Dividimos y No Conquistamos (D&!C)

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#include <bits/stdc++.h>

4

1 Template

1.1 C++ Template

```
using namespace std;
   #define TESTS
   #define LOCAL
   #define ll long long
   #define ii pair<11, 11>
   #define F first
   #define S second
   #define formi(i, o, n) for (int i = o; i < n; i ++)
   #define forn(i, n) forni(i, 0, n)
   #define pub push_back
   #define popf pop_front
13
   #ifdef LOCAL
   #define DBG(x) cout << "[" << x << "]";
   #else
   #define DBG(x) 42
   #endif
19
20
21
   void solve(){
22
23
^{24}
25
26
   int main(){
27
       ios::sync_with_stdio(0);cin.tie(0);
28
   #ifdef LOCAL
29
       freopen("in.txt","r", stdio);
30
       freopen("out.txt","w", stdout);
31
   #endif
32
       int tt = 1;
33
   #ifdef TESTS
34
       cin >> tt;
35
   #endif
36
       while(tt--)solve();
37
  |}
38
```

2 Data structures

2.1 BIT

```
#define LSOne(S) (S & -S)
   struct BIT {
       vector<int> B;
4
       int n;
5
       BIT(int n = 1): B(n + 1), n(n+1){}
6
       BIT(vector<int> &v): B(v.size()+1), n(v.size()+1) {
            for (int i = 1; i \le n; i \leftrightarrow ++){
8
                B[i] += v[i-1];
                if (i + LSOne(i) <= n){</pre>
10
                    B[i + LSOne(i)] += B[i];
11
12
           }
13
14
       void update(int i, int x){
15
            while (i \le n){
16
                B[i] += x;
17
                i += LSOne(i);
18
           }
19
       }
20
       int sum(int i){
21
            int res = 0;
22
            while (i > 0){
23
                res += B[i];
24
                i -= LSOne(i);
25
           }
26
            return res;
27
       }
28
       int range_sum(int 1, int r){
29
            return sum(r) - sum(1 - 1);
30
31
32 };
                                        DSU
                                  2.2
struct DSU {
       vector<int> par, sz;
2
       int n;
3
```

DSU(int n = 1): par(n), sz(n, 1), n(n){

struct Node {

```
for (int i = 0; i < n; i ++) par[i] = i;
       }
6
       int find(int a){
7
           return a == par[a] ? a : par[a] = find(par[a]);
8
       }
9
       void join(int a, int b){
10
           a=find(a);
11
           b=find(b);
12
           if (a != b){
13
               if (sz[b] > sz[a]) swap(a,b);
14
               par[b] = a;
15
               sz[a] += sz[b];
16
           }
17
18
19 };
                                Sparse Table
  int log2_floor(unsigned long long i) {
       return i ? __builtin_clzll(1) - __builtin_clzll(i) : -1;
2
3
   const int MAXN = 10;
   int K = log2_floor(MAXN);
   int st[K + 1][MAXN];
   // Load Array to st[0][i]
   std::copy(array.begin(), array.end(), st[0]);
11
   // Build
  for (int i = 1; (1 << i) <= n; i ++){
       for (int j = 0; j + (1 << (i - 1)) < n; j ++){}
           st[i][j] = min(st[i-1][j], st[i-1][j + (1 << (i - 1))]);
15
       }
16
17
18
   // Query
19
   int min_range(int 1, int r){
20
       int C = log2\_floor(r - l + 1);
21
       return min(st[C][1], st[C][r - (1 << C) + 1]);
^{22}
23 }
                            2.4 Segment tree
```

```
long long sum = 0;
2
       long long min_val = LLONG_MAX;
3
       long long max_val = LLONG_MIN;
4
       long long lazy = 0;
5
6
       // Merge function to combine two nodes
7
       void merge(const Node& left, const Node& right) {
8
           sum = left.sum + right.sum;
           min_val = min(left.min_val, right.min_val);
           max_val = max(left.max_val, right.max_val);
11
       }
12
13
       // Update function for lazy propagation
       void apply(int 1, int r, long long value) {
15
           sum += (r - 1 + 1) * value;
16
           min_val += value;
17
           max_val += value;
           lazy += value;
19
       }
20
   };
21
22
   struct SegTree {
23
       int n;
24
       vector<Node> tree;
25
26
       SegTree(int n) : n(n) {
27
           tree.resize(4 * n + 5);
28
       }
29
30
       SegTree(vector<int>& arr) : n(arr.size()) {
31
           tree.resize(4 * n + 5);
32
           build(arr, 0, 0, n-1);
33
       }
34
35
       // Push lazy value to children
36
       void push(int id, int 1, int r) {
37
           if (tree[id].lazy == 0) return;
38
39
           int mid = (1 + r) >> 1;
40
           tree[2*id + 1].apply(1, mid, tree[id].lazy);
41
           tree[2*id + 2].apply(mid+1, r, tree[id].lazy);
42
           tree[id].lazy = 0;
43
```

```
}
44
45
       void build(vector<int>& arr, int id, int 1, int r) {
46
           if (1 == r) {
47
               tree[id].sum = arr[1];
48
               tree[id].min_val = arr[1];
49
               tree[id].max_val = arr[1];
               return;
51
           }
52
53
           int mid = (1 + r) >> 1;
54
           build(arr, 2*id + 1, 1, mid);
55
           build(arr, 2*id + 2, mid+1, r);
56
           tree[id].merge(tree[2*id + 1], tree[2*id + 2]);
57
       }
58
59
       // Range update with lazy propagation
60
       void update(int id, int 1, int r, int q1, int qr, long long val) {
61
           if (ql > r || qr < l) return;
62
63
           if (ql <= l && r <= qr) {
64
               tree[id].apply(l, r, val);
65
               return;
66
           }
67
68
           push(id, 1, r);
69
           int mid = (1 + r) >> 1;
70
           update(2*id + 1, 1, mid, ql, qr, val);
71
           update(2*id + 2, mid+1, r, ql, qr, val);
72
           tree[id].merge(tree[2*id + 1], tree[2*id + 2]);
73
       }
74
75
       // Range query
76
       Node query(int id, int 1, int r, int ql, int qr) {
77
           if (ql > r || qr < l) return Node();</pre>
78
79
           if (ql <= l && r <= qr) {
80
               return tree[id]:
81
           }
82
83
           push(id, 1, r);
84
           int mid = (1 + r) >> 1;
85
           Node left = query(2*id + 1, 1, mid, ql, qr);
86
```

```
Node right = query(2*id + 2, mid+1, r, ql, qr);
88
89
           Node result:
           result.merge(left, right);
           return result;
91
       }
92
93
       // Public interface
       void update(int 1, int r, long long val) {
           update(0, 0, n-1, 1, r, val);
       }
97
98
       Node query(int 1, int r) {
99
           return query(0, 0, n-1, 1, r);
100
       }
101
102 };
```

3 Dynamic Programming

3.1 Knapsack

```
int knapsack(vector<int>& values, vector<int>& weights, int W) {
       int n = values.size();
       vector<vector<int>> dp(n + 1, vector<int>(W + 1, 0));
4
       for(int i = 1; i <= n; i++) {
5
           for(int w = 0: w \le W: w++) {
6
               if(weights[i-1] <= w) {</pre>
7
                   dp[i][w] = max(dp[i-1][w],
                                  dp[i-1][w-weights[i-1]] + values[i-1]);
               } else {
10
                   dp[i][w] = dp[i-1][w];
11
12
           }
13
       return dp[n][W];
15
16 }
```

