   28
11
   // Build
                                                                                              }
                                                                                   29
12
   for (int i = 1; (1 << i) <= n; i ++){
                                                                                          }
                                                                                   30
       for (int j = 0; j + (1 << (i - 1)) < n; j ++){}
                                                                                          void push(int v, int L, int R){
                                                                                   31
14
           st[i][j] = min(st[i-1][j], st[i-1][j + (1 << (i - 1))]);
                                                                                              if (lz[v]){
15
                                                                                   32
       }
                                                                                                  if (L != R){
16
                                                                                   33
   }
                                                                                                       st[LC(v)].mx += lz[v];
17
                                                                                   34
                                                                                                       st[RC(v)].mx += lz[v];
18
                                                                                                       lz[LC(v)] += lz[v];
   // Query
19
   int min_range(int 1, int r){
                                                                                                       lz[RC(v)] += lz[v];
20
                                                                                   37
       int C = log2\_floor(r - 1 + 1);
                                                                                                  }
21
                                                                                   38
       return min(st[C][1], st[C][r - (1 << C) + 1]);
                                                                                                  lz[v] = 0;
22
                                                                                   39
23 }
                                                                                              }
                                                                                   40
                                                                                          }
                                                                                   41
                            3.5 Segment tree
                                                                                          void update(int v, int L, int R, int ql, int qr, ll w){
                                                                                   42
                                                                                              if (ql > R || qr < L) return;
                                                                                   43
                                                                                              push(v, L, R);
   #define LC(v) (v<<1)
                                                                                   44
   #define RC(v) ((v<<1)|1)
                                                                                              if (ql == L \&\& qr == R){
                                                                                   45
                                                                                                   st[v].mx += w;
   #define MD(L, R) (L+((R-L)>>1))
                                                                                                  lz[v] += w;
   struct node {
                                                                                   47
                                                                                                   push(v, L, R);
       11 mx;
5
                                                                                                   return;
       11 cant;
                                                                                   49
6
                                                                                              }
  };
                                                                                   50
7
                                                                                              int m = MD(L, R);
   struct ST {
                                                                                   51
                                                                                              update(LC(v), L, m, ql, min(qr, m), w);
                                                                                   52
       vec<node> st;
9
                                                                                              update(RC(v), m + 1, R, max(m + 1, ql), qr, w);
       vec<ll> lz;
                                                                                   53
10
                                                                                              st[v] = merge(st[LC(v)], st[RC(v)]);
                                                                                   54
       int n;
11
       ST(int n = 1): st(4 * n + 10, {oo, oo}), lz(4 * n + 10, 0), n(n) {
                                                                                   55
12
                                                                                          node query(int v, int L, int R, int ql, int qr){
           build(1, 0, n - 1);
                                                                                   56
13
                                                                                              if (q1 > R || qr < L) return {oo, oo};
       }
                                                                                   57
14
                                                                                              push(v, L, R);
       node merge(node a, node b){
                                                                                   58
15
                                                                                              if (ql == L \&\& qr == R){
           if (a.mx == oo) return b;
                                                                                   59
16
                                                                                                   return st[v];
           if (b.mx == oo) return a;
                                                                                   60
17
                                                                                              }
           if (a.mx == b.mx) return {a.mx, a.cant + b.cant};
                                                                                   61
18
                                                                                              int m = MD(L, R);
                                                                                   62
           return \{\max(a.mx, b.mx), a.mx > b.mx ? a.cant : b.cant\};
19
                                                                                              return merge(query(LC(v), L, m, ql, min(m, qr)), query(RC(v), m
       }
                                                                                   63
20
                                                                                                   + 1, R, max(m + 1, ql), qr);
       void build(int v, int L, int R){
21
           if (L == R){
                                                                                   64
22
                                                                                          node query(int 1, int r){
               st[v] = \{0, 1\};
                                                                                   65
23
                                                                                              return query(1, 0, n - 1, 1, r);
           } else {
                                                                                   66
24
               int m = MD(L, R);
                                                                                   67
25
                                                                                          void update(int 1, int r, ll w){
               build(LC(v), L, m);
                                                                                   68
26
```

```
69 update(1, 0, n - 1, 1, r, w);
70 }
71 };
```

### 4 Dynamic Programming

### 4.1 Knapsack

```
int knapsack(vector<int>& values, vector<int>& weights, int W) {
       int n = values.size();
2
       vector<vector<int>> dp(n + 1, vector<int>(W + 1, 0));
3
       for(int i = 1: i <= n: i++) {
5
           for(int w = 0: w \le W: w++) {
               if(weights[i-1] <= w) {</pre>
                   dp[i][w] = max(dp[i-1][w],
                                  dp[i-1][w-weights[i-1]] + values[i-1]);
9
               } else {
                   dp[i][w] = dp[i-1][w];
11
               }
12
           }
13
       }
14
       return dp[n][W];
15
16 }
```

#### 4.2 LIS

```
vector<int> getLIS(vector<int>& arr) {
       int n = arr.size();
2
       vector<int> dp(n + 1, INT_MAX); // dp[i] = smallest value that ends
3
            an LIS of length i
       vector<int> len(n);
                                        // Length of LIS ending at each
4
           position
       dp[0] = INT_MIN;
5
6
       for(int i = 0; i < n; i++) {
           int j = upper_bound(dp.begin(), dp.end(), arr[i]) - dp.begin();
8
           dp[j] = arr[i];
9
           len[i] = j;
10
       }
11
12
       // Find maxLen and reconstruct sequence
13
       int maxLen = 0;
14
```

```
for(int i = n-1; i \ge 0; i--) maxLen = max(maxLen, len[i]);
16
17
       vector<int> lis;
       for(int i = n-1, currLen = maxLen; i \ge 0; i--) {
           if(len[i] == currLen) {
19
               lis.push_back(arr[i]);
               currLen--;
21
           }
22
       }
23
       reverse(lis.begin(), lis.end());
       return lis;
25
26 }
                           4.3 Edit Distance
1
   //3. Edit Distance - O(n*m)
   int editDistance(string& s1, string& s2) {
       int n = s1.length(), m = s2.length();
       vector<vector<int>> dp(n + 1, vector<int>(m + 1));
5
6
       // Base cases
       for(int i = 0; i \le n; i++) dp[i][0] = i;
       for(int j = 0; j \le m; j++) dp[0][j] = j;
       for(int i = 1; i <= n; i++) {
11
           for(int j = 1; j <= m; j++) {
12
               if(s1[i-1] == s2[i-1]) {
13
                   dp[i][j] = dp[i-1][j-1];
               } else {
                   dp[i][j] = 1 + min({dp[i-1][j]},
                                                      // deletion
16
                                     dp[i][j-1],
                                                      // insertion
17
                                     dp[i-1][j-1]}); // replacement
               }
19
           }
20
21
       return dp[n][m];
22
23 }
                              4.4 Kadane
pair<int, pair<int,int>> kadane(vector<int>& arr) {
       int maxSoFar = arr[0], maxEndingHere = arr[0];
```

int start = 0, end = 0, s = 0;

