Dividimos y No Conquistamos (D&!C)

Contents

1 Template

1.1 C++ Template

```cpp
#include <bits/stdc++.h>
using namespace std;

#define L(i, j, n) for (int i = (j); i < (int)n; i ++)
#define SZ(x) int((x).size())
#define ALL(x) begin(x),end(x)
#define vec vector
#define pb push_back
#define eb emplace_back

using ll = long long;
using ld = long double;

void solve(){}

int main(){
    ios::sync_with_stdio(0);cin.tie(0);
    int TT = 1;
    //cin >> TT;
    while (TT--) {
        solve();
    }
}
```

1.2 Fast Python

```python
import os, sys, io
finput = io.BytesIO(os.read(0, os.fstat(0).st_size)).readline
fprint = sys.stdout.write
```

1.3 Policy Based

```cpp
#include <ext/pb_ds/assoc_container.hpp>
using namespace __gnu_pbds;
template<typename Key, typename Val=null_type>
using indexed_set = tree<Key, Val, less<Key>, rb_tree_tag,
                         tree_order_statistics_node_update>;
// indexed_set<char> s;
// char val = *s.find_by_order(0); // acceso por indice
// int idx = s.order_of_key('a'); // busca indice del valor
template<class Key,class Val=null_type>using htable=gp_hash_table<Key,
    Val>;
// como unordered_map (o unordered_set si Val es vacio), pero sin metodo
     count
```

2 Search

2.1 Ternary

```cpp
// Minimo de `f` en `(l,r)`.
template<class Fun>ll ternary(Fun f, ll l, ll r) {
  for (ll d = r-l; d > 2; d = r-l) {
    ll a = l + d/3, b = r - d/3;
    if (f(a) > f(b)) l = a; else r = b;
  }
  return l + 1;
}
// para error < EPS, usar iters=log((r-l)/EPS)/log(1.618)
template<class Fun>double golden(Fun f, double l, double r, int iters){
  double const ratio = (3-sqrt(5))/2;
  double x1=l+(r-l)*ratio, x2=r-(r-l)*ratio, f1=f(x1), f2=f(x2);
  while (iters--) {
    if (f1 > f2) l=x1, x1=x2, f1=f2, x2=r-(r-l)*ratio, f2=f(x2);
    else         r=x2, x2=x1, f2=f1, x1=l+(r-l)*ratio, f1=f(x1);
  }
  return (l+r)/2;
}
```

2.2 Simulated Annealing

```cpp
using my_clock = chrono::steady_clock;
struct Random {
  mt19937_64 engine;
  Random(): engine(my_clock::now().time_since_epoch().count()) {}
  template<class Int>Int integer(Int n) {return integer<Int>(0, n);} //
      `[0,n)`
  template<class Int>Int integer(Int l, Int r)
    {return uniform_int_distribution{l, r-1}(engine);} // `[l,r)`
  double real() {return uniform_real_distribution{}(engine);} // `[0,1)`
} rng;
struct Timer {
  using time = my_clock::time_point;
  time start = my_clock::now();
```

```
  double elapsed() { // Segundos desde el inicio.
    time now = my_clock::now();
    return chrono::duration<double>(now - start).count();
  }
} timer;
template<class See,class Upd>struct Annealing {
  using energy = invoke_result_t<See>;
  energy curr, low;
  See see;
  Upd upd;
  Annealing(See _see, Upd _upd): see{_see}, upd{_upd}
    {curr = low = see(), upd();}
  void simulate(double s, double mult=1) { // Simula por `s` segundos.
    double t0 = timer.elapsed();
    for (double t = t0; t-t0 < s; t = timer.elapsed()) {
      energy near = see();
      auto delta = double(curr - near);
      if (delta >= 0) upd(), curr = near, low = min(low, curr);
      else {
        double temp = mult * (1 - (t-t0)/s);
        if (exp(delta/temp) > rng.real()) upd(), curr = near;
      }
    }
  }
};
auto see = [&] -> double {
    l = rng.integer(gsz);r = rng.integer(gsz);
    swap(groups[l], groups[r]);
    int ans = 0, rem =0;
    L(i,0,gsz){
        if (groups[i] > rem) {
            rem = x;
            ans ++;
        }
        rem -= groups[i];
    }
    swap(groups[l], groups[r]);
    return ans;
};
auto upd = [&] {swap(groups[l], groups[r]);};
```

3 Data structures

3.1 BIT

```
#define LSO(S) (S & -S)
struct BIT {
    vec<int> B;
    int n;
    BIT(int n = 1): B(n + 1), n(n+1){}
    BIT(vec<int> &v): B(SZ(v)+1), n(SZ(v)+1) {
        LI(i, 1, n){
            B[i] += v[i-1];
            if (i + LSO(i) <= n){
                B[i + LSO(i)] += B[i];
            }
        }
    }
    void update(int i, int x){
        while (i <= n){
            B[i] += x;
            i += LSO(i);
        }
    }
    int sum(int i){
        int res = 0;
        while (i > 0){
            res += B[i];
            i -= LSO(i);
        }
        return res;
    }
    int range_sum(int l, int r){
        return sum(r) - sum(l - 1);
    }
};
```

3.2 BIT - 2D

```
#define LSO(S) (S & -S)
int f[N][N]; // Initial Matrix
struct BIT {
    vec<vec<int>> B;
    int n;
```

```
    BIT(int n_ = 1): B(n_+1,vec<int>(n_+1)), sz(n_) { // N * N * log(N) * log(N);
        for (int i = 1; i <= n_; i ++)
            for (int j = 1; j <= n_; j ++)
                add(i, j, f[i-1][j-1]);
    }
    void add(int i, int j, int delta){ // log(N) * log(N)
        for (int x = i; x <= n; x += LSO(x))
            for (int y = j; y <= n; y += LSO(y))
                B[x][y] += delta;
    }
    int sum(int i, int j){ // log(N) * log(N)
        int tot = 0;
        for (int x = i; x > 0; x -= LSO(x))
            for(int y = j; y > 0; y -= LSO(y))
                tot += B[x][y];
        return tot;
    }
};
```