3.2 LIS

```
vector<int> getLIS(vector<int>& arr) {
   int n = arr.size();
   vector<int> dp(n + 1, INT_MAX); // dp[i] = smallest value that ends
   an LIS of length i
```

 $dp[i][j] = 1 + min({dp[i-1][j]},$

16

// deletion

```
vector<int> len(n);
                                         // Length of LIS ending at each
                                                                                                                         dp[i][j-1],
                                                                                                                                          // insertion
4
                                                                                   17
           position
                                                                                                                         dp[i-1][j-1]}); // replacement
                                                                                   18
       dp[0] = INT_MIN;
                                                                                   19
5
                                                                                              }
6
                                                                                  20
       for(int i = 0; i < n; i++) {
                                                                                          }
                                                                                  21
                                                                                          return dp[n][m];
           int j = upper_bound(dp.begin(), dp.end(), arr[i]) - dp.begin();
8
                                                                                   22
                                                                                  23 }
           dp[j] = arr[i];
9
           len[i] = j;
10
                                                                                                                  3.4 Kadane
       }
11
12
                                                                                   pair<int, pair<int,int>> kadane(vector<int>& arr) {
       // Find maxLen and reconstruct sequence
13
                                                                                          int maxSoFar = arr[0], maxEndingHere = arr[0];
       int maxLen = 0;
14
                                                                                          int start = 0, end = 0, s = 0;
       for(int i = n-1; i \ge 0; i--) maxLen = max(maxLen, len[i]);
                                                                                   3
15
                                                                                   4
16
                                                                                          for(int i = 1; i < arr.size(); i++) {</pre>
       vector<int> lis;
                                                                                   5
17
       for(int i = n-1, currLen = maxLen; i \ge 0; i--) {
                                                                                              if(maxEndingHere + arr[i] < arr[i]) {</pre>
                                                                                   6
18
                                                                                                  maxEndingHere = arr[i];
           if(len[i] == currLen) {
                                                                                   7
19
                                                                                                  s = i;
               lis.push_back(arr[i]);
                                                                                   8
20
                                                                                              } else {
               currLen--;
                                                                                   9
21
                                                                                                  maxEndingHere += arr[i];
           }
                                                                                   10
22
       }
                                                                                              }
                                                                                  11
23
       reverse(lis.begin(), lis.end());
                                                                                  12
24
                                                                                              if(maxEndingHere > maxSoFar) {
       return lis;
                                                                                  13
25
                                                                                                  maxSoFar = maxEndingHere;
26 }
                                                                                  14
                                                                                                  start = s;
                                                                                  15
                                 Edit Distance
                                                                                                  end = i;
                                                                                  16
                                                                                              }
                                                                                  17
                                                                                   18
1
                                                                                          return {maxSoFar, {start, end}}; // max, 1, r
    //3. Edit Distance - O(n*m)
                                                                                   19
                                                                                  20 }
   int editDistance(string& s1, string& s2) {
3
       int n = s1.length(), m = s2.length();
4
                                                                                                                       Strings
       vector<vector<int>> dp(n + 1, vector<int>(m + 1));
5
6
                                                                                                                4.1 Prefix Tree
       // Base cases
7
       for(int i = 0; i \le n; i++) dp[i][0] = i;
8
       for(int j = 0; j \le m; j++) dp[0][j] = j;
                                                                                     #include <bits/stdc++.h>
9
                                                                                   2
10
       for(int i = 1: i <= n: i++) {
                                                                                      using namespace std;
11
                                                                                   3
           for(int j = 1; j <= m; j++) {
                                                                                   4
12
               if(s1[i-1] == s2[j-1]) {
                                                                                      struct TrieNodeStruct {
13
                    dp[i][j] = dp[i-1][j-1];
                                                                                          TrieNodeStruct* children[26];
14
                                                                                   6
               } else {
                                                                                   7
                                                                                          bool isEndOfWord;
15
```

8

```
TrieNodeStruct() {
9
           isEndOfWord = false;
10
           for(int i = 0; i < 26; i++) {
11
                children[i] = nullptr;
12
           }
13
       }
14
15
16
   struct TrieStruct {
17
       TrieNodeStruct* root;
18
19
       TrieStruct() {
20
           root = new TrieNodeStruct();
21
       }
22
23
       void insert(string word) {
24
           TrieNodeStruct* current = root;
25
           for(char c : word) {
26
               int index = c - 'a';
27
               if(current->children[index] == nullptr) {
28
                    current->children[index] = new TrieNodeStruct();
29
               }
30
               current = current->children[index];
31
32
           current->isEndOfWord = true;
33
       }
34
35
       bool search(string word) {
36
           TrieNodeStruct* current = root;
37
           for(char c : word) {
38
               int index = c - 'a';
39
               if(current->children[index] == nullptr) {
40
                    return false:
41
42
               current = current->children[index];
43
44
           return current->isEndOfWord;
45
       }
46
47
       bool startsWithDirect(string prefix) {
48
           TrieNodeStruct* current = root;
49
           for(char c : prefix) {
50
               int index = c - 'a';
51
```

```
if(current->children[index] == nullptr) {
    return false;
}

current = current->children[index];

return true;
}

return true;
}

;
```

4.2 HASHING

```
struct StrHash { // Hash polinomial con exponentes decrecientes.
     static constexpr ll ms[] = {1'000'000'007, 1'000'000'403};
     static constexpr 11 b = 500'000'000;
     vector<11> hs[2], bs[2];
     StrHash(string const& s) {
5
       int n = sz(s);
6
       forn(k, 2) {
7
         hs[k].resize(n+1), bs[k].resize(n+1, 1);
         forn(i, n) {
           hs[k][i+1] = (hs[k][i] * b + s[i]) % ms[k];
10
           bs[k][i+1] = bs[k][i] * b
                                               % ms[k]:
11
         }
       }
13
     }
14
     ll get(int idx, int len) const { // Hashes en 's[idx, idx+len)'.
15
       ll h[2];
16
       forn(k, 2) {
17
         h[k] = hs[k][idx+len] - hs[k][idx] * bs[k][len] % ms[k];
18
         if (h[k] < 0) h[k] += ms[k];
19
20
       return (h[0] << 32) | h[1];
21
22
   };
23
24
   pll union_hash(pll l, pll r, ll len_r){ //pll = pair<11,11>
    1.first = ((1.first * binpow(b, len_r, ms[0])) % ms[0] + r.first) % ms
26
     1.second = ((1.second * binpow(b, len_r, ms[1])) % ms[1] + r.second) %
          ms[1]:
28
    return 1;
29
30 }
```

23 }

4.3 KMP

```
#include <bits/stdc++.h>
   using namespace std;
2
  vector<int> kmp(string pat, string sec){ //geeks4geeks implementation
       with some changes
     int m = pat.length();
     int n = sec.length();
     cout << m << "" << n << endl;
     vector<int> lps = getLps(pat);
     vector<int> res;
10
11
     int i = 0;
12
     int j = 0;
13
14
     while((n - i) >= (m - j)){
15
       if(pat[j] == sec[i]){
16
         i++;
17
         j++;
18
19
       if(j == m){
20
         res.push_back(i - j);
21
         j = lps[j - 1];
22
23
       else{
24
         if(i < n && pat[j] != sec[i]){</pre>
25
           if(j != 0) j = lps[ j - 1 ];
26
           else i = i + 1;
27
28
29
     }
30
31
     return res;
32
33
   vector<int> getLps(string pat){ //geek4geeks implementatio with some
35
       changes
     vector<int> lps(pat.length(), 0);
36
     int len = 0;
37
     int i = 1;
38
     lps[0] = 0;
```

```
while(i < pat.length()){</pre>
40
        if(pat[i] == pat[len]){
41
          len++;
42
          lps[i] = len;
43
          i++;
44
45
        else //pat[i] != pat[len]
46
47
          lps[i] = 0;
48
          i++;
50
     }
51
52
     return lps;
53
54 }
```