```
4
       for(int i = 1; i < arr.size(); i++) {
5
            if(maxEndingHere + arr[i] < arr[i]) {</pre>
6
                maxEndingHere = arr[i];
                s = i;
           } else {
9
                maxEndingHere += arr[i];
10
11
^{12}
           if(maxEndingHere > maxSoFar) {
13
                maxSoFar = maxEndingHere;
14
                start = s;
15
                end = i;
16
           }
17
18
       return {maxSoFar, {start, end}}; // max, 1, r
19
20 }
```

## 5 Strings

#### 5.1 Prefix Trie

```
#include <bits/stdc++.h>
   using namespace std;
3
   struct TrieNodeStruct {
       TrieNodeStruct* children[26];
6
       bool isEndOfWord;
7
8
       TrieNodeStruct() {
9
           isEndOfWord = false;
10
           for(int i = 0; i < 26; i++) {
11
                children[i] = nullptr;
12
13
14
15
16
   struct TrieStruct {
17
       TrieNodeStruct* root;
18
19
       TrieStruct() {
20
           root = new TrieNodeStruct();
21
```

```
}
22
23
       void insert(string word) {
24
           TrieNodeStruct* current = root;
25
           for(char c : word) {
26
               int index = c - 'a';
27
               if(current->children[index] == nullptr) {
28
                   current->children[index] = new TrieNodeStruct();
29
               }
30
               current = current->children[index];
32
           current->isEndOfWord = true;
33
       }
35 };
                               5.2 Hashing
  | static constexpr ll ms[] = {1'000'000'007, 1'000'000'403};
  static constexpr 11 b = 500'000'000;
   struct StrHash { // Hash polinomial con exponentes decrecientes.
     vector<11> hs[2], bs[2];
    StrHash(string const& s) {
       int n = SZ(s);
      L(k, 0, 2) {
         hs[k].resize(n+1), bs[k].resize(n+1, 1);
         L(i, 1, n) {
           hs[k][i+1] = (hs[k][i] * b + s[i]) % ms[k];
           bs[k][i+1] = bs[k][i] * b
                                               % ms[k];
11
12
       }
13
14
     ll get(int idx, int len) const { // Hashes en 's[idx, idx+len)'.
15
       ll h[2];
16
       L(k, 0, 2) {
17
         h[k] = hs[k][idx+len] - hs[k][idx] * bs[k][len] % ms[k];
         if (h[k] < 0) h[k] += ms[k];
19
20
       return (h[0] << 32) | h[1];
21
22
   };
23
```

pll union\_hash(vector<pll> hs, vector<ll> lens){ //use arrays makes it

slower

```
11 len = 0;
26
     for(int i = hs.size()-1; i > 0; i--){
27
       len += lens[i];
28
       pll& [11, 12] = hs[i];
29
       pll& [r1, r2] = hs[i-1];
30
       11 = ((11 * binpow(b, len, ms[0])) \% ms[0] + r1) \% ms[0];
31
       12 = ((12 * binpow(b, len, ms[1])) \% ms[1] + r2) \% ms[1];
32
33
34
     return hs[0];
35
  |}
36
```

#### 5.3 KMP

```
#include <bits/stdc++.h>
   using namespace std;
2
   vector<int> kmp(string pat, string sec){ //geeks4geeks implementation
       with some changes
     int m = pat.length();
     int n = sec.length();
     cout << m << "" << n << endl;
8
     vector<int> lps = getLps(pat);
9
     vector<int> res:
10
11
     int i = 0;
12
     int j = 0;
13
14
     while((n - i) >= (m - j)){
15
       if(pat[j] == sec[i]){
16
         i++;
17
         j++;
18
19
       if(i == m){
20
         res.push_back(i - j);
^{21}
         j = lps[j - 1];
^{22}
       }
23
       else{
24
         if(i < n && pat[j] != sec[i]){</pre>
25
           if(j != 0) j = lps[ j - 1 ];
26
           else i = i + 1;
27
28
```

```
}
29
     }
30
31
     return res;
32
33 }
                                 5.4 LPS
#include <bits/stdc++.h>
   using namespace std;
   vector<int> getLps(string pat){ //geek4geeks implementatio with some
       changes
     vector<int> lps(pat.length(), 0);
     int len = 0;
     int i = 1:
     lps[0] = 0;
     while(i < pat.length()){</pre>
9
       if(pat[i] == pat[len]){
10
         len++;
11
         lps[i] = len;
12
         i++;
13
       }
14
       else //pat[i] != pat[len]
15
       {
16
         lps[i] = 0;
17
         i++;
18
19
     }
20
21
     return lps;
22
23 | }
                           5.5 Z-FUNCTION
  template<class Char=char>vector<int> zfun(const basic_string<Char>& w) {
     int n = SZ(w), l = 0, r = 0; vector<int> z(n);
     z[0] = w.length();
     L(i, 1, n) {
4
       if (i \le r) \{z[i] = min(r - i + 1, z[i - 1]);\}
5
       while (i + z[i] < n \&\& w[z[i]] == w[i + z[i]]) \{++z[i];\}
       if (i + z[i] - 1 > r) \{l = i, r = i + z[i] - 1;\}
7
     }
8
     return z;
```

```
10 |}
```

## 6 Graph

#### 6.1 Tarjan

```
const int N = 10;
2
   vector<int> G[N];
   vector<int> dfs_low(N, -1), dfs_num(N, -1), ap(N, 0); // ap for
       Articulation Points
  int dfs_count = 0;
   int root = -1; // For AP
   void dfs(int u, int p = -1){
       dfs_low[u] =dfs_num[u] =dfs_count++;
9
       int child = 0;
10
       for (int v: G[u]){
11
           if (v == p) continue;
12
           if (dfs_num[v] == -1){
13
               child ++;
14
               dfs(v, u);
15
               dfs_low[u] = min(dfs_low[u], dfs_low[v]);
16
               if (dfs_low[v] > dfs_num[u]){
17
                 // Bridge from u -> v
18
                  cout << "Bridge_" << u << "_->_" << v << "\n";
19
20
               if (dfs_low[v] >= dfs_num[u]) {
21
                   // u is AP
^{22}
                    ap[u] = 1;
23
24
           } else dfs_low[u] = min(dfs_low[u], dfs_num[v]);
25
       }
26
       if (u == root){
27
           ap[u] = child > 1;
28
       }
29
30 | }
```

#### 6.2 Bellman Ford