3.3 DSU

```
struct DSU {
    vec<int> par, sz; int n;
    DSU(int n = 1): par(n), sz(n, 1), n(n) { iota(ALL(par), 0); }
    int find(int a){return a == par[a] ? a : par[a] = find(par[a]);}
    void join(int a, int b){
        a=find(a);b=find(b);
        if (a == b) return;
        if (sz[b] > sz[a]) swap(a,b);
        par[b] = a;
        sz[a] += sz[b];
    }
};
```

3.4 Index Compession

```
template<class T>
struct Index{ // If only 1 use Don't need to copy T type
    vec<T> d;
    int sz;
    Index(const vec<T> &a): d(ALL(a)){
        sort(ALL(d)); // Sort
        d.erase(unique(ALL(d)), end(d)); // Erase continuous duplicates
        sz = SZ(d); }
    inline int of(T e) const{return lower_bound(ALL(d), e) - begin(d);}
        // get index
    inline T at(int i) const{return d[i];} // get value of index
};
```

3.5 Sparse Table

```
struct SPT {
    vec<vec<int>> st;
    int K;
    SPT(vec<int> &a): K(0) {
        int n = SZ(a);
        while((1<<K)<=n) K ++;
        st = vec<vec<int>>(K, vec<int>(n));
        L(i,0,n) st[0][i] = a[i];
        L(i,1,K) {
            for (int j = 0; j + (1 << i) <= n; j ++) {
                st[i][j] = min(st[i-1][j], st[i - 1][j + (1 << (i - 1))
                    ]); // change op
            }
        }
    }
    int get(int l, int r) {
        int bit = log2(r - l + 1);
        return min(st[bit][l], st[bit][r - (1<<bit) + 1]); // change op
    }
};
```

3.6 Segment tree

```
#define LC(v) (v<<1)
#define RC(v) ((v<<1)|1)
#define MD(L, R) (L+((R-L)>>1))
struct node {
    ll mx;
    ll cant; };
struct ST {
    vec<node> st;
    vec<ll> lz; int n;
    ST(int n = 1): st(4 * n + 10, {oo, oo}), lz(4 * n + 10, 0), n(n) {
        build(1, 0, n - 1);}
    node merge(node a, node b){
        if (a.mx == oo) return b;
```

```
        if (b.mx == oo) return a;
        if (a.mx == b.mx) return {a.mx, a.cant + b.cant};
        return {max(a.mx, b.mx), a.mx > b.mx ? a.cant : b.cant};
    }
    void build(int v, int L, int R){
        if (L == R){
            st[v] = {0, 1};
        } else {
            int m = MD(L, R);
            build(LC(v), L, m);
            build(RC(v), m + 1, R);
            st[v] = merge(st[LC(v)], st[RC(v)]);
        }
    }
    void push(int v, int L, int R){
        if (lz[v]){
            if (L != R){
                st[LC(v)].mx += lz[v]; // Apply to left
                st[RC(v)].mx += lz[v]; // And right
                lz[LC(v)] += lz[v];
                lz[RC(v)] += lz[v];
            }
            lz[v] = 0;
        }
    }
    void update(int v, int L, int R, int ql, int qr, ll w){
        if (ql > R || qr < L) return;
        push(v, L, R);
        if (ql == L && qr == R){
            st[v].mx += w;
            lz[v] += w;
            push(v, L, R);
            return;
        }
        int m = MD(L, R);
        update(LC(v), L, m, ql, min(qr, m), w);
        update(RC(v), m + 1, R, max(m + 1, ql), qr, w);
        st[v] = merge(st[LC(v)], st[RC(v)]);
    }
    node query(int v, int L, int R, int ql, int qr){
        if (ql > R || qr < L) return {oo, oo};
        push(v, L, R);
        if (ql == L && qr == R) return st[v];
        int m = MD(L, R);
        return merge(query(LC(v), L, m, ql, min(m, qr)), query(RC(v), m
            + 1, R, max(m + 1, ql), qr));
    }
    node query(int l, int r){return query(1, 0, n - 1, l, r);}
    void update(int l, int r, ll w){update(1, 0, n - 1, l, r, w);}
};
```

3.7 Segment Tree Iterativo

```
struct STI {
    vec<ll> st;
    int n, K;
    STI(vec<ll> &a): n(SZ(a)), K(1) {
        while(K<=n) K<<=1;
        st.assign(2*K, 0); // 0 default
        L(i,0,n) st[K+i] = a[i];
        for (int i = K - 1; i > 0; i --) st[i] = st[i*2] + st[i*2+1];
    }
    void upd(int pos, ll w) {
        pos += K;
        st[pos] += w;
        while((pos>>=1) > 0) st[pos] = st[pos * 2] + st[pos * 2 + 1];
    }
    ll query(int l, int r) { // [l, r)
        ll res = 0;
        l += K; r += K;
        while (l < r) {
            if (l & 1) res += st[l++];
            if (r & 1) res += st[--r];
            l>>=1;r>>=1;
        }
        return res;
    }
};
```

4 Graph

4.1 Bellman Ford

```
struct Edge {int a, b, cost;};
vector<Edge> edges;
int solve(int s) // Source
```

```
{
    vector<int> d(n, INF);
    d[s] = 0;
    for (int i = 0; i < n - 1; ++i)
        for (Edge e : edges)
            if (d[e.a] < INF)
                d[e.b] = min(d[e.b], d[e.a] + e.cost);
}
```

4.2 SCC

```
vec<int> dfs_num(N, -1), dfs_low(N, -1), in_stack(N);
int dfs_count = 0;
int numSCC = 0;
stack<int> st;
void dfs(int u){
  dfs_low[u]=dfs_num[u]=dfs_count++;
  st.push(u);
  in_stack[u] = 1;
  for(int v: G[u]) {
    if (dfs_num[v] == -1) dfs(v);
    if (in_stack[v]) dfs_low[u] = min(dfs_low[u], dfs_low[v]);
  }
  if (dfs_num[u] == dfs_low[u]){
    numSCC ++;
    while(1){
      int v = st.top(); st.pop();
      in_stack[v] = 0;
      if (u == v) break;
    }
  }
}
```