4.4 LPS

```
#include <bits/stdc++.h>
   using namespace std;
   vector<int> getLps(string pat){ //geek4geeks implementatio with some
       changes
     vector<int> lps(pat.length(), 0);
     int len = 0:
     int i = 1;
     lps[0] = 0;
     while(i < pat.length()){</pre>
9
       if(pat[i] == pat[len]){
10
         len++;
11
         lps[i] = len;
12
         i++;
13
14
       else //pat[i] != pat[len]
15
16
         lps[i] = 0;
17
         i++:
18
19
     }
20
21
22
     return lps;
```

4.5 Z FUNCTION

```
#include <bits/stdc++.h>
2
   using namespace std;
3
   vector<int> z_function(string s) {
       int n = s.length();
       vector<int> z(n, 0);
       z[0] = n;
9
10
       int 1 = 0, r = 0;
11
12
       for(int i = 1; i < n; i++) {
13
           if(i <= r) {
14
               z[i] = min(r - i + 1, z[i - 1]);
15
           }
16
17
           while(i + z[i] < n && s[z[i]] == s[i + z[i]]) {
18
                z[i]++;
19
           }
20
21
           if(i + z[i] - 1 > r) {
22
               1 = i:
23
               r = i + z[i] - 1;
24
           }
25
       }
26
27
28
       return z;
29
30
31
   vector<int> find_pattern(string text, string pattern) {
32
33
       string s = pattern + "$" + text;
34
       vector<int> z = z_function(s);
35
       vector<int> result:
36
37
38
       for(int i = pattern.length() + 1; i < s.length(); i++) {</pre>
39
           if(z[i] == pattern.length()) {
40
                result.push_back(i - pattern.length() - 1);
41
```

```
}
       }
43
44
       return result;
45
46 }
                                    Graph
                                5.1 Tarjan
_{1} | const int N = 10;
3 | vector<int> G[N];
   vector<int> dfs_low(N, -1), dfs_num(N, -1), ap(N, 0); // ap for
       Articulation Points
5 int dfs_count = 0;
   int root = -1; // For AP
   void dfs(int u, int p = -1){
       dfs_low[u]=dfs_num[u]=dfs_count++;
       int child = 0;
10
       for (int v: G[u]){
           if (v == p) continue;
12
           if (dfs_num[v] == -1){
               child ++;
14
               dfs(v, u);
               dfs_low[u] = min(dfs_low[u], dfs_low[v]);
16
               if (dfs_low[v] > dfs_num[u]){
17
                 // Bridge from u -> v
18
                 cout << "Bridge," << u << ",,->,," << v << "\n";
19
20
               if (dfs_low[v] >= dfs_num[u]) {
21
                   // u is AP
22
                   ap[u] = 1;
23
24
           } else dfs_low[u] = min(dfs_low[u], dfs_num[v]);
25
26
       if (u == root){
27
           ap[u] = child > 1;
28
29
30 }
```

Bellman Ford

```
struct Edge {
                                                                                   1 // Complexity (V * V * E);
                                                                                     struct Dinic {
       int a, b, cost;
  };
                                                                                         struct Edge {
                                                                                  3
3
                                                                                             int to, rev;
4
                                                                                  4
                                                                                             long long cap, flow;
  int n, m, v;
  vector<Edge> edges;
                                                                                             Edge(int to, int rev, long long cap) :
                                                                                                 to(to), rev(rev), cap(cap), flow(0) {}
   const int INF = 1000000000;
                                                                                         };
                                                                                  8
   void solve()
9
                                                                                  9
   {
                                                                                         vector<vector<Edge>> g;
10
       vector<int> d(n, INF);
                                                                                         vector<int> level, ptr;
                                                                                  11
11
       d[v] = 0;
                                                                                         queue<int> q;
12
                                                                                  12
       for (int i = 0; i < n - 1; ++i)
                                                                                         int n, source, sink;
                                                                                  13
           for (Edge e : edges)
                                                                                         const long long INF = 1e18;
               if (d[e.a] < INF)
                                                                                  15
15
                   d[e.b] = min(d[e.b], d[e.a] + e.cost);
                                                                                         Dinic(int n, int s, int t) : n(n), source(s), sink(t) {
                                                                                  16
16
                                                                                             g.resize(n);
17 | }
                                                                                  17
                                                                                             level.resize(n);
                                 5.3 SCC
                                                                                             ptr.resize(n);
                                                                                  19
                                                                                         }
                                                                                  20
   vector<int> dfs_num(N, -1), dfs_low(N, -1), visited(N);
                                                                                  21
   int dfs_count = 0;
                                                                                         void add_edge(int from, int to, long long cap) {
                                                                                  22
   int numSCC = 0;
                                                                                             g[from].emplace_back(to, g[to].size(), cap);
                                                                                  23
   stack<int> st;
                                                                                             g[to].emplace_back(from, g[from].size()-1, 0); // Reverse edge
                                                                                  24
  void dfs(int u){
                                                                                         }
                                                                                  25
     dfs_low[u] =dfs_num[u] =dfs_count++;
                                                                                  26
     st.push(u);
7
                                                                                         bool bfs() {
                                                                                  27
     visited[u] = 1:
8
                                                                                             while(!q.empty()) {
                                                                                  28
     for(int v: G[u]) {
                                                                                                 q.pop();
9
                                                                                  29
       if (dfs_num[v] == -1) dfs(v);
10
                                                                                             }
                                                                                  30
       if (visited[v]) dfs_low[u] = min(dfs_low[u], dfs_low[v]);
                                                                                             fill(level.begin(), level.end(), -1);
11
                                                                                  31
     }
12
                                                                                  32
     if (dfs_num[u] == dfs_low[u]){
                                                                                             q.push(source);
13
                                                                                  33
       numSCC ++;
                                                                                             level[source] = 0;
14
                                                                                  34
       while(1){
15
                                                                                  35
         int v = st.top(); st.pop();
                                                                                             while(!q.empty() && level[sink] == -1) {
16
                                                                                  36
         visited[v] = 0;
                                                                                                  int v = q.front();
17
                                                                                  37
         if (u == v) break;
18
                                                                                                  q.pop();
                                                                                  38
       }
19
                                                                                  39
     }
                                                                                                 for(const Edge& e : g[v]) {
20
                                                                                  40
21 }
                                                                                                      if(level[e.to] == -1 \&\& e.flow < e.cap) {
                                                                                  41
                                                                                                          level[e.to] = level[v] + 1;
                                                                                  42
                                 5.4 Flow
```