```
struct Edge {
int a, b, cost;
};
```

```
4
5 | int n, m, v;
  vector<Edge> edges;
   const int INF = 1000000000;
   void solve()
   {
10
       vector<int> d(n, INF);
       d[v] = 0;
12
       for (int i = 0; i < n - 1; ++i)
13
           for (Edge e : edges)
14
               if (d[e.a] < INF)
15
                   d[e.b] = min(d[e.b], d[e.a] + e.cost);
16
17 }
                                6.3 SCC
vector<int> dfs_num(N, -1), dfs_low(N, -1), visited(N);
  int dfs_count = 0;
  int numSCC = 0;
   stack<int> st;
   void dfs(int u){
     dfs_low[u] = dfs_num[u] = dfs_count++;
     st.push(u);
     visited[u] = 1;
8
     for(int v: G[u]) {
       if (dfs_num[v] == -1) dfs(v);
10
       if (visited[v]) dfs_low[u] = min(dfs_low[u], dfs_low[v]);
11
    }
12
     if (dfs_num[u] == dfs_low[u]){
13
       numSCC ++;
       while(1){
15
         int v = st.top(); st.pop();
         visited[v] = 0;
17
         if (u == v) break;
19
    }
20
21 }
              6.4 Bipartite Matching Hopcroft-Karp
1 | int L_S, R_S;
vec<int> G[S_MX]; // S_MX (Maxima cantidad de nodos de un lado)
int mat[S_MX]; // matching [0,L_S) -> [0,R_S)
```

cover[1].assign(R\_S,false); // R\_S right size

4

```
4 int inv[S_MX]; // matching [0,R_S) \rightarrow [0,L_S)
                                                                                         int size = hopkarp(); // alternativamente, tambien funciona con
  int hopkarp() {
       fill(mat,mat+L_S,-1);
                                                                                         auto dfs = [&](auto&& me, int u) -> void {
6
                                                                                   6
       fill(inv,inv+R_S,-1);
                                                                                             cover[0][u] = false;
                                                                                             for (auto v : g[u]) if (!cover[1][v]) {
       int size = 0;
                                                                                   8
8
                                                                                                  cover[1][v] = true;
       vector<int> d(L_S);
9
                                                                                   9
       auto bfs = [\&] {
                                                                                                  me(me,inv[v]);
                                                                                  10
10
                                                                                             }
           bool aug = false;
                                                                                  11
11
           queue<int> q;
                                                                                         };
12
                                                                                  12
           L(u, 0, L_S) if (mat[u] < 0) q.push(u); else d[u] = -1;
                                                                                         L(u,0,L_S) if (mat[u] < 0) dfs(dfs,u);
                                                                                  13
13
           while (!q.empty()) {
                                                                                         return size;
14
                                                                                  14
               int u = q.front();
                                                                                  15 }
15
               q.pop();
16
                                                                                                                6.6 Hungarian
               for (auto v : G[u]) {
17
                   if (inv[v] < 0) aug = true;</pre>
18
                   else if (d[inv[v]] < 0) d[inv[v]] = d[u] + 1, q.push(inv
19
                                                                                   using vi = vec<int>;
                        [v]);
                                                                                     using vd = vec<ld>;
               }
20
                                                                                     const ld INF = 1e100;
                                                                                                             // Para max asignacion, INF = 0, y negar costos
           }
                                                                                     bool zero(ld x) {return fabs(x) < 1e-9;} // Para int/ll: return x==0;
21
           return aug;
                                                                                     vec<pii> ans; // Guarda las aristas usadas en el matching: [0..n)x[0..m)
22
       };
                                                                                     struct Hungarian{
23
       auto dfs = [\&] (auto&& me, int u) -> bool {
24
                                                                                       int n; vec<vd> cs; vi vL, vR;
           for (auto v : G[u]) if (inv[v] < 0) {
                                                                                       Hungarian(int N, int M): n(max(N,M)), cs(n,vd(n)), vL(n), vR(n){
25
               mat[u] = v, inv[v] = u;
                                                                                         L(x, 0, N) L(y, 0, M) cs[x][y] = INF;
26
                                                                                  9
               return true;
                                                                                       }
27
                                                                                  10
                                                                                       void set(int x, int y, ld c) { cs[x][y] = c; }
28
                                                                                  11
           for (auto v : G[u]) if (d[inv[v]] > d[u] \&\& me(me,inv[v])) {
                                                                                       ld assign(){
29
                                                                                  12
               mat[u] = v, inv[v] = u;
                                                                                         int mat = 0; vd ds(n), u(n), v(n); vi dad(n), sn(n);
30
                                                                                  13
               return true;
                                                                                         L(i, 0, n) u[i] = *min_element(ALL(cs[i]));
31
                                                                                  14
           }
32
                                                                                         L(i, 0, n){
                                                                                  15
           d[u] = 0;
                                                                                           v[i] = cs[0][i]-u[0];
33
                                                                                  16
           return false;
                                                                                           L(i, 1, n) v[j] = min(v[j], cs[i][j] - u[i]);
34
                                                                                  17
       }:
35
                                                                                  18
       while (bfs()) L(u, 0, L_S) if (mat[u] < 0) size += dfs(dfs, u);
                                                                                         vL = vR = vi(n, -1);
36
                                                                                  19
       return size:
                                                                                         L(i,0, n) L(i,0,n) if (vR[i] == -1 \text{ and } zero(cs[i][i] - u[i] - v[i])
37
                                                                                  20
38 | }
                                                                                             ){
                                                                                            vL[i] = j; vR[j] = i; mat++; break;
                                                                                  21
                 6.5 Konig Theorem Min V.Cover
                                                                                  22
                                                                                         for(: mat < n: mat ++){</pre>
                                                                                  23
  vec<int> cover[2]; // if cover[i][j] = 1 -> node i, j is part of cover
                                                                                           int s = 0, j = 0, i;
                                                                                  24
  int konig() {
                                                                                           while(vL[s] != -1) s++;
                                                                                  25
2
       cover[0].assign(L_S,true); // L_S left size
                                                                                           fill(ALL(dad), -1); fill(ALL(sn), 0);
                                                                                  26
3
```

27

L(k, 0, n) ds[k] = cs[s][k]-u[s]-v[k];

g.resize(n);

level.resize(n);

17

18