4.3 Bipartite Matching Hopcroft-Karp - With Konig

```
mt19937 rng((int) chrono::steady_clock::now().time_since_epoch().count()
    );
struct hopcroft_karp {
  int n, m; // n is Left Partition Size, m is Right Partition Size
  vec<vec<int>> g;
  vec<int> dist, nxt, ma, mb;
  hopcroft_karp(int n_, int m_) : n(n_), m(m_), g(n),
    dist(n), nxt(n), ma(n, -1), mb(m, -1) {}
  void add(int a, int b) { g[a].pb(b); }
  bool dfs(int i) {
    for (int &id = nxt[i]; id < g[i].size(); id++) {
      int j = g[i][id];
      if (mb[j] == -1 or (dist[mb[j]] == dist[i]+1 and dfs(mb[j]))) {
        ma[i] = j, mb[j] = i;
        return true;
      }
    }
    return false;
  }
  bool bfs() {
    for (int i = 0; i < n; i++) dist[i] = n;
    queue<int> q;
    for (int i = 0; i < n; i++) if (ma[i] == -1) {
      dist[i] = 0;
      q.push(i);
    }
    bool rep = 0;
    while (q.size()) {
      int i = q.front(); q.pop();
      for (int j : g[i]) {
        if (mb[j] == -1) rep = 1;
        else if (dist[mb[j]] > dist[i] + 1) {
          dist[mb[j]] = dist[i] + 1;
          q.push(mb[j]);
        }
      }
    }
    return rep;
  }
  int matching() {
    int ret = 0;
    for (auto& i : g) shuffle(ALL(i), rng);
    while (bfs()) {
      for (int i = 0; i < n; i++) nxt[i] = 0;
      for (int i = 0; i < n; i++)
        if (ma[i] == -1 and dfs(i)) ret++;
    }
    return ret;
  }
  vec<int> cover[2]; // if cover[i][j] = 1 -> node i, j is part of cover
  int konig() {
    cover[0].assign(n,1); // n left size
```

```
    cover[1].assign(m,0); // m right size
    auto go = [&](auto&& me, int u) -> void {
      cover[0][u] = false;
      for (auto v : g[u]) if (!cover[1][v]) {
        cover[1][v] = true;
        me(me,mb[v]);
      }
    };
    L(u,0,n) if (ma[u] < 0) go(go,u);
    return size;
  }
};
```

4.4 Hungarian

```
using vi = vec<int>;
using vd = vec<ld>;
const ld INF = 1e100;    // Para max asignacion, INF = 0, y negar costos
bool zero(ld x) {return fabs(x) < 1e-9;}  // Para int/ll: return x==0;
vec<pii> ans; // Guarda las aristas usadas en el matching: [0..n)x[0..m)
struct Hungarian{
  int n; vec<vd> cs; vi vL, vR;
  Hungarian(int N, int M) : n(max(N,M)), cs(n,vd(n)), vL(n), vR(n){
    L(x, 0, N) L(y, 0, M) cs[x][y] = INF;
  }
  void set(int x, int y, ld c) { cs[x][y] = c; }
  ld assign(){
    int mat = 0; vd ds(n), u(n), v(n); vi dad(n), sn(n);
    L(i, 0, n) u[i] = *min_element(ALL(cs[i]));
    L(j, 0, n){
      v[j] = cs[0][j]-u[0];
      L(i, 1, n) v[j] = min(v[j], cs[i][j] - u[i]);
    }
    vL = vR = vi(n, -1);
    L(i,0, n) L(j, 0, n) if(vR[j] == -1 and zero(cs[i][j] - u[i] - v[j])
        ){
      vL[i] = j; vR[j] = i; mat++; break;
    }
    for(; mat < n; mat ++){
      int s = 0, j = 0, i;
      while(vL[s] != -1) s++;
      fill(ALL(dad), -1); fill(ALL(sn), 0);
      L(k, 0, n) ds[k] = cs[s][k]-u[s]-v[k];
      while(true){
        j = -1;
        L(k, 0, n) if(!sn[k] and (j == -1 or ds[k] < ds[j])) j = k;
        sn[j] = 1; i = vR[j];
        if(i == -1) break;
        L(k, 0, n) if(!sn[k]){
          auto new_ds = ds[j] + cs[i][k] - u[i]-v[k];
          if(ds[k] > new_ds) ds[k]=new_ds, dad[k]=j;
        }
      }
      L(k, 0, n) if(k!=j and sn[k]){
        auto w = ds[k]-ds[j]; v[k] += w, u[vR[k]] -= w;
      }
      u[s] += ds[j];
      while(dad[j] >= 0){ int d = dad[j]; vR[j] = vR[d]; vL[vR[j]] = j;
          j = d; }
      vR[j] = s; vL[s] = j;
    }
    ld value = 0; L(i, 0, n) value += cs[i][vL[i]], ans.pb({i, vL[i]});
    return value;
  }
};
```