q.push(e.to);

```
}
44
                                                                                     87
                }
                                                                                                     }
45
                                                                                     88
           }
                                                                                                 }
46
                                                                                     89
           return level[sink] != -1;
                                                                                                 return flow;
47
                                                                                     90
       }
                                                                                             }
                                                                                     91
48
49
                                                                                     92
       long long dfs(int v, long long pushed) {
                                                                                             // Find minimum cut
                                                                                     93
50
           if(v == sink || pushed == 0) return pushed;
                                                                                             vector<bool> min_cut() {
51
                                                                                     94
                                                                                                 vector<bool> reachable(n, false);
52
                                                                                     95
           for(int& i = ptr[v]; i < (int)g[v].size(); i++) {</pre>
                                                                                                 queue<int> q;
53
                Edge& e = g[v][i];
                                                                                                 q.push(source);
54
                                                                                     97
                                                                                                 reachable[source] = true;
55
                                                                                     98
                if(level[e.to] != level[v] + 1 || e.flow >= e.cap) continue;
56
                                                                                     99
                                                                                                 while(!q.empty()) {
                                                                                     100
57
                long long flow = dfs(e.to, min(pushed, e.cap - e.flow));
                                                                                                     int v = q.front();
                                                                                     101
58
                if(flow == 0) continue;
                                                                                                     q.pop();
59
                                                                                     102
60
                                                                                     103
                e.flow += flow;
                                                                                                     for(const Edge& e : g[v]) {
61
                                                                                     104
                g[e.to][e.rev].flow -= flow;
                                                                                                         if(!reachable[e.to] && e.flow < e.cap) {</pre>
                                                                                     105
62
                return flow;
                                                                                                              reachable[e.to] = true;
63
           }
                                                                                                              q.push(e.to);
                                                                                     107
64
                                                                                                         }
           return 0;
65
                                                                                     108
       }
                                                                                                     }
                                                                                     109
66
                                                                                    110
67
       long long max_flow() {
                                                                                                 return reachable;
                                                                                    111
68
           long long flow = 0;
                                                                                             }
                                                                                    112
69
                                                                                    113 };
70
           while(bfs()) {
                                                                                    114
71
               fill(ptr.begin(), ptr.end(), 0);
                                                                                        // Example usage:
                                                                                    115
72
                while(long long pushed = dfs(source, INF)) {
                                                                                        /*
                                                                                    116
73
                    flow += pushed;
                                                                                        int main() {
74
                }
                                                                                             // Example: 6 vertices, source = 0, sink = 5
                                                                                    118
75
           }
                                                                                            int n = 6:
                                                                                    119
76
           return flow;
                                                                                             Dinic flow(n, 0, 5);
                                                                                    120
77
       }
                                                                                    121
78
                                                                                     122
                                                                                            // Add edges: (from, to, capacity)
79
       // Get the actual flow passing through each edge
                                                                                             flow.add_edge(0, 1, 16);
                                                                                    123
80
       vector<vector<long long>> get_flow() {
                                                                                             flow.add_edge(0, 2, 13);
                                                                                    124
81
           vector<vector<long long>> flow(n, vector<long long>(n, 0));
                                                                                             flow.add_edge(1, 2, 10);
                                                                                    125
82
           for(int v = 0; v < n; v++) {
                                                                                    126
                                                                                             flow.add_edge(1, 3, 12);
83
                for(const Edge& e : g[v]) {
                                                                                             flow.add_edge(2, 1, 4);
                                                                                    127
84
                    if(e.cap > 0) { // Only original edges, not residual
                                                                                            flow.add_edge(2, 4, 14);
                                                                                    128
85
                                                                                            flow.add_edge(3, 2, 9);
                        flow[v][e.to] = e.flow;
                                                                                    129
86
```

```
9 struct MaxFlow {
        flow.add_edge(3, 5, 20);
130
        flow.add_edge(4, 3, 7);
                                                                                             vector<Edge> edges;
                                                                                      10
131
        flow.add_edge(4, 5, 4);
                                                                                             vector<vector<int>> adj;
                                                                                      11
132
                                                                                             vector<int> level, ptr;
                                                                                      12
133
        // Calculate maximum flow
                                                                                             int n;
                                                                                      13
134
        long long max_flow = flow.max_flow();
                                                                                             queue<int> q;
135
        cout << "Maximum flow: " << max_flow << "\n";</pre>
136
                                                                                      15
                                                                                             MaxFlow(int n) : n(n) {
137
                                                                                      16
        // Get minimum cut
                                                                                                 adj.resize(n);
138
                                                                                      17
        vector<bool> cut = flow.min_cut();
                                                                                                 level.resize(n);
139
        cout << "Vertices on source side of min cut: ";</pre>
                                                                                                 ptr.resize(n);
140
                                                                                      19
        for(int i = 0; i < n; i++) {
                                                                                             }
141
                                                                                      20
            if(cut[i]) cout << i << " ":
142
                                                                                     21
        }
                                                                                             void add_edge(int from, int to, ll cap) {
                                                                                     22
143
        cout << "\n":
                                                                                                  edges.emplace_back(from, to, cap);
                                                                                     23
144
                                                                                                 edges.emplace_back(to, from, 0);
145
                                                                                      24
        // Get flow through each edge
                                                                                                 adj[from].push_back(edges.size() - 2);
146
                                                                                      25
        auto flow_matrix = flow.get_flow();
                                                                                                 adj[to].push_back(edges.size() - 1);
147
                                                                                      26
        cout << "Flow matrix:\n":</pre>
                                                                                             }
                                                                                      27
148
        for(int i = 0; i < n; i++) {
                                                                                      28
149
            for(int j = 0; j < n; j++) {
                                                                                             bool bfs(int s, int t) {
                                                                                     29
150
                if(flow_matrix[i][j] > 0) {
                                                                                                 while(!q.empty()) q.pop();
151
                                                                                      30
                     cout << i << " -> " << j << ": " << flow_matrix[i][j] <</pre>
                                                                                                 fill(level.begin(), level.end(), -1);
152
                                                                                     31
                          "\n";
                                                                                      32
                                                                                                 q.push(s);
                                                                                      33
153
                                                                                                 level[s] = 0;
                                                                                      34
154
155
                                                                                      35
                                                                                                 while(!q.empty() && level[t] == -1) {
                                                                                      36
156
                                                                                                      int v = q.front();
        return 0;
157
                                                                                      37
                                                                                                      q.pop();
158
                                                                                      38
159
                                                                                      39
                                                                                                      for(int id : adj[v]) {
                                                                                      40
                                  Ford Fulkerson
                                                                                                          if(level[edges[id].to] == -1 && edges[id].cap - edges[id
                                                                                      41
                                                                                                              1.flow > 0) {
                                                                                                              level[edges[id].to] = level[v] + 1;
   #define ll long long
                                                                                      42
                                                                                                              q.push(edges[id].to);
   const 11 INF = (11)4e18;
                                                                                      43
                                                                                                          }
   struct Edge {
                                                                                      44
                                                                                                      }
        int from, to;
                                                                                      45
 4
        ll cap, flow;
                                                                                      46
 5
                                                                                                 return level[t] != -1;
        Edge(int from, int to, ll cap) : from(from), to(to), cap(cap), flow
                                                                                      47
 6
            (0) \{ \}
                                                                                      48
7 | };
                                                                                      49
```