```
while(true){
                                                                                               ptr.resize(n);
28
                                                                                   19
                                                                                           }
           i = -1;
                                                                                   20
29
           L(k, 0, n) if(!sn[k] and (j == -1 or ds[k] < ds[j])) j = k;
                                                                                   21
30
           sn[j] = 1; i = vR[j];
                                                                                           void add_edge(int from, int to, long long cap) {
                                                                                   22
31
           if(i == -1) break;
                                                                                               g[from].emplace_back(to, g[to].size(), cap);
                                                                                   23
32
                                                                                               g[to].emplace_back(from, g[from].size()-1, 0); // Reverse edge
           L(k, 0, n) if(!sn[k]){
33
                                                                                   24
             auto new_ds = ds[j] + cs[i][k] - u[i]-v[k];
                                                                                           }
                                                                                   25
34
             if(ds[k] > new_ds) ds[k]=new_ds, dad[k]=j;
                                                                                   26
35
           }
                                                                                           bool bfs() {
                                                                                   27
36
                                                                                               while(!q.emptv()) {
                                                                                   28
37
         L(k, 0, n) if (k!=j \text{ and } sn[k])
                                                                                                   q.pop();
38
                                                                                   29
           auto w = ds[k]-ds[j]; v[k] += w, u[vR[k]] -= w;
39
                                                                                   30
         }
                                                                                               fill(level.begin(), level.end(), -1);
40
                                                                                   31
         u[s] += ds[j];
                                                                                   32
         while (dad[i] >= 0) { int d = dad[i]; vR[i] = vR[d]; vL[vR[i]] = i;
                                                                                               q.push(source);
                                                                                   33
              i = d: 
                                                                                               level[source] = 0;
                                                                                   34
         vR[j] = s; vL[s] = j;
43
                                                                                   35
       }
                                                                                               while(!q.empty() && level[sink] == -1) {
44
       ld value = 0; L(i, 0, n) value += cs[i][vL[i]], ans.pb({i, vL[i]});
                                                                                                   int v = q.front();
                                                                                   37
45
       return value;
                                                                                                   q.pop();
46
                                                                                   38
     }
47
                                                                                   39
48 };
                                                                                                   for(const Edge& e : g[v]) {
                                                                                                       if(level[e.to] == -1 \&\& e.flow < e.cap) {
                                                                                   41
                                 6.7 Flow
                                                                                                           level[e.to] = level[v] + 1;
                                                                                   42
                                                                                                           q.push(e.to);
                                                                                   43
                                                                                                       }
   // Complexity (V * V * E);
                                                                                   44
                                                                                                   }
   struct Dinic {
                                                                                   45
       struct Edge {
                                                                                   46
3
                                                                                               return level[sink] != -1;
                                                                                   47
           int to, rev;
4
                                                                                           }
           long long cap, flow;
                                                                                   48
5
           Edge(int to, int rev, long long cap) :
                                                                                   49
6
                                                                                           long long dfs(int v, long long pushed) {
                                                                                   50
               to(to), rev(rev), cap(cap), flow(0) {}
7
                                                                                               if(v == sink || pushed == 0) return pushed;
       };
                                                                                   51
8
                                                                                   52
9
                                                                                               for(int& i = ptr[v]; i < (int)g[v].size(); i++) {</pre>
                                                                                   53
       vector<vector<Edge>> g;
10
                                                                                                   Edge& e = g[v][i];
                                                                                   54
       vector<int> level, ptr;
11
       queue<int> q;
                                                                                   55
12
                                                                                                   if(level[e.to] != level[v] + 1 || e.flow >= e.cap) continue;
       int n, source, sink;
                                                                                   56
13
       const long long INF = 1e18;
                                                                                   57
14
                                                                                                   long long flow = dfs(e.to, min(pushed, e.cap - e.flow));
                                                                                   58
15
                                                                                                   if(flow == 0) continue;
       Dinic(int n, int s, int t) : n(n), source(s), sink(t) {
                                                                                   59
16
```

60

61

e.flow += flow;

```
g[e.to][e.rev].flow -= flow;
                                                                                                           if(!reachable[e.to] && e.flow < e.cap) {</pre>
62
                                                                                      105
                return flow;
                                                                                                               reachable[e.to] = true;
                                                                                     106
63
            }
                                                                                                               q.push(e.to);
64
                                                                                      107
            return 0;
                                                                                                          }
65
                                                                                      108
        }
                                                                                                      }
66
                                                                                      109
                                                                                                  }
67
                                                                                     110
       long long max_flow() {
                                                                                                  return reachable;
                                                                                     111
68
            long long flow = 0;
                                                                                     112
69
                                                                                         };
                                                                                     113
70
            while(bfs()) {
                                                                                     114
71
                fill(ptr.begin(), ptr.end(), 0);
                                                                                          // Example usage:
72
                                                                                     115
                while(long long pushed = dfs(source, INF)) {
73
                                                                                     116
                     flow += pushed;
                                                                                         int main() {
74
                                                                                     117
                }
                                                                                              // Example: 6 vertices, source = 0, sink = 5
                                                                                     118
75
            }
                                                                                             int n = 6;
                                                                                     119
76
                                                                                             Dinic flow(n, 0, 5);
            return flow;
                                                                                      120
77
        }
78
                                                                                      121
                                                                                             // Add edges: (from, to, capacity)
                                                                                      122
79
        // Get the actual flow passing through each edge
                                                                                              flow.add_edge(0, 1, 16);
                                                                                      123
80
        vector<vector<long long>> get_flow() {
                                                                                              flow.add_edge(0, 2, 13);
                                                                                      124
81
            vector<vector<long long>> flow(n, vector<long long>(n, 0));
                                                                                              flow.add_edge(1, 2, 10);
                                                                                     125
82
            for(int v = 0; v < n; v++) {
                                                                                              flow.add_edge(1, 3, 12);
                                                                                      126
83
                for(const Edge& e : g[v]) {
                                                                                              flow.add_edge(2, 1, 4);
                                                                                     127
84
                     if(e.cap > 0) { // Only original edges, not residual
                                                                                              flow.add_edge(2, 4, 14);
                                                                                      128
85
                         flow[v][e.to] = e.flow;
                                                                                              flow.add_edge(3, 2, 9);
                                                                                      129
86
                     }
                                                                                              flow.add_edge(3, 5, 20);
                                                                                     130
87
                }
                                                                                              flow.add_edge(4, 3, 7);
                                                                                     131
88
            }
                                                                                              flow.add_edge(4, 5, 4);
                                                                                     132
89
            return flow;
                                                                                     133
90
        }
                                                                                     134
                                                                                              // Calculate maximum flow
91
                                                                                              long long max_flow = flow.max_flow();
                                                                                     135
92
                                                                                             cout << "Maximum flow: " << max_flow << "\n";</pre>
        // Find minimum cut
                                                                                     136
93
        vector<bool> min cut() {
                                                                                     137
94
            vector<bool> reachable(n, false);
                                                                                             // Get minimum cut
                                                                                     138
95
            queue<int> q;
                                                                                              vector<bool> cut = flow.min cut():
                                                                                     139
96
            q.push(source);
                                                                                              cout << "Vertices on source side of min cut: ";</pre>
                                                                                     140
97
            reachable[source] = true;
                                                                                              for(int i = 0; i < n; i++) {
98
                                                                                     141
                                                                                                  if(cut[i]) cout << i << " ":
                                                                                     142
99
            while(!q.empty()) {
                                                                                     143
100
                                                                                              cout << "\n";
                int v = q.front();
                                                                                     144
101
                q.pop();
                                                                                     145
102
                                                                                              // Get flow through each edge
                                                                                     146
103
                                                                                             auto flow_matrix = flow.get_flow();
                for(const Edge& e : g[v]) {
                                                                                     147
104
```

```
cout << "Flow matrix:\n":</pre>
148
        for(int i = 0; i < n; i++) {
149
             for(int j = 0; j < n; j++) {
150
                 if(flow_matrix[i][j] > 0) {
151
                      cout << i << " -> " << j << ": " << flow_matrix[i][j] <</pre>
152
                           "\n":
153
154
        }
155
156
        return 0;
157
158
159
```

#### 6.8 Ford Fulkerson