4.5 Flow - Dinics

```
struct Dinic {
  bool scaling = false; // com scaling -> O(nm log(MAXCAP)),
  int lim;                    // com constante alta
  struct edge {
    int to, cap, rev, flow;
    bool res;
    edge(int to_, int cap_, int rev_, bool res_)
      : to(to_), cap(cap_), rev(rev_), flow(0), res(res_) {}
  };
  vec<vec<edge>> g;
  vec<int> lev, beg;
  ll F;
  Dinic(int n) : g(n), F(0) {}
  void add(int a, int b, int c) {
    g[a].emplace_back(b, c, g[b].size(), false);
    g[b].emplace_back(a, 0, g[a].size()-1, true);
  }
  bool bfs(int s, int t) {
```

```
    lev = vector<int>(g.size(), -1); lev[s] = 0;
    beg = vector<int>(g.size(), 0);
    queue<int> q; q.push(s);
    while (q.size()) {
      int u = q.front(); q.pop();
      for (auto& i : g[u]) {
        if (lev[i.to] != -1 or (i.flow == i.cap)) continue;
        if (scaling and i.cap - i.flow < lim) continue;
        lev[i.to] = lev[u] + 1;
        q.push(i.to);
      }
    }
    return lev[t] != -1;
  }
  int dfs(int v, int s, int f = oo) {
    if (!f or v == s) return f;
    for (int& i = beg[v]; i < g[v].size(); i++) {
      auto& e = g[v][i];
      if (lev[e.to] != lev[v] + 1) continue;
      int foi = dfs(e.to, s, min(f, e.cap - e.flow));
      if (!foi) continue;
      e.flow += foi, g[e.to][e.rev].flow -= foi;
      return foi;
    }
    return 0;
  }
  ll max_flow(int s, int t) {
    for (lim = scaling ? (1<<30) : 1; lim; lim /= 2)
      while (bfs(s, t)) while (int ff = dfs(s, t)) F += ff;
    return F;
  }
};
vec<pair<int, int>> get_cut(Dinic& g, int s, int t) {
  g.max_flow(s, t);
  vec<pair<int, int>> cut;
  vec<int> vis(g.g.size(), 0), st = {s};
  vis[s] = 1;
  while (st.size()) {
    int u = st.back(); st.pop_back();
    for (auto e : g.g[u]) if (!vis[e.to] and e.flow < e.cap)
      vis[e.to] = 1, st.push_back(e.to);
  }
  for (int i = 0; i < g.g.size(); i++) for (auto e : g.g[i])
    if (vis[i] and !vis[e.to] and !e.res) cut.emplace_back(i, e.to);
  return cut;
}
```

4.6 Flow - MinCostMaxFlow

```
// O(nm + f * m log n)
// const ll oo = (ll)1e18;
template<typename T> struct mcmf {
  struct edge {
    int to, rev, flow, cap; // para, id da reversa, fluxo, capacidade
    bool res; // se eh reversa
    T cost; // custo da unidade de fluxo
    edge() : to(0), rev(0), flow(0), cap(0), cost(0), res(false) {}
    edge(int to_, int rev_, int flow_, int cap_, T cost_, bool res_)
      : to(to_), rev(rev_), flow(flow_), cap(cap_), res(res_), cost(
          cost_) {}
  };
  vec<vec<edge>> g;
  vec<int> par_idx, par;
  T inf;
  vec<T> dist;
  mcmf(int n) : g(n), par_idx(n), par(n), inf(numeric_limits<T>::max()
      /3) {}
  void add(int u, int v, int w, T cost) { // de u pra v com cap w e
      custo cost
    edge a = edge(v, g[v].size(), 0, w, cost, false);
    edge b = edge(u, g[u].size(), 0, 0, -cost, true);
    g[u].push_back(a);
    g[v].push_back(b);
  }
  vec<T> spfa(int s) { // nao precisa se nao tiver custo negativo
    deque<int> q;
    vec<bool> is_inside(g.size(), 0);
    dist = vec<T>(g.size(), inf);
    dist[s] = 0;
    q.push_back(s);
    is_inside[s] = true;
    while (!q.empty()) {
      int v = q.front();
      q.pop_front();
      is_inside[v] = false;
      for (int i = 0; i < g[v].size(); i++) {
```

```
        auto [to, rev, flow, cap, res, cost] = g[v][i];
        if (flow < cap and dist[v] + cost < dist[to]) {
          dist[to] = dist[v] + cost;

          if (is_inside[to]) continue;
          if (!q.empty() and dist[to] > dist[q.front()]) q.push_back(to)
              ;
          else q.push_front(to);
          is_inside[to] = true;
        }
      }
    }
    return dist;
  }
  bool dijkstra(int s, int t, vec<T>& pot) {
    priority_queue<pair<T, int>, vec<pair<T, int>>, greater<>> q;
    dist = vec<T>(g.size(), inf);
    dist[s] = 0;
    q.emplace(0, s);
    while (q.size()) {
      auto [d, v] = q.top();
      q.pop();
      if (dist[v] < d) continue;
      for (int i = 0; i < g[v].size(); i++) {
        auto [to, rev, flow, cap, res, cost] = g[v][i];
        cost += pot[v] - pot[to];
        if (flow < cap and dist[v] + cost < dist[to]) {
          dist[to] = dist[v] + cost;
          q.emplace(dist[to], to);
          par_idx[to] = i, par[to] = v;
        }
      }
    }
    return dist[t] < inf;
  }
  pair<int, T> min_cost_flow(int s, int t, int flow = (int)1e9) {
    vec<T> pot(g.size(), 0);
    pot = spfa(s); // mudar algoritmo de caminho minimo aqui
    int f = 0;
    T ret = 0;
    while (f < flow and dijkstra(s, t, pot)) {
      for (int i = 0; i < g.size(); i++)
        if (dist[i] < inf) pot[i] += dist[i];
      int mn_flow = flow - f, u = t;
      while (u != s){
        mn_flow = min(mn_flow,
          g[par[u]][par_idx[u]].cap - g[par[u]][par_idx[u]].flow);
        u = par[u];
      }
      ret += pot[t] * mn_flow;
      u = t;
      while (u != s) {
        g[par[u]][par_idx[u]].flow += mn_flow;
        g[u][g[par[u]][par_idx[u]].rev].flow -= mn_flow;
        u = par[u];
      }
      f += mn_flow;
    }
    return make_pair(f, ret);
  }
  // Opcional: retorna as arestas originais por onde passa flow = cap
  vec<pair<int,int>> recover() {
    vec<pair<int,int>> used;
    for (int i = 0; i < g.size(); i++) for (edge e : g[i])
      if(e.flow == e.cap && !e.res) used.push_back({i, e.to});
    return used;
  }
};
```