11 dfs(int v, int t, ll pushed) {

```
if(v == t || pushed == 0)
51
                return pushed;
52
53
            for(; ptr[v] < (int)adj[v].size(); ptr[v]++) {</pre>
54
                int id = adj[v][ptr[v]];
55
                int u = edges[id].to;
56
57
                if(level[u] != level[v] + 1) continue;
58
59
                ll tr = dfs(u, t, min(pushed, edges[id].cap - edges[id].flow
60
                    ));
                if(tr > 0) {
61
                     edges[id].flow += tr;
62
                     edges[id ^ 1].flow -= tr;
63
                    return tr;
64
                }
65
            }
66
            return 0;
67
       }
68
69
       11 max_flow(int s, int t) {
70
            11 \text{ flow} = 0;
71
            while(bfs(s, t)) {
72
                fill(ptr.begin(), ptr.end(), 0);
73
                while(ll pushed = dfs(s, t, LLONG_MAX)) {
74
                    flow += pushed;
75
                }
76
77
            return flow;
78
       }
79
80
       vector<ll> get_flows() {
81
            vector<ll> flows:
82
           for(int i = 0; i < edges.size(); i += 2) {</pre>
83
                flows.push_back(edges[i].flow);
84
            }
85
            return flows;
86
87
88 };
```

6 Math

6.1 BINARY POW

```
1 #include <iostream>
   using namespace std;
   typedef long long 11;
   11 \mod = 1e9+7;
   ll binary_pow(ll base, ll exp) {
       ll result = 1;
       base %= mod;
9
       while (exp > 0) {
           if (\exp \% 2 == 1) {
11
               result = (result * base) % mod;
12
           }
13
           base = (base * base) % mod;
           exp /= 2;
15
      }
16
17
      return result;
18
19 }
                            6.2 CATALAN
1 | ll catalan(ll n) {
       if (n == 0) return 1;
       11 catalan_num = (fact[2 * n] * inv_fact[n] % MOD) * inv_fact[n + 1]
            % MOD;
      return catalan_num;
4
5 }
                            COMBINATORICS
vector<ll> fact, inv_fact;
  void precompute_factorials(ll n, ll mod) {
      fact.resize(n + 1);
       inv_fact.resize(n + 1);
4
      fact[0] = inv_fact[0] = 1;
5
      for (ll i = 1; i <= n; i++) {
           fact[i] = (fact[i - 1] * i) % mod;
7
      }
8
```

inv_fact[n] = mod_inverse(fact[n], mod);

6.4 EUCLIDEAN EXTENDED

```
| 11 extendedGCD(11 a, 11 b, 11 &x, 11 &y) {
       if (b == 0) {
           x = 1;
3
           v = 0;
           return a;
5
       }
6
       ll x1, y1;
       ll gcd = extendedGCD(b, a % b, x1, y1);
       x = y1;
9
       y = x1 - (a / b) * y1;
10
       return gcd;
11
12
13
   bool findSolutionWithConstraints(ll a, ll b, ll c, ll x_min, ll y_min,
       11 &x, 11 &y) {
       11 g = extendedGCD(a, b, x, y);
15
16
       if (c % g != 0) return false;
17
18
       x *= c / g;
19
       y *= c / g;
20
21
       // Ajustamos las variables a/g y b/g para mover las soluciones
22
       a /= g;
23
       b /= g;
^{24}
25
       if (x < x min) {
26
           ll k = (x_min - x + b - 1) / b; // Redondeo hacia arriba
27
           x += k * b;
28
           y -= k * a;
29
       } else if (x > x_min) {
30
```

```
11 k = (x - x min) / b:
31
           x -= k * b;
32
           y += k * a;
33
34
35
       if (y < y_min) {
           ll k = (y_min - y + a - 1) / a; // Redondeo hacia arriba
37
           x += k * b;
           y -= k * a;
       } else if (y > y_min) {
           11 k = (y - y_min) / a;
41
           x -= k * b;
42
           y += k * a;
43
       }
44
45
       return x >= x_min && y >= y_min;
46
47 }
```

6.5 EULER TOTIENT

```
#include <bits/stdc++.h>
  using namespace std;
   typedef long long 11;
   vector<ll> compute_totients(ll n) {
       vector<ll> phi(n + 1);
7
       for (ll i = 0; i <= n; i++) {
8
           phi[i] = i;
9
       }
10
11
       for (ll i = 2; i <= n; i++) {
12
           if (phi[i] == i) { // i es primo
13
               for (ll j = i; j <= n; j += i) {
14
                   phi[j] = phi[j] * (i - 1) / i;
15
16
           }
17
       }
18
19
       return phi;
20
21 }
```

6.6 JOSEPHUS

```
#include <iostream>
   using namespace std;
2
    typedef long long 11;
   ll josephus_iterative(ll n, ll k) {
       11 result = 0;
       for (ll i = 2; i <= n; ++i) {
8
            result = (result + k) % i;
9
       }
10
       return result;
11
12
13
14
   ll josephus_recursive(ll n, ll k) {
15
16
       if (n == 1)
17
            return 0;
18
19
       return (josephus_recursive(n - 1, k) + k) % n;
20
21
^{22}
23
   11 josephus_power_of_2(11 n) {
24
25
       11 power = 1;
26
       while (power <= n) {</pre>
27
            power <<= 1;
28
       }
29
       power >>= 1;
30
31
32
       return 2 * (n - power);
33
34 | }
```

6.7 MOBIUS

```
#include <bits/stdc++.h>
using namespace std;
typedef long long ll;

vector<ll> compute_mobius(ll n) {
```

```
vector<ll> mu(n + 1, 1);
       vector<bool> is_prime(n + 1, true);
8
9
       for (ll i = 2; i <= n; i++) {
10
           if (is_prime[i]) { // i es un primo
11
                for (ll j = i; j <= n; j += i) {
12
                    mu[j] *= -1; // Multiplicamos por -1 para cada primo
13
                    is_prime[j] = false;
               }
15
                for (ll j = i * i; j \le n; j += i * i) {
                    mu[j] = 0; // Si tiene un cuadrado de un primo, se pone
17
               }
18
           }
       }
20
21
22
       return mu;
23
24
25
   ll mobius(ll x) {
       11 count = 0;
27
       for (ll i = 2; i * i <= x; i++) {
28
           if (x \% (i * i) == 0)
29
               return 0;
30
           if (x \% i == 0) {
31
                count++;
                x /= i;
           }
34
       }
35
36
       if (x > 1) count++;
37
38
       return (count % 2 == 0) ? 1 : -1;
39
40 }
```