```
#define ll long long
   const 11 \text{ INF} = (11)4e18;
   struct Edge {
       int from, to;
       ll cap, flow;
5
       Edge(int from, int to, 11 cap): from(from), to(to), cap(cap), flow
            (0) {}
  };
7
   struct MaxFlow {
       vector<Edge> edges;
10
       vector<vector<int>> adj;
11
       vector<int> level, ptr;
12
       int n;
13
       queue<int> q;
14
15
       MaxFlow(int n) : n(n) {
16
           adj.resize(n);
17
           level.resize(n);
18
           ptr.resize(n);
19
       }
20
21
       void add_edge(int from, int to, ll cap) {
22
           edges.emplace_back(from, to, cap);
23
           edges.emplace_back(to, from, 0);
24
           adj[from].push_back(edges.size() - 2);
25
           adj[to].push_back(edges.size() - 1);
26
```

```
}
27
28
       bool bfs(int s, int t) {
29
            while(!q.empty()) q.pop();
30
           fill(level.begin(), level.end(), -1);
31
32
            q.push(s);
33
            level[s] = 0;
34
35
            while(!q.empty() && level[t] == -1) {
                int v = q.front();
37
                q.pop();
38
39
                for(int id : adj[v]) {
40
                    if(level[edges[id].to] == -1 && edges[id].cap - edges[id
41
                        1.flow > 0) {
                        level[edges[id].to] = level[v] + 1;
                        q.push(edges[id].to);
                    }
44
                }
            }
46
            return level[t] != -1;
47
       }
48
49
       11 dfs(int v, int t, ll pushed) {
50
            if(v == t \mid \mid pushed == 0)
51
                return pushed;
52
53
           for(; ptr[v] < (int)adj[v].size(); ptr[v]++) {</pre>
54
                int id = adj[v][ptr[v]];
55
                int u = edges[id].to;
56
57
                if(level[u] != level[v] + 1) continue;
58
59
                11 tr = dfs(u, t, min(pushed, edges[id].cap - edges[id].flow
60
                    )):
                if(tr > 0) {
61
                    edges[id].flow += tr;
                    edges[id ^ 1].flow -= tr;
63
                    return tr;
64
                }
65
            }
66
            return 0;
67
```

```
}
68
69
       11 max_flow(int s, int t) {
70
            11 \text{ flow} = 0;
71
            while(bfs(s, t)) {
72
                fill(ptr.begin(), ptr.end(), 0);
73
                while(ll pushed = dfs(s, t, LLONG_MAX)) {
74
                     flow += pushed;
75
                }
76
77
            return flow;
78
       }
79
80
       vector<ll> get_flows() {
81
            vector<ll> flows;
82
            for(int i = 0; i < edges.size(); i += 2) {</pre>
83
                flows.push_back(edges[i].flow);
84
            }
85
            return flows;
86
       }
87
88 };
```

### Math

#### 7.1 Euclidean Extended

```
1 | 11 extendedGCD(11 a, 11 b, 11 &x, 11 &y) {
       if (b == 0) {
2
           x = 1;
3
           v = 0;
4
           return a;
5
       }
6
       ll x1, y1;
7
       11 gcd = extendedGCD(b, a % b, x1, y1);
8
       x = y1;
9
       y = x1 - (a / b) * y1;
10
       return gcd;
11
12
13
  |bool findSolutionWithConstraints(ll a, ll b, ll c, ll x_min, ll y_min,
       11 &x, 11 &y) {
       11 g = extendedGCD(a, b, x, y);
15
16
```

```
if (c % g != 0) return false;
17
18
       x *= c / g;
19
       y *= c / g;
20
21
       // Ajustamos las variables a/g y b/g para mover las soluciones
       a /= g;
23
       b /= g;
       if (x < x_min) {</pre>
           ll k = (x_min - x + b - 1) / b; // Redondeo hacia arriba
27
           x += k * b;
           y -= k * a;
29
       } else if (x > x_min) {
           11 k = (x - x_min) / b;
           x -= k * b;
           y += k * a;
33
       }
35
       if (y < y_min) {</pre>
           ll k = (y_min - y + a - 1) / a; // Redondeo hacia arriba
37
           x += k * b;
           y -= k * a;
39
       } else if (y > y_min) {
           ll k = (y - y_min) / a;
41
           x -= k * b;
42
           y += k * a;
43
44
45
       return x >= x_min && y >= y_min;
47 }
                                 Euler Totient
```

```
| #include <bits/stdc++.h>
  using namespace std;
  typedef long long 11;
  vector<ll> compute_totients(ll n) {
      vector<ll> phi(n + 1);
      for (ll i = 0; i <= n; i++) {
8
9
          phi[i] = i;
```

```
}
10
11
       for (ll i = 2; i <= n; i++) {
12
           if (phi[i] == i) { // i es primo
13
                for (ll j = i; j <= n; j += i) {
14
                    phi[j] = phi[j] * (i - 1) / i;
15
16
           }
17
       }
18
19
       return phi;
20
21 | }
```

### 7.3 Josephus

```
#include <iostream>
   using namespace std;
   typedef long long 11;
   ll josephus_iterative(ll n, ll k) {
       11 result = 0;
7
       for (ll i = 2; i <= n; ++i) {
8
           result = (result + k) % i;
9
       }
10
       return result;
11
12
13
14
   ll josephus_recursive(ll n, ll k) {
15
16
       if (n == 1)
17
           return 0;
18
19
       return (josephus_recursive(n - 1, k) + k) % n;
20
^{21}
^{22}
23
   11 josephus_power_of_2(11 n) {
24
25
       11 power = 1;
26
       while (power <= n) {</pre>
27
           power <<= 1;
28
```

```
}
29
       power >>= 1;
30
31
32
       return 2 * (n - power);
33
34 }
                                7.4 Mobius
1 | #include <bits/stdc++.h>
   using namespace std;
   typedef long long 11;
   vector<ll> compute_mobius(ll n) {
       vector<ll> mu(n + 1, 1);
7
       vector<bool> is_prime(n + 1, true);
9
       for (ll i = 2; i <= n; i++) {
           if (is_prime[i]) { // i es un primo
11
               for (ll j = i; j <= n; j += i) {
                   mu[j] *= -1; // Multiplicamos por -1 para cada primo
13
                    is_prime[j] = false;
14
15
               for (ll j = i * i; j \le n; j += i * i) {
16
                    mu[j] = 0; // Si tiene un cuadrado de un primo, se pone
17
                        en 0
               }
18
           }
19
       }
20
21
22
       return mu;
23
^{24}
25
   11 mobius(ll x) {
       11 count = 0;
27
       for (ll i = 2; i * i <= x; i++) {
28
           if (x \% (i * i) == 0)
29
               return 0;
30
           if (x % i == 0) {
31
               count++;
32
```

x /= i;

33

for (int len = 2; len <= n; len <<= 1) {

33