4.7 2 Sat

```
struct TwoSat {
    int n, v_n;
    vec<bool> vis, assign;
    vec<int> order, comp;
    vec<vec<int>> g, g_t;
    TwoSat(int n_): n(n_), v_n(2 * n_), vis(v_n) , assign(n_),  comp(v_n
        , - 1), g(v_n), g_t(v_n) {
        order.reserve(v_n);
    }
    void add_disj(int a, bool na, int b, bool nb) { // negated_a,
        negated_b
        a = 2 * a ^ na;
        b = 2 * b ^ nb;
        int neg_a = a ^ 1;
        int neg_b = b ^ 1;
```

```
        g[neg_a].pb(b);
        g[neg_b].pb(a);
        g_t[a].pb(neg_b);
        g_t[b].pb(neg_a);
    }
    void dfs1(int u){
        vis[u] = 1;
        for (int v: g[u]) if (!vis[v]) dfs1(v);
        order.pb(u);
    }
    void dfs2(int u, int cc) {
        comp[u] = cc;
        for (int v: g_t[u]) if (comp[v] == -1) dfs2(v, cc);
    }
    bool solve() {
        order.clear();
        vis.assign(v_n, 0);
        L(i,0, v_n) if (!vis[i]) dfs1(i);
        comp.assign(v_n, - 1);
        int cc = 0;
        L(i, 0, v_n) {
            int v = order[v_n - 1 - i];
            if (comp[v] == -1) dfs2(v, cc ++);
        }
        assign.assign(n, false);
        for (int i = 0;i < v_n; i += 2) {
            if (comp[i] == comp[i+1]) return false;
            assign[i / 2] = comp[i] > comp[i + 1];
        }
        return true;
    }
};
```

4.8 Euler Tour

```
// Directed version (uncomment commented code for undirected)
struct edge {
    int y;
//    list<edge>::iterator rev;
    edge(int y):y(y){}
};
list<edge> g[N];
void add_edge(int a, int b){
    g[a].push_front(edge(b));//auto ia=g[a].begin();
//    g[b].push_front(edge(a));auto ib=g[b].begin();
//    ia->rev=ib;ib->rev=ia;
}
vec<int> p;
void go(int x){
    while(g[x].size()){
        int y=g[x].front().y;
        //g[y].erase(g[x].front().rev);
        g[x].pop_front();
        go(y);
    }
    p.push_back(x);
}
vec<int> get_path(int x){ // get a path that begins in x
// check that a path exists from x before calling to get_path!
    p.clear();go(x);reverse(p.begin(),p.end());
    return p;
}
```

5 Trees

5.1 Heavy Light Decomposition

```
int ans[N], par[N], depth[N], head[N], pos[N];
vec<int> heavy(N, - 1);
int t = 0;
vec<int> g[N];
int dfs(int u) {
    int size = 1;
    int max_size = 0;
    for (int v: g[u]) if (v != par[u]) {
        par[v] = u;
        depth[v] = depth[u] + 1;
        int cur_size = dfs(v);
        size += cur_size;
        if (cur_size > max_size) {
            max_size = cur_size;
            heavy[u] = v;
        }
    }
    return size;
}
```

```
void decompose(int u, int h){
    head[u] = h;
    pos[u] = t ++;
    if (heavy[u] != -1){ decompose(heavy[u], h); }
    for (int v: G[u]) if (v != par[u] && v != heavy[u]) {
        decompose(v, v);
    }
}
int query(int a, int b) {
    int resp = -1;
    for (; head[a] != head[b]; b = par[head[b]]){ // Subi todo el heavy path y a su padre // Next
        if (depth[head[a]] > depth[head[b]]) swap(a, b);
        resp = max(resp, st.query(pos[head[b]], pos[b])); // pos[head[b]] < pos[b]
    }
    if (depth[a] > depth[b]) swap(a, b); // Una vez misma path(head) entonces es una query [a,b]
    resp = max(resp, st.query(pos[a], pos[b]));
    return resp;
}
dfs(root);
decompose(root, root);
```

5.2 Centroid

```
int sz[N];
bool removed[N];
int getSize(int u, int p){
    sz[u] = 1;
    for(int v: G[u]) if (v != p && !removed[v]){
        sz[u] += getSize(v, u);
    }
    return sz[u];
}
int centroid(int u, int p, int tz){
    for (int v: g[u])
        if (v != p && !removed[v] && sz[v] * 2 > tz) return centroid(v, u, tz);
    return u;
}
int build(int u){
    int c = centroid(u, -1, getSize(u, -1));
    removed[c] = 1;
    for (int v: G[c]) if (!removed[v]) { build(v); }
    return c;
}
```

5.3 LCA - Binary exponentiation

```
vec<int> g[N];
int K; // K should be (1<<K) > n
int jump[20][N];
int depth[N];

void dfs(int u, int p){
    for (int v: g[u]) if (v != p) {
        jump[0][v] = u;
        L(i, 1, K + 1) {
            jump[i][v] = -1;
            if (jump[i - 1][v] != -1) {
                jump[i][v] = jump[i - 1][jump[i - 1][v]];
            }
        }
        depth[v] = depth[u] + 1;
        dfs(v, u);
    }
}

int LCA(int u, int v){
    if (depth[u] < depth[v]) swap(u, v); // Make u the deepest
    for (int i = K; i >= 0; i --){ // make them same depth
        if (jump[i][u] != -1 && depth[jump[i][u]] >= depth[v]){
            u = jump[i][u];
        }
    }
    if (u == v) return u; // u is parent of v
    for (int i = K; i>= 0; i --){
        if (jump[i][u] != jump[i][v] && jump[i][u] != -1 && jump[i][v] != -1){
            u = jump[i][u];
            v = jump[i][v];
        }
    }
    return jump[0][u];
}
```