6.8 NTT

```
#include <bits/stdc++.h>
using namespace std;
using cd = complex<double>;
typedef long long l1;
const l1 mod = 998244353;
```

```
6 const ll root = 31;
   const ll root_1 = inverse(root, mod);
   const 11 root_pw = 1 << 23;</pre>
   ll inverse(ll a, ll m) {
       11 \text{ res} = 1, \exp = m - 2;
11
       while (exp) {
12
           if (exp % 2 == 1) res = (1LL * res * a) % m;
13
           a = (1LL * a * a) % m;
14
           exp /= 2;
15
       }
16
       return res;
17
18
19
   void ntt(vector<11> & a, bool invert) {
20
       int n = a.size();
21
22
       for (int i = 1, j = 0; i < n; i++) {
23
           int bit = n \gg 1:
24
           for (; j & bit; bit >>= 1)
25
                j ^= bit;
26
           j ^= bit;
27
28
           if (i < j)
29
                swap(a[i], a[j]);
30
       }
31
32
       for (int len = 2; len <= n; len <<= 1) {
33
           int wlen = invert ? root_1 : root;
34
           for (int i = len; i < root_pw; i <<= 1)</pre>
35
                wlen = (int)(1LL * wlen * wlen % mod);
36
37
           for (int i = 0; i < n; i += len) {
38
                int w = 1:
39
                for (int j = 0; j < len / 2; j++) {
40
                    int u = a[i+j], v = (int)(1LL * a[i+j+len/2] * w % mod);
41
                    a[i+j] = u + v < mod ? u + v : u + v - mod;
42
                    a[i+j+len/2] = u - v >= 0 ? u - v : u - v + mod;
43
                    w = (int)(1LL * w * wlen % mod);
44
45
           }
46
       }
47
48
```

```
if (invert) {
49
           int n_1 = inverse(n, mod);
50
           for (auto & x : a)
51
                x = (int)(1LL * x * n_1 \% mod);
52
       }
53
   }
54
55
   vector<ll> multiply(vector<ll> const &a, vector<ll> const &b) {
       vector<ll> fa(a.begin(), a.end()), fb(b.begin(), b.end());
57
       11 n = 1;
       while (n < a.size() + b.size())</pre>
59
           n <<= 1:
       fa.resize(n);
       fb.resize(n):
       ntt(fa, false);
       ntt(fb, false);
       for (ll i = 0; i < n; i++)
           fa[i] = (fa[i] * fb[i]) % mod;
67
       ntt(fa, true);
       vector<ll> result(n);
       for (ll i = 0; i < n; i++)
71
           result[i] = fa[i];
72
       return result;
73
74 }
```

6.9 PRIME FACTORIZATION

```
1
   vector<pair<11, 11>> prime_factors(11 n) {
       vector<pair<11, 11>> factors;
3
       for (ll i = 2; i * i <= n; i++) {
4
           if (n % i == 0) {
5
                11 count = 0;
6
                while (n \% i == 0) {
7
                    n /= i;
8
9
                    count++;
10
                factors.push_back({i, count});
11
           }
12
       }
13
       if (n > 1) factors.push_back(\{n, 1\});
```

```
return factors;
15
   }
16
17
18
   vector<ll> divisors(ll n) {
19
       vector<ll> divs;
20
       for (ll i = 1; i * i <= n; i++) {
21
            if (n % i == 0) {
^{22}
                divs.push_back(i);
23
                if (i != n / i) { // Evita duplicar si n es un cuadrado
^{24}
                    perfecto
                    divs.push_back(n / i);
25
                }
26
           }
27
       }
28
       sort(divs.begin(), divs.end()); // Ordena los divisores en orden
29
            ascendente
       return divs;
30
31 }
```

6.10 **SIEVE**

```
#include <iostream>
   #include <vector>
   using namespace std;
   typedef long long 11;
   vector<ll> sieve_of_eratosthenes(ll n) {
8
       vector<ll> primes;
9
       vector<ll> primoRel(n,0);
10
       for(int i = 2; i < n; i++){
11
           if(!primoRel[i]){
12
               primes.push_back(i);
13
               for(int j = i*i; j < n; j+=i){
14
                    primoRel[j] = i;
15
               }
16
           }
17
       }
18
19
       return primes;
20
21 | }
```

6.11 fft

```
#include <bits/stdc++.h>
   using namespace std;
   using cd = complex<double>;
   typedef long long 11;
   const double PI = acos(-1);
   void fft(vector<cd> &a, bool invert) {
       ll n = a.size();
       if (n == 1)
           return;
10
       vector<cd> a0(n / 2), a1(n / 2);
11
       for (ll i = 0; 2 * i < n; i++) {
12
           a0[i] = a[2 * i];
13
           a1[i] = a[2 * i + 1];
14
15
       fft(a0, invert);
16
       fft(a1, invert);
       double ang = 2 * PI / n * (invert ? -1 : 1);
18
       cd w(1), wn(cos(ang), sin(ang));
       for (ll i = 0; 2 * i < n; i++) {
20
           a[i] = a0[i] + w * a1[i];
           a[i + n / 2] = a0[i] - w * a1[i];
           if (invert) {
                a[i] /= 2;
24
                a[i + n / 2] /= 2;
25
           }
26
           w *= wn;
27
28
29
30
   vector<ll> multiply(vector<ll> const &a, vector<ll> const &b) {
31
       vector<cd> fa(a.begin(), a.end()), fb(b.begin(), b.end());
32
       11 n = 1;
33
       while (n < a.size() + b.size())</pre>
34
           n <<= 1;
35
       fa.resize(n):
36
       fb.resize(n);
37
38
       fft(fa, false);
39
       fft(fb, false);
40
       for (ll i = 0; i < n; i++)
41
```

```
fa[i] *= fb[i]:
42
                                                                               85
       fft(fa, true);
                                                                               86
43
                                                                                      for (int i = 1, j = 0; i < n; ++i) {
                                                                               87
44
       vector<ll> result(n);
                                                                                          int bit = n \gg 1;
45
      for (ll i = 0; i < n; i++)
                                                                                          for (; j & bit; bit >>=1)
                                                                               89
46
           result[i] = round(fa[i].real());
                                                                                              j ^= bit;
47
                                                                                          j ^= bit;
       return result;
48
                                                                                          if (i < j)
49
                                                                                              swap(a[i], a[j]);
50
                                                                                      }
51
   //Exponensiacion binommial-----
                                                                               95
52
                                                                                      // Cooley-Tukey FFT
                                                                               96
   vector<ll> binomial_exponentiation(const vector<ll> &a, int exp) {
                                                                                      for (int len = 2; len <= n; len <<=1) {
                                                                               97
       vector<ll> result = {1};
                                                                                          ld ang = 2 * PI / len * (invert ? -1 : 1);
55
       vector<ll> base = a;
                                                                                          cd wlen(cosl(ang), sinl(ang));
56
                                                                                          for (int i = 0; i < n; i += len) {
57
       while (exp > 0) {
                                                                                              cd w(1);
58
           if (exp % 2 == 1) {
                                                                                              int len2 = len >> 1;
59
               result = multiply(result, base);
                                                                                              for (int j = 0; j < len2; ++j) {
                                                                               103
60
                                                                                                  cd u = a[i + j];
61
           base = multiply(base, base);
                                                                                                  cd v = a[i + j + len2] * w;
62
           exp /= 2;
                                                                                                  a[i + j] = u + v;
63
       }
                                                                                                  a[i + j + len2] = u - v;
                                                                               107
64
                                                                                                  w *= wlen;
                                                                               108
65
       while (result.size() > 1 && result.back() == 0) {
                                                                                              }
                                                                               109
66
           result.pop_back();
                                                                               110
67
       }
                                                                                      }
                                                                               111
68
                                                                               112
69
       return result;
                                                                               113
70
                                                                                      if (invert) {
71
                                                                                          for (cd & x : a)
                                                                               115
72
   //FFT PRECISO -----
                                                                                              x /= n;
                                                                               116
                                                                                      }
                                                                               117
74
   #define ll long long
                                                                               118
   using namespace std;
                                                                               119
   const double pi = acos(-1);
                                                                                  vector<ll> multiply(const vector<ll>& a, const vector<ll>& b) {
                                                                                      const 11 BASE = 1e6;
                                                                               121
   typedef long double ld;
                                                                               122
   typedef complex<ld> cd;
                                                                                      int n = 1;
                                                                               123
   const ld PI = acos(-1);
                                                                                      while(n < (int)(a.size() + b.size()))</pre>
                                                                                          n <<= 1;
                                                                               125
   void fft(vector<cd>& a, bool invert) {
                                                                               126
       int n = a.size();
                                                                                      vector<cd> al(n), ah(n), bl(n), bh(n);
84
                                                                              127
```

```
for (size_t i = 0; i < a.size(); ++i) {
128
            al[i] = a[i] \% BASE;
129
            ah[i] = a[i] / BASE;
130
        }
131
        for (size_t i = 0; i < b.size(); ++i) {</pre>
132
            bl[i] = b[i] \% BASE;
133
            bh[i] = b[i] / BASE;
134
        }
135
136
        fft(al, false);
137
        fft(ah, false);
138
        fft(bl, false);
139
        fft(bh. false):
140
141
        vector<cd> lx(n), lh(n), hl(n), hh(n);
142
        for (int i = 0; i < n; ++i) {
143
            lx[i] = al[i] * bl[i];
144
            lh[i] = al[i] * bh[i];
145
            hl[i] = ah[i] * bl[i]:
146
            hh[i] = ah[i] * bh[i];
147
        }
148
149
        fft(lx, true);
150
        fft(lh, true);
151
        fft(hl, true);
152
        fft(hh, true);
153
154
        vector<ll> result(n);
155
        for (int i = 0; i < n; ++i) {
156
            11 temp_ll = llround(lx[i].real());
157
            ll temp_lh = llround(lh[i].real());
158
            ll temp_hl = llround(hl[i].real());
159
            11 temp_hh = llround(hh[i].real());
160
161
            result[i] = temp_ll +
162
                          ((temp_lh + temp_hl) * BASE) +
163
                          (temp_hh * BASE * BASE);
164
        }
165
166
        return result;
167
168
169
    // mejor version
```

```
171
    typedef long long 11;
    typedef complex<double> C;
    typedef vector<double> vd;
    typedef vector<ll> vll;
    const double PI = acos(-1);
177
    void fft(vector<C>& a) {
        int n = a.size(), L = 31 - __builtin_clz(n);
179
        static vector<C> R(2, 1);
180
        static vector<C> rt(2, 1):
181
        for (static int k = 2: k < n: k *= 2) {
            R.resize(n); rt.resize(n);
183
            auto x = polar(1.0, PI / k);
            for (int i = k; i < 2 * k; i++)
185
                rt[i] = R[i] = i & 1 ? R[i / 2] * x : R[i / 2];
186
        }
187
        vector<int> rev(n);
188
        for (int i = 0; i < n; i++) rev[i] = (rev[i / 2] | (i & 1) << L) /
        for (int i = 0; i < n; i++) if (i < rev[i]) swap(a[i], a[rev[i]]);
        for (int k = 1; k < n; k *= 2)
            for (int i = 0; i < n; i += 2 * k) for (int j = 0; j < k; j++) {
192
                auto x = (double*)&rt[j + k], v = (double*)&a[i + j + k];
193
                C z(x[0] * y[0] - x[1] * y[1], x[0] * y[1] + x[1] * y[0]);
194
                a[i + j + k] = a[i + j] - z;
195
                a[i + j] += z;
196
            }
197
198
199
    vll multiply(const vll& a, const vll& b) {
200
        if (a.empty() || b.empty()) return {};
201
        vd fa(a.begin(), a.end()), fb(b.begin(), b.end());
202
        int L = 32 - \_builtin\_clz(fa.size() + fb.size() - 1), n = 1 << L;
203
        vector<C> in(n), out(n);
204
205
        for (int i = 0; i < a.size(); i++) in[i] = C(fa[i], 0);
206
        for (int i = 0; i < b.size(); i++) in[i].imag(fb[i]);</pre>
207
208
        fft(in);
209
        for (C\& x : in) x *= x;
210
```

```
for (int i = 0; i < n; i++) out[i] = in[-i & (n - 1)] - conj(in[i]);
211
               // Corregido aqui
        fft(out):
^{212}
213
        vll res(a.size() + b.size() - 1);
214
        for (int i = 0; i < res.size(); i++) {</pre>
215
            res[i] = llround(imag(out[i]) / (4 * n));
216
        }
217
        return res;
218
219 }
```