```
}
                                                                                                int wlen = invert ? root_1 : root;
34
                                                                                    34
       }
                                                                                                for (int i = len; i < root_pw; i <<= 1)</pre>
                                                                                    35
35
                                                                                                    wlen = (int)(1LL * wlen * wlen % mod);
36
                                                                                    36
       if (x > 1) count++;
37
                                                                                    37
                                                                                                for (int i = 0; i < n; i += len) {
38
                                                                                    38
       return (count % 2 == 0) ? 1 : -1;
                                                                                                    int w = 1;
39
                                                                                    39
40 }
                                                                                                    for (int j = 0; j < len / 2; j++) {
                                                                                    40
                                                                                                        int u = a[i+j], v = (int)(1LL * a[i+j+len/2] * w % mod);
                                                                                    41
                                 7.5 NTT
                                                                                                        a[i+j] = u + v < mod ? u + v : u + v - mod;
                                                                                    42
                                                                                                        a[i+j+len/2] = u - v >= 0 ? u - v : u - v + mod;
                                                                                                        w = (int)(1LL * w * wlen % mod);
  #include <bits/stdc++.h>
                                                                                    44
                                                                                                    }
   using namespace std;
                                                                                    45
                                                                                                }
   using cd = complex<double>;
                                                                                    46
                                                                                            }
   typedef long long 11;
                                                                                    47
   const 11 mod = 998244353;
                                                                                            if (invert) {
   const ll root = 31;
                                                                                                int n_1 = inverse(n, mod);
   const ll root_1 = inverse(root, mod);
                                                                                                for (auto & x : a)
   const ll root_pw = 1 << 23;</pre>
                                                                                                    x = (int)(1LL * x * n 1 \% mod):
                                                                                    52
                                                                                            }
   ll inverse(ll a, ll m) {
                                                                                    53
                                                                                       }
       11 \text{ res} = 1, \exp = m - 2;
                                                                                    54
11
       while (exp) {
                                                                                    55
12
                                                                                       vector<ll> multiply(vector<ll> const &a, vector<ll> const &b) {
           if (exp % 2 == 1) res = (1LL * res * a) % m;
13
                                                                                            vector<ll> fa(a.begin(), a.end()), fb(b.begin(), b.end());
           a = (1LL * a * a) \% m:
                                                                                    57
14
                                                                                            11 n = 1;
                                                                                    58
           exp /= 2;
15
                                                                                            while (n < a.size() + b.size())</pre>
       }
16
                                                                                                n <<= 1;
                                                                                    60
       return res;
17
                                                                                            fa.resize(n);
                                                                                    61
18
                                                                                            fb.resize(n);
19
   void ntt(vector<11> & a, bool invert) {
20
                                                                                            ntt(fa, false);
       int n = a.size();
21
                                                                                            ntt(fb, false);
22
                                                                                            for (ll i = 0; i < n; i++)
       for (int i = 1, j = 0; i < n; i++) {
                                                                                    66
23
                                                                                                fa[i] = (fa[i] * fb[i]) % mod;
           int bit = n >> 1;
                                                                                    67
^{24}
                                                                                            ntt(fa, true);
                                                                                    68
           for (; j & bit; bit >>= 1)
25
                                                                                    69
               j ^= bit;
26
                                                                                            vector<ll> result(n);
           j ^= bit;
27
                                                                                            for (11 i = 0: i < n: i++)
28
                                                                                                result[i] = fa[i];
           if (i < j)
                                                                                    72
29
                                                                                    73
                                                                                            return result;
                swap(a[i], a[j]);
30
                                                                                    74 }
       }
31
32
```

7.6 FFT

```
1 typedef long long 11;
                                                                                  42
   typedef complex<double> C;
                                                                                          vll res(a.size() + b.size() - 1);
                                                                                  43
   typedef vector<double> vd;
                                                                                         for (int i = 0; i < res.size(); i++) {</pre>
                                                                                  44
   typedef vector<ll> vll;
                                                                                              res[i] = llround(imag(out[i]) / (4 * n));
   const double PI = acos(-1);
                                                                                         }
                                                                                  46
                                                                                         return res;
                                                                                  47
                                                                                  48 }
   void fft(vector<C>& a) {
       int n = a.size(), L = 31 - __builtin_clz(n);
8
                                                                                                                    7.7 Rho
       static vector<C> R(2, 1);
9
       static vector<C> rt(2, 1);
10
       for (static int k = 2; k < n; k *= 2) {
                                                                                   1 //RECOMENDADO USAR UNSIGNED LONG LONG
11
           R.resize(n); rt.resize(n);
12
                                                                                     | 11 mulmod(11 a, 11 b, 11 m) { return 11(__int128(a) * b % m); }
           auto x = polar(1.0, PI / k);
13
           for (int i = k; i < 2 * k; i++)
                                                                                     ll expmod(ll b, ll e, ll m) {
               rt[i] = R[i] = i & 1 ? R[i / 2] * x : R[i / 2];
                                                                                      ll ret = 1;
15
       }
16
                                                                                       while (e) {
       vector<int> rev(n):
17
                                                                                         if (e%2) ret = mulmod(ret, b, m);
       for (int i = 0; i < n; i++) rev[i] = (rev[i / 2] | (i & 1) << L) /
18
                                                                                         b = mulmod(b, b, m);
                                                                                         e /= 2;
                                                                                   9
       for (int i = 0; i < n; i++) if (i < rev[i]) swap(a[i], a[rev[i]]);
19
                                                                                  10
       for (int k = 1; k < n; k *= 2)
20
                                                                                  11
                                                                                       return ret;
           for (int i = 0; i < n; i += 2 * k) for (int j = 0; j < k; j++) {
21
                                                                                  12 }
               auto x = (double*)&rt[j + k], y = (double*)&a[i + j + k];
                                                                                     bool miller(ll n) {
22
                                                                                  13
               C z(x[0] * y[0] - x[1] * y[1], x[0] * y[1] + x[1] * y[0]);
                                                                                          if (n < 2) return false:
23
                                                                                  14
               a[i + j + k] = a[i + j] - z;
                                                                                         for (11 p : {2, 3, 5, 7, 11, 13, 17, 19}) {
24
                                                                                  15
               a[i + j] += z;
                                                                                              if (n \% p == 0)
25
                                                                                  16
           }
                                                                                                  return (n == p);
26
                                                                                  17
27
                                                                                  18
                                                                                         if (n < 529) return true; // 23^2 = 529
28
   vll multiply(const vll& a, const vll& b) {
                                                                                         int s = 0;
       if (a.empty() || b.empty()) return {};
                                                                                         11 d = n - 1;
30
                                                                                  21
       vd fa(a.begin(), a.end()), fb(b.begin(), b.end());
31
                                                                                          while ((d \& 1) == 0) {
       int L = 32 - \_builtin\_clz(fa.size() + fb.size() - 1), n = 1 << L;
                                                                                              d >>= 1;
32
                                                                                  23
       vector<C> in(n). out(n):
33
                                                                                  24
                                                                                              s++;
34
                                                                                  25
       for (int i = 0; i < a.size(); i++) in[i] = C(fa[i], 0);
35
                                                                                          auto witness = [&](ll a) {
                                                                                  26
       for (int i = 0; i < b.size(); i++) in[i].imag(fb[i]);</pre>
36
                                                                                              11 x = expmod(a \% n, d, n);
                                                                                  27
                                                                                              if (x == 1 \mid | x == n - 1) return false:
37
                                                                                  28
       fft(in);
                                                                                              for (int i = 1; i < s; i++) {
38
                                                                                  29
       for (C\& x : in) x *= x;
                                                                                                  x = mulmod(x, x, n):
39
                                                                                  30
       for (int i = 0; i < n; i++) out[i] = in[-i & (n - 1)] - conj(in[i]);
                                                                                                  if (x == n - 1) return false;
40
                                                                                  31
             // Corregido aqui
                                                                                  32
       fft(out);
41
                                                                                              return true; // "true" => 'a' es testigo => n es compuesto
                                                                                  33
```

// Multiplica por h/3

return (h / 3.0L) \* s;

14

15

30