5.4 LCA - Const Time

```cpp
struct LCA {
    vec<int> depth, in, euler;
    vec<vec<int>> g, st;
    int K, n;
    inline int Min(int i, int j) {return depth[i] <= depth[j] ? i : j;}
    void dfs(int u, int p) {
        in[u] = SZ(euler);
        euler.pb(u);
        for (int v: g[u]) if (v != p){
            depth[v] = depth[u] + 1;
            dfs(v, u);
            euler.pb(u);
        }
    }
    LCA(int n_): depth(n_), g(vec<vec<int>>(n_)), K(0), n(n_), in(n_) {
        euler.reserve(2 * n); }
    void add_edge(int u, int v) {g[u].pb(v);}
    void build(int root){
        dfs(root, -1);
        int ln = SZ(euler);
        while((1<<K)<=ln)K++;
        st = vec<vec<int>> (K, vec<int>(ln));
        L(i,0,ln) st[0][i] = euler[i];
        for (int i = 1; (1 << i) <= ln; i ++) {
            for (int j = 0; j + (1<<i) <= ln; j ++) {
                st[i][j] = Min(st[i-1][j], st[i-1][j + (1<<(i-1))]);
            }
        }
    }
    int get(int u, int v) {
        int su = in[u];
        int sv = in[v];
        if (sv < su) swap(sv, su);
        int bit = log2(sv - su + 1);
        return Min(st[bit][su], st[bit][sv - (1<<bit) + 1]);
    }
};
```

6 Dynamic Programming

6.1 Knapsack

```cpp
int knapsack(vector<int>& values, vector<int>& weights, int W) {
    int n = values.size();
    vector<vector<int>> dp(n + 1, vector<int>(W + 1, 0));

    for(int i = 1; i <= n; i++) {
        for(int w = 0; w <= W; w++) {
            if(weights[i-1] <= w) {
                dp[i][w] = max(dp[i-1][w],
                               dp[i-1][w-weights[i-1]] + values[i-1]);
            } else {
                dp[i][w] = dp[i-1][w];
            }
        }
    }
    return dp[n][W];
}
```

6.2 LIS

```cpp
vector<int> getLIS(vector<int>& arr) {
    int n = arr.size();
    vector<int> dp(n + 1, INT_MAX);  // dp[i] = smallest value that ends
        an LIS of length i
    vector<int> len(n);              // Length of LIS ending at each
        position
    dp[0] = INT_MIN;
    for(int i = 0; i < n; i++) {
        int j = upper_bound(dp.begin(), dp.end(), arr[i]) - dp.begin();
        dp[j] = arr[i];
        len[i] = j;
    }
    // Find maxLen and reconstruct sequence
    int maxLen = 0;
    for(int i = n-1; i >= 0; i--) maxLen = max(maxLen, len[i]);
    vector<int> lis;
    for(int i = n-1, currLen = maxLen; i >= 0; i--) {
        if(len[i] == currLen) {
            lis.push_back(arr[i]);
            currLen--;
```

```
        }
    }
    reverse(lis.begin(), lis.end());
    return lis;
}
```

6.3 Edit Distance

```
int editDistance(string& s1, string& s2) {
    int n = s1.length(), m = s2.length();
    vector<vector<int>> dp(n + 1, vector<int>(m + 1));
    for(int i = 0; i <= n; i++) dp[i][0] = i;
    for(int j = 0; j <= m; j++) dp[0][j] = j;
    for(int i = 1; i <= n; i++) {
        for(int j = 1; j <= m; j++) {
            if(s1[i-1] == s2[j-1]) {
                dp[i][j] = dp[i-1][j-1];
            } else {
                dp[i][j] = 1 + min({dp[i-1][j],    // deletion
                                    dp[i][j-1],    // insertion
                                    dp[i-1][j-1]}); // replacement
            }
        }
    }
    return dp[n][m];
}
```

6.4 Kadane

```
pair<int, pair<int,int>> kadane(vector<int>& arr) {
    int maxSoFar = arr[0], maxEndingHere = arr[0];
    int start = 0, end = 0, s = 0;

    for(int i = 1; i < arr.size(); i++) {
        if(maxEndingHere + arr[i] < arr[i]) {
            maxEndingHere = arr[i];
            s = i;
        } else {
            maxEndingHere += arr[i];
        }

        if(maxEndingHere > maxSoFar) {
            maxSoFar = maxEndingHere;
            start = s;
            end = i;
        }
    }
    return {maxSoFar, {start, end}}; // max, l, r
}
```

7 Strings

7.1 Prefix Trie

```
struct Trie {
    map<char, int> ch;
    bool eee;
    Trie(): eee(0) {}
};
vec<Trie> t;
void initTrie(){t.clear();t.pb(Trie());}
void insert(string &word) {
    int v = 0;
    for(char c : word) {
        if(!t[v].ch[c]) {
            t[v].ch[c] = SZ(t);
            t.pb(Trie());
        }
        v = t[v].ch[c];
    }
    t[v].eee = 1;
}
```

7.2 Hashing

```
static constexpr ll ms[] = {1'000'000'007, 1'000'000'403};
static constexpr ll b = 500'000'000;
struct StrHash { // Hash polinomial con exponentes decrecientes.
  vec<ll> hs[2], bs[2];
  StrHash(string const& s) {
    int n = SZ(s);
    L(k, 0, 2) {
      hs[k].resize(n+1), bs[k].resize(n+1, 1);
      L(i, 0, n) {
        hs[k][i+1] = (hs[k][i] * b + s[i]) % ms[k];
        bs[k][i+1] =  bs[k][i] * b         % ms[k];
      }
```

```
    }
  }
  ll get(int idx, int len) const { // Hashes en `s[idx, idx+len)`.
    ll h[2];
    L(k, 0, 2) {
      h[k] = hs[k][idx+len] - hs[k][idx] * bs[k][len] % ms[k];
      if (h[k] < 0) h[k] += ms[k];
    }
    return (h[0] << 32) | h[1];
  }
};

pll union_hash(vec<pll> hs, vec<ll> lens){ //use arrays makes it slower
  ll len = 0;
  for(int i = hs.size()-1; i > 0; i--){
    len += lens[i];
    pll& [l1, l2] = hs[i];
    pll& [r1, r2] = hs[i-1];
    l1 = ((l1 * binpow(b, len, ms[0])) % ms[0] + r1) % ms[0];
    l2 = ((l2 * binpow(b, len, ms[1])) % ms[1] + r2) % ms[1];
  }

  return hs[0];
}
```