7 Geometry

7.1 CONVEX HULL

```
#include <iostream>
   #include <vector>
   #include <algorithm>
   using namespace std;
   typedef long long 11;
   typedef pair<11, 11> Point;
   11 cross_product(Point 0, Point A, Point B) {
       return (A.first - O.first) * (B.second - O.second) - (A.second - O.
10
           second) * (B.first - 0.first);
11
12
   vector<Point> convex_hull(vector<Point>& points) {
13
       sort(points.begin(), points.end());
14
       points.erase(unique(points.begin(), points.end()), points.end());
15
       vector<Point> hull;
16
17
       // Parte inferior
18
       for (const auto& p : points) {
19
           while (hull.size() >= 2 && cross_product(hull[hull.size() - 2],
20
               hull[hull.size() - 1], p) < 0)
               hull.pop_back();
21
           if (hull.empty() || hull.back() != p) {
22
               hull.push_back(p);
23
           }
24
       }
25
26
```

```
// Parte superior
27
       int t = hull.size() + 1;
28
       for (int i = points.size() - 1; i >= 0; --i) {
29
            while (hull.size() >= t && cross_product(hull[hull.size() - 2],
30
                hull[hull.size() - 1], points[i]) < 0)</pre>
                hull.pop_back();
31
           if (hull.empty() || hull.back() != points[i]) {
32
                hull.push_back(points[i]);
33
           }
34
       }
35
36
       hull.pop_back();
37
       return hull:
38
39 }
```

7.2 OPERATIONS

```
#include <bits/stdc++.h>
   using namespace std;
   typedef long long 11;
   11 cross_product(pair<11, 11> P1, pair<11, 11> P2, pair<11, 11> P3) {
       11 x1 = P2.first - P1.first:
       11 y1 = P2.second - P1.second;
9
       11 x2 = P3.first - P1.first;
10
       11 y2 = P3.second - P1.second;
11
       return x1 * y2 - y1 * x2;
12
   }
13
14
15
   double distancia(pair<11, 11> P1, pair<11, 11> P2) {
       return sqrt((P2.first - P1.first) * (P2.first - P1.first) +
17
                     (P2.second - P1.second) * (P2.second - P1.second));
18
19
20
21
   11 dot_product(pair<11, 11> P1, pair<11, 11> P2, pair<11, 11> P3) {
       11 x1 = P2.first - P1.first;
23
       11 y1 = P2.second - P1.second;
24
       11 \times 2 = P3.first - P1.first;
25
       11 \text{ y2} = P3.\text{second} - P1.\text{second};
26
```

28 }

return x1 * x2 + y1 * y2;

```
POLYGON AREA
   #include <iostream>
   #include <vector>
   #include <cmath>
   using namespace std;
5
   typedef long long 11;
   typedef pair<11, 11> Point;
   double polygon_area(const vector<Point>& polygon) {
10
       11 \text{ area} = 0:
11
       int n = polygon.size();
12
       for (int i = 0; i < n; ++i) {
13
           11 j = (i + 1) \% n;
14
           area += (polygon[i].first * polygon[j].second - polygon[i].
15
               second * polygon[j].first);
16
       return abs(area) / 2.0:
17
18 }
```