```
16 }
       };
34
       // Bases para 64 bits
35
       for (11 b : {2, 325, 9375, 28178, 450775, 9780504, 1795265022}) {
36
           if (b % n == 0) return true;
                                                                                   _{19} | 1d a = 0.0L, b = 3.0L;
37
           if (witness(b)) return false;
38
       }
39
       return true; // "probable primo"
40
41
   11 rho(11 n) {
     if(n \% 2 == 0) return 2;
     11 x = 2, y = 2, d = 1;
     ll c = rand() % n + 1;
45
                                                                                    1 #include <iostream>
     while(d == 1)  {
                                                                                      #include <vector>
       x = (mulmod(x, x, n) + c) \% n;
                                                                                      #include <algorithm>
       y = (mulmod(y, y, n) + c) \% n;
                                                                                      using namespace std;
       y = (mulmod(y, y, n) + c) \% n;
49
       d = \_gcd(x - y, n);
50
                                                                                       typedef long long 11;
51
     return d == n ? rho(n) : d;
52
53
   void fact(ll n, map<ll, int>& F) { // Agrega los factores de n en F
54
     if (n == 1) return;
55
     if (miller(n)) {F[n]++; return;}
                                                                                   11
     ll q = rho(n); fact(q, F); fact(n / q, F);
57
58 }
                                                                                   14
                               7.8 Simpson
                                                                                   15
                                                                                           vector<Point> hull;
                                                                                   16
   ld simpsonRule(function<ld(ld)> f, ld a, ld b, int n) {
                                                                                   17
                                                                                           // Parte inferior
       // Asegurarse de que n sea par
2
                                                                                   18
       if (n % 2 != 0) {
                                                                                   19
3
           n++;
                                                                                   20
4
5
       1d h = (b - a) / n;
                                                                                   21
       1d s = f(a) + f(b);
7
                                                                                   22
                                                                                   23
8
       // Suma de terminos interiores con los factores apropiados
                                                                                   24
9
       for (int i = 1; i < n; i++) {
                                                                                           }
                                                                                   25
10
           1d x = a + i * h:
                                                                                   26
11
           s += (i \% 2 == 1 ? 4.0L : 2.0L) * f(x);
                                                                                          // Parte superior
                                                                                   27
12
       }
13
```

```
17 // Ejemplo: integrar la funcion x^2 entre 0 y 3
auto f = [k](ld x)\{ return x * x; \};
int n = 1000; // numero de subintervalos
ld resultado = simpsonRule(f, a, b, n);
                                Geometry
                           8.1 Convex Hull
   typedef pair<11, 11> Point;
   11 cross_product(Point O, Point A, Point B) {
       return (A.first - O.first) * (B.second - O.second) - (A.second - O.
           second) * (B.first - O.first);
   vector<Point> convex_hull(vector<Point>& points) {
       sort(points.begin(), points.end());
       points.erase(unique(points.begin(), points.end()), points.end());
       for (const auto& p : points) {
           while (hull.size() >= 2 && cross_product(hull[hull.size() - 2],
               hull[hull.size() - 1], p) < 0)
               hull.pop_back();
          if (hull.empty() || hull.back() != p) {
               hull.push_back(p);
       int t = hull.size() + 1;
28
       for (int i = points.size() - 1; i \ge 0; --i) {
29
```

while (hull.size() >= t && cross\_product(hull[hull.size() - 2],

```
hull[hull.size() - 1], points[i]) < 0)</pre>
               hull.pop_back();
31
           if (hull.empty() || hull.back() != points[i]) {
32
               hull.push_back(points[i]);
33
           }
34
       }
35
36
       hull.pop_back();
37
       return hull;
38
39 | }
                                    Operations
                              8.2
   #include <bits/stdc++.h>
   using namespace std;
   typedef long long 11;
5
6
   11 cross_product(pair<11, 11> P1, pair<11, 11> P2, pair<11, 11> P3) {
       11 \times 1 = P2.first - P1.first;
8
       11 y1 = P2.second - P1.second;
9
       11 \times 2 = P3.first - P1.first;
10
       11 y2 = P3.second - P1.second;
11
       return x1 * y2 - y1 * x2;
12
13
14
   double distancia(pair<11, 11> P1, pair<11, 11> P2) {
       return sqrt((P2.first - P1.first) * (P2.first - P1.first) +
                    (P2.second - P1.second) * (P2.second - P1.second));
18
19
20
21
   11 dot_product(pair<11, 11> P1, pair<11, 11> P2, pair<11, 11> P3) {
^{22}
       11 x1 = P2.first - P1.first;
23
       11 y1 = P2.second - P1.second;
^{24}
       11 x2 = P3.first - P1.first;
25
       11 y2 = P3.second - P1.second;
26
       return x1 * x2 + y1 * y2;
27
28 }
                            8.3 Polygon Area
```

```
1 #include <iostream>
   #include <vector>
   #include <cmath>
   using namespace std;
   typedef long long 11;
   typedef pair<11, 11> Point;
   double polygon_area(const vector<Point>& polygon) {
       11 \text{ area} = 0;
11
       int n = polygon.size();
12
       for (int i = 0; i < n; ++i) {
13
           11 j = (i + 1) \% n;
           area += (polygon[i].first * polygon[j].second - polygon[i].
15
               second * polygon[j].first);
       }
16
       return abs(area) / 2.0;
18 }
```

#### 8.4 Ray Casting

```
1 #include <iostream>
   #include <vector>
   using namespace std;
   typedef long long 11;
   typedef pair<11, 11> Point;
   bool is_point_in_polygon(const vector<Point>& polygon, Point p) {
       bool inside = false;
10
       int n = polygon.size();
11
       for (int i = 0, j = n - 1; i < n; j = i++) {
12
           if ((polygon[i].second > p.second) != (polygon[j].second > p.
13
                second) &&
               p.first < (polygon[j].first - polygon[i].first) * (p.second</pre>
14
                    - polygon[i].second) /
                          (polygon[j].second - polygon[i].second) + polygon[
15
                              i].first) {
               inside = !inside:
16
           }
17
18
```

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```
return inside;
```