7.3 KMP

```
vec<int> kmp(string pat, string sec){ //geeks4geeks implementation with
    some changes
  int m = pat.length();
  int n = sec.length();
  cout << m << "␣" << n << endl;

  vec<int> lps = getLps(pat);
  vec<int> res;

  int i = 0;
  int j = 0;

  while((n - i) >= (m - j)){
    if(pat[j] == sec[i]){
      i++;
      j++;
    }
    if(j == m){
      res.push_back(i - j);
      j = lps[j - 1];
    }
    else{
      if(i < n && pat[j] != sec[i]){
        if(j != 0) j = lps[ j - 1 ];
        else i = i + 1;
      }
    }
  }

  return res;
}
```

7.4 LPS

```
vec<int> getLps(string pat){ //geek4geeks implementatio with some
    changes
  vec<int> lps(pat.length(), 0);
  int len = 0;
  int i = 1;
  lps[0] = 0;
  while(i < pat.length()){
    if(pat[i] == pat[len]){
      len++;
      lps[i] = len;
      i++;
    }
    else //pat[i] != pat[len]
    {
      lps[i] = 0;
      i++;
    }
  }
  return lps;
}
```

7.5 Z-FUNCTION

```
template<class Char=char>vec<int> zfun(const basic_string<Char>& w) {
  int n = SZ(w), l = 0, r = 0; vec<int> z(n);
  z[0] = w.length();
```

```
  L(i, 1, n) {
    if (i <= r) {z[i] = min(r - i + 1, z[i - l]);}
    while (i + z[i] < n && w[z[i]] == w[i + z[i]]) {++z[i];}
    if (i + z[i] - 1 > r) {l = i, r = i + z[i] - 1;}
  }
  return z;
}
```

7.6 Aho-Corasick

```
bool vis[N], r[N];
struct ACvertex {
  map<char,int> next,go;
  int p,link;
  char pch;
  vec<int> leaf;
  ACACvertex(int p=-1, char pch=-1):p(p),pch(pch),link(-1){}
};
vec<ACvertex> t;
void aho_init(){ //do not forget!!
  t.clear();t.pb(ACvertex());
}
void add_string(string &s, int id){
  int v=0;
  for(char c:s){
    if(!t[v].next.count(c)){
      t[v].next[c]=t.size();
      t.pb(ACvertex(v,c));
    }
    v=t[v].next[c];
  }
  t[v].leaf.pb(id);
}
int go(int v, char c);
int get_link(int v){ // Failure link
  if(t[v].link<0)
    if(!v||!t[v].p)t[v].link=0;
    else t[v].link=go(get_link(t[v].p),t[v].pch);
  return t[v].link;
}
int go(int v, char c){ // state = go(state, ch) this state is ACvertex
    id
  if(!t[v].go.count(c))
    if(t[v].next.count(c))t[v].go[c]=t[v].next[c];
    else t[v].go[c]=v==0?0:go(get_link(v),c);
  return t[v].go[c];
}
void proc(int x){
    if (x == - 1|| vis[x]) return;
    vis[x] = 1;
    L(i,0,SZ(t[x].leaf)) r[t[x].leaf[i]] = 1;
    proc(get_link(x));
}
```

8 Math

8.1 Euclidean Extended

```
ll extendedGCD(ll a, ll b, ll &x, ll &y) {
    if (b == 0) {
        x = 1;
        y = 0;
        return a;
    }
    ll x1, y1;
    ll gcd = extendedGCD(b, a % b, x1, y1);
    x = y1;
    y = x1 - (a / b) * y1;
    return gcd;
}

bool findSolutionWithConstraints(ll a, ll b, ll c, ll x_min, ll y_min,
    ll &x, ll &y) {
    ll g = extendedGCD(a, b, x, y);

    if (c % g != 0) return false;

    x *= c / g;
    y *= c / g;

    // Ajustamos las variables a/g y b/g para mover las soluciones
    a /= g;
    b /= g;

    if (x < x_min) {
        ll k = (x_min - x + b - 1) / b;  // Redondeo hacia arriba
```

```
        x += k * b;
        y -= k * a;
    } else if (x > x_min) {
        ll k = (x - x_min) / b;
        x -= k * b;
        y += k * a;
    }

    if (y < y_min) {
        ll k = (y_min - y + a - 1) / a;  // Redondeo hacia arriba
        x += k * b;
        y -= k * a;
    } else if (y > y_min) {
        ll k = (y - y_min) / a;
        x -= k * b;
        y += k * a;
    }

    return x >= x_min && y >= y_min;
}
```

8.2 Euler Totient

```
#include <bits/stdc++.h>
using namespace std;
typedef long long ll;


vector<ll> compute_totients(ll n) {
    vector<ll> phi(n + 1);
    for (ll i = 0; i <= n; i++) {
        phi[i] = i;
    }

    for (ll i = 2; i <= n; i++) {
        if (phi[i] == i) { // i es primo
            for (ll j = i; j <= n; j += i) {
                phi[j] = phi[j] * (i - 1) / i;
            }
        }
    }

    return phi;
}
```

8.3 Josephus

```
#include <iostream>
using namespace std;

typedef long long ll;

ll josephus_iterative(ll n, ll k) {
    ll result = 0;
    for (ll i = 2; i <= n; ++i) {
        result = (result + k) % i;
    }
    return result;
}


ll josephus_recursive(ll n, ll k) {

    if (n == 1)
        return 0;

    return (josephus_recursive(n - 1, k) + k) % n;
}


ll josephus_power_of_2(ll n) {

    ll power = 1;
    while (power <= n) {
        power <<= 1;
    }
    power >>= 1;


    return 2 * (n - power);
}
```