7.4 RAY CASTING

```
#include <iostream>
   #include <vector>
   using namespace std;
   typedef long long 11;
   typedef pair<11, 11> Point;
   bool is_point_in_polygon(const vector<Point>& polygon, Point p) {
9
       bool inside = false;
10
       int n = polygon.size();
11
       for (int i = 0, j = n - 1; i < n; j = i++) {
12
           if ((polygon[i].second > p.second) != (polygon[j].second > p.
13
               p.first < (polygon[j].first - polygon[i].first) * (p.second</pre>
14
                    - polygon[i].second) /
```

8 Trees

8.1 Centroid

```
#include <bits/stdc++.h>
   using namespace std;
   #define L(i, j, n) for (int i = (j); i < int(n); i ++)
   #define ii pair<int, int>
   const int inf = 1e9;
   const int N = 1e5;
   vector<int> G[N];
   int ct[N];
   set<ii> dist[N];
   int up[N][18];
   int colors[N];
   int depth[N];
   int sz[N];
   bool removed[N];
   int n, root, L;
18
   int getSize(int u, int p){
       int szi = 1;
20
       for(int v: G[u]){
21
           if (p == v || removed[v]) continue;
22
           szi += getSize(v, u);
23
24
       return sz[u] = szi;
25
   }
26
27
   int centroid(int u, int tree_size, int p){
28
       for (int v: G[u]){
29
           if (v == p || removed[v]) continue;
30
           if (sz[v] * 2 > tree_size) return centroid(v, tree_size, u);
31
```

```
}
32
       return u;
33
   }
34
35
   void build(int node, int tree_size, int p)
37
       getSize(node, - 1);
38
       int cen = centroid(node, tree_size, -1);
39
       removed[cen] = 1;
40
       ct[cen] = p;
41
       if (p == -1) root = cen;
42
43
       if (tree_size == 1) return;
44
45
       for (int v: G[cen]){
46
           if (removed[v]) continue;
47
           build(v, sz[v], cen);
48
       }
49
50
51
52
   void update(int v){
53
       int u = v;
54
       while(v != -1){
55
           dist[v].insert(distance(u, v), v);
56
           v = par[v];
57
       }
58
       return res;
59
60
61
   int query(int v){
62
       int u = v;
63
       int res = INT MAX:
64
       while(v != -1){
65
           res = min(res, distance(u, v), dist[v].begin()->first); //
66
                Minimun
           v = par[v];
67
       }
68
       return res;
69
70 }
```

```
#include <bits/stdc++.h>
  using namespace std;
   #define L(i, j, n) for (int i = (j); i < int(n); i ++)
   #define ii pair<int, int>
   const int inf = 1e9;
   const int N = 1e5;
   vector<int> G[N], ct[N];
   set<ii> dist[N];
  int up[N][18];
   int colors[N];
  int depth[N];
   int sz[N];
  bool removed[N];
   int n, root, L;
   int getSize(int u, int p){
       int szi = 1;
19
       for(int v: G[u]){
           if (p == v || removed[v]) continue;
21
           szi += getSize(v, u);
22
23
       return sz[u] = szi;
24
   }
25
26
   int centroid(int u, int tree_size, int p){
27
       for (int v: G[u]){
28
           if (v == p || removed[v]) continue;
29
           if (sz[v] * 2 > tree_size) return centroid(v, tree_size, u);
30
       }
31
       return u;
32
   }
33
34
   void build(int node, int tree_size, int p)
35
   {
36
       getSize(node, - 1);
37
       int cen = centroid(node, tree_size, -1);
       removed[cen] = 1;
39
       if (p != -1){
40
           ct[cen].push_back(p);
41
       } else root = cen;
42
43
```

```
87 }
       if (tree_size == 1) return;
44
                                                                                      88
45
       for (int v: G[cen]){
                                                                                         void uup(int u, int node){
46
                                                                                      89
            if (removed[v]) continue;
                                                                                             dist[u].insert({dis(u, node), node});
47
           build(v, sz[v], cen);
                                                                                             for (int v: ct[u])
                                                                                     91
48
       }
                                                                                                  uup(v, node);
49
                                                                                      92
                                                                                         }
                                                                                      93
50
51
                                                                                      94
                                                                                         void update(int node){
52
   void dfs(int u, int p){
                                                                                             dist[node].insert({0, node});
53
                                                                                             for (int v: ct[node])
       up[u][0] = p;
54
                                                                                      97
       for (int i = 1; i \le L; i \leftrightarrow \}
                                                                                                  uup(v, node);
55
                                                                                      98
            if (up[u][i-1] != -1) up[u][i] = up[up[u][i-1]][i-1];
                                                                                         }
                                                                                      99
56
            else up[u][i] = -1;
                                                                                     100
57
       }
                                                                                         int qup(int u, int node){
58
       for (int v: G[u]){
                                                                                             int mn = dis(node, u) + dist[u].begin()->first;
                                                                                     102
59
                                                                                             for (int v: ct[u]) mn = min(mn, qup(v, node));
           if (v == p) continue;
60
                                                                                     103
           depth[v] = depth[u] + 1;
                                                                                             return mn;
61
                                                                                     104
           dfs(v, u);
                                                                                         }
                                                                                     105
62
       }
                                                                                     106
63
                                                                                         int query(int node){
                                                                                     107
64
                                                                                             int mn = dist[node].begin()->first;
65
   int LCA(int u, int v){
                                                                                             for (int v: ct[node]) mn = min(mn, qup(v, node));
                                                                                     109
66
       if (depth[u] < depth[v]) swap(u, v);</pre>
                                                                                             return mn;
                                                                                     110
67
       for (int i = L; i \ge 0; i --){
                                                                                         }
                                                                                     111
68
           if (up[u][i] != -1 && depth[up[u][i]] >= depth[v]){
                                                                                     112
69
                u = up[u][i];
                                                                                         int main()
                                                                                     113
70
           }
                                                                                         {
                                                                                     114
71
       }
                                                                                             ios::sync_with_stdio(0);cin.tie(0);
                                                                                     115
72
       if (u == v) return u:
                                                                                             int m; cin >> n >> m;
                                                                                     116
73
       for (int i = L; i >= 0; i --){
                                                                                             L = log2(n);
                                                                                     117
74
           if (up[u][i] != up[v][i] \&\& up[u][i] != -1 \&\& up[v][i] != -1){
                                                                                             L(i, 1, n){
                                                                                     118
75
                u = up[u][i];
                                                                                                 int u, v; cin >> u >> v;
                                                                                     119
76
                v = up[v][i];
                                                                                                 u --; v --;
                                                                                     120
77
           }
                                                                                                  G[u].push_back(v);
                                                                                     121
78
                                                                                                 G[v].push_back(u);
                                                                                     122
79
       return up[u][0];
                                                                                             }
                                                                                     123
80
                                                                                             L(i, 0, n){
                                                                                     124
81
                                                                                                 dist[i].insert({inf, i});
                                                                                     125
82
   int dis(int u, int v){
                                                                                             }
                                                                                     126
83
                                                                                             build(0, n, -1);
       int cmm = LCA(u, v);
                                                                                     127
84
       // cout << u << " " << v << " " << cmm << "\n";
                                                                                             L(i, 0, L + 1) up[root][i] = -1;
                                                                                     128
85
       return depth[u] + depth[v] - (2 * depth[cmm]);
                                                                                             run(root, -1);
86
                                                                                     129
```

```
update(0);
130
        L(_q, 0, m){
131
            int op, node; cin >> op >> node;
132
            if (op == 2){
133
                cout << query(node-1) << '\n';</pre>
134
            } else {
135
                update(node-1); // Log Log
136
137
        }
138
139 }
```