20 }

#### 9 Trees

#### 9.1 Centroid

```
#include <bits/stdc++.h>
   using namespace std;
   #define L(i, j, n) for (int i = (j); i < int(n); i ++)
   #define ii pair<int, int>
   const int inf = 1e9;
   const int N = 1e5;
   vector<int> G[N];
   int ct[N];
   set<ii> dist[N];
   int up[N][18];
   int colors[N];
   int depth[N];
   int sz[N];
   bool removed[N];
   int n, root, L;
17
18
   int getSize(int u, int p){
19
       int szi = 1;
20
       for(int v: G[u]){
21
           if (p == v || removed[v]) continue;
22
           szi += getSize(v, u);
23
24
       return sz[u] = szi;
25
26
27
   int centroid(int u, int tree_size, int p){
28
       for (int v: G[u]){
29
           if (v == p || removed[v]) continue;
30
           if (sz[v] * 2 > tree_size) return centroid(v, tree_size, u);
31
       }
32
       return u;
33
34
35
  void build(int node, int tree_size, int p)
```

```
37 {
       getSize(node, - 1);
38
       int cen = centroid(node, tree_size, -1);
39
       removed[cen] = 1;
40
       ct[cen] = p;
41
       if (p == -1) root = cen;
42
43
       if (tree_size == 1) return;
44
45
       for (int v: G[cen]){
            if (removed[v]) continue;
47
            build(v, sz[v], cen);
48
       }
49
50
51
   void update(int v){
       int u = v;
       while(v != -1){
55
           dist[v].insert(distance(u, v), v);
            v = par[v];
57
       }
58
       return res;
59
60
61
   int query(int v){
       int u = v;
63
       int res = INT_MAX;
64
       while(v != -1){
65
            res = min(res, distance(u, v), dist[v].begin()->first); //
66
                Minimun
            v = par[v];
67
68
       return res;
69
70 }
```

#### 9.2 LCA

```
#include <bits/stdc++.h>
using namespace std;

#define L(i, j, n) for (int i = (j); i < int(n); i ++)
#define ii pair<int, int>
```

```
6 const int inf = 1e9;
                                                                                           }
                                                                                    49
   const int N = 1e5;
                                                                                   50
                                                                                   51
   vector<int> G[N], ct[N];
                                                                                    52
   set<ii> dist[N];
                                                                                      void dfs(int u, int p){
   int up[N][18];
                                                                                           up[u][0] = p;
                                                                                           for (int i = 1; i \le L; i ++){
   int colors[N];
   int depth[N];
                                                                                               if (up[u][i-1] != -1) up[u][i] = up[up[u][i-1]][i-1];
   int sz[N];
                                                                                               else up[u][i] = -1;
                                                                                   57
                                                                                           }
   bool removed[N];
   int n, root, L;
                                                                                           for (int v: G[u]){
                                                                                   59
                                                                                               if (v == p) continue;
17
                                                                                               depth[v] = depth[u] + 1;
   int getSize(int u, int p){
                                                                                   61
18
       int szi = 1;
                                                                                               dfs(v, u);
                                                                                   62
19
       for(int v: G[u]){
                                                                                           }
                                                                                    63
20
           if (p == v || removed[v]) continue;
                                                                                       }
21
                                                                                    64
           szi += getSize(v, u);
22
       }
                                                                                       int LCA(int u, int v){
23
       return sz[u] = szi:
                                                                                           if (depth[u] < depth[v]) swap(u, v);</pre>
                                                                                   67
24
                                                                                           for (int i = L; i \ge 0; i --){
25
                                                                                               if (up[u][i] != -1 && depth[up[u][i]] >= depth[v]){
26
                                                                                   69
   int centroid(int u, int tree_size, int p){
                                                                                                   u = up[u][i];
27
                                                                                   70
       for (int v: G[u]){
                                                                                               }
28
                                                                                   71
           if (v == p || removed[v]) continue;
                                                                                           }
                                                                                   72
29
           if (sz[v] * 2 > tree_size) return centroid(v, tree_size, u);
                                                                                           if (u == v) return u;
                                                                                   73
30
       }
                                                                                           for (int i = L; i \ge 0; i --){
                                                                                   74
31
                                                                                               if (up[u][i] != up[v][i] && up[u][i] != -1 && up[v][i] != -1){
       return u;
                                                                                   75
32
                                                                                                   u = up[u][i];
                                                                                   76
33
                                                                                                   v = up[v][i];
                                                                                   77
34
   void build(int node, int tree_size, int p)
                                                                                               }
35
                                                                                    78
36
                                                                                   79
                                                                                           return up[u][0];
       getSize(node, - 1);
37
                                                                                    80
       int cen = centroid(node, tree_size, -1);
                                                                                       }
                                                                                   81
38
       removed[cen] = 1:
                                                                                    82
39
       if (p != -1){
                                                                                       int dis(int u, int v){
40
           ct[cen].push_back(p);
                                                                                           int cmm = LCA(u, v);
41
                                                                                           // cout << u << " " << v << " " << cmm << "\n";
       } else root = cen;
42
                                                                                   85
                                                                                           return depth[u] + depth[v] - (2 * depth[cmm]);
                                                                                    86
43
       if (tree_size == 1) return;
                                                                                       }
                                                                                   87
44
45
       for (int v: G[cen]){
                                                                                       void uup(int u, int node){
46
           if (removed[v]) continue;
                                                                                           dist[u].insert({dis(u, node), node});
47
           build(v, sz[v], cen);
                                                                                           for (int v: ct[u])
48
                                                                                   91
```

```
uup(v, node);
92
   }
93
94
    void update(int node){
95
        dist[node].insert({0, node});
96
        for (int v: ct[node])
97
            uup(v, node);
98
99
100
    int qup(int u, int node){
101
        int mn = dis(node, u) + dist[u].begin()->first;
102
        for (int v: ct[u]) mn = min(mn, qup(v, node));
103
        return mn;
104
105
106
    int query(int node){
107
        int mn = dist[node].begin()->first;
108
        for (int v: ct[node]) mn = min(mn, qup(v, node));
109
        return mn;
110
111
112
    int main()
113
114
        ios::sync_with_stdio(0);cin.tie(0);
115
        int m; cin >> n >> m;
116
        L = log2(n);
117
        L(i, 1, n){
118
            int u, v; cin >> u >> v;
119
            u --; v --;
120
            G[u].push_back(v);
121
            G[v].push_back(u);
122
        }
123
        L(i, 0, n){
124
            dist[i].insert({inf, i});
125
        }
126
        build(0, n, -1);
127
        L(i, 0, L + 1) up[root][i] = -1;
128
        run(root, -1);
129
        update(0);
130
        L(_q, 0, m){
131
            int op, node; cin >> op >> node;
132
            if (op == 2){
133
                 cout << query(node-1) << '\n';</pre>
134
```