8.4 Mobius

```
#include <bits/stdc++.h>
using namespace std;
typedef long long ll;
```

```cpp

vector<ll> compute_mobius(ll n) {
    vector<ll> mu(n + 1, 1);
    vector<bool> is_prime(n + 1, true);

    for (ll i = 2; i <= n; i++) {
        if (is_prime[i]) { // i es un primo
            for (ll j = i; j <= n; j += i) {
                mu[j] *= -1; // Multiplicamos por -1 para cada primo
                is_prime[j] = false;
            }
            for (ll j = i * i; j <= n; j += i * i) {
                mu[j] = 0; // Si tiene un cuadrado de un primo, se pone
                    en 0
            }
        }
    }

    return mu;
}


ll mobius(ll x) {
    ll count = 0;
    for (ll i = 2; i * i <= x; i++) {
        if (x % (i * i) == 0)
            return 0;
        if (x % i == 0) {
            count++;
            x /= i;
        }
    }

    if (x > 1) count++;

    return (count % 2 == 0) ? 1 : -1;
}
```

8.5 NTT

```cpp
#include <bits/stdc++.h>
using namespace std;
using cd = complex<double>;
typedef long long ll;
const ll mod = 998244353;
const ll root = 31;
const ll root_1 = inverse(root, mod);
const ll root_pw = 1 << 23;

ll inverse(ll a, ll m) {
    ll res = 1, exp = m - 2;
    while (exp) {
        if (exp % 2 == 1) res = (1LL * res * a) % m;
        a = (1LL * a * a) % m;
        exp /= 2;
    }
    return res;
}

void ntt(vector<ll> & a, bool invert) {
    int n = a.size();

    for (int i = 1, j = 0; i < n; i++) {
        int bit = n >> 1;
        for (; j & bit; bit >>= 1)
            j ^= bit;
        j ^= bit;

        if (i < j)
            swap(a[i], a[j]);
    }

    for (int len = 2; len <= n; len <<= 1) {
        int wlen = invert ? root_1 : root;
        for (int i = len; i < root_pw; i <<= 1)
            wlen = (int)(1LL * wlen * wlen % mod);

        for (int i = 0; i < n; i += len) {
            int w = 1;
            for (int j = 0; j < len / 2; j++) {
                int u = a[i+j], v = (int)(1LL * a[i+j+len/2] * w % mod);
                a[i+j] = u + v < mod ? u + v : u + v - mod;
                a[i+j+len/2] = u - v >= 0 ? u - v : u - v + mod;
                w = (int)(1LL * w * wlen % mod);
            }
```

```
        }
    }

    if (invert) {
        int n_1 = inverse(n, mod);
        for (auto & x : a)
            x = (int)(1LL * x * n_1 % mod);
    }
}

vector<ll> multiply(vector<ll> const &a, vector<ll> const &b) {
    vector<ll> fa(a.begin(), a.end()), fb(b.begin(), b.end());
    ll n = 1;
    while (n < a.size() + b.size())
        n <<= 1;
    fa.resize(n);
    fb.resize(n);

    ntt(fa, false);
    ntt(fb, false);
    for (ll i = 0; i < n; i++)
        fa[i] = (fa[i] * fb[i]) % mod;
    ntt(fa, true);

    vector<ll> result(n);
    for (ll i = 0; i < n; i++)
        result[i] = fa[i];
    return result;
}
```

8.6 FFT

```
typedef long long ll;
typedef complex<double> C;
typedef vector<double> vd;
typedef vector<ll> vll;
const double PI = acos(-1);

void fft(vector<C>& a) {
    int n = a.size(), L = 31 - __builtin_clz(n);
    static vector<C> R(2, 1);
    static vector<C> rt(2, 1);
    for (static int k = 2; k < n; k *= 2) {
        R.resize(n); rt.resize(n);
        auto x = polar(1.0, PI / k);
        for (int i = k; i < 2 * k; i++)
            rt[i] = R[i] = i & 1 ? R[i / 2] * x : R[i / 2];
    }
    vector<int> rev(n);
    for (int i = 0; i < n; i++) rev[i] = (rev[i / 2] | (i & 1) << L) /
        2;
    for (int i = 0; i < n; i++) if (i < rev[i]) swap(a[i], a[rev[i]]);
    for (int k = 1; k < n; k *= 2)
        for (int i = 0; i < n; i += 2 * k) for (int j = 0; j < k; j++) {
            auto x = (double*)&rt[j + k], y = (double*)&a[i + j + k];
            C z(x[0] * y[0] - x[1] * y[1], x[0] * y[1] + x[1] * y[0]);
            a[i + j + k] = a[i + j] - z;
            a[i + j] += z;
        }
}

vll multiply(const vll& a, const vll& b) {
    if (a.empty() || b.empty()) return {};
    vd fa(a.begin(), a.end()), fb(b.begin(), b.end());
    int L = 32 - __builtin_clz(fa.size() + fb.size() - 1), n = 1 << L;
    vector<C> in(n), out(n);

    for (int i = 0; i < a.size(); i++) in[i] = C(fa[i], 0);
    for (int i = 0; i < b.size(); i++) in[i].imag(fb[i]);

    fft(in);
    for (C& x : in) x *= x;
    for (int i = 0; i < n; i++) out[i] = in[-i & (n - 1)] - conj(in[i]);
          // Corregido aqui
    fft(out);

    vll res(a.size() + b.size() - 1);
    for (int i = 0; i < res.size(); i++) {
        res[i] = llround(imag(out[i]) / (4 * n));
    }
    return res;
}
```

8.7 Rho

```
//RECOMENDADO USAR UNSIGNED LONG LONG
```

```cpp
static inline ll mulmod(ll a, ll b, ll m) {
    return (ll)((__int128)a * b % m);
}

static inline ll powmod(ll b, ll e, ll m) {
    ll r = 1;
    while (e) {
        if (e & 1) r = mulmod(r, b, m);
        b = mulmod(b, b, m);
        e >>= 1;
    }
    return r;
}

// RNG rapido
static inline ll splitmix64(ll x) {
    x += 0x9e3779b97f4a7c15ULL;
    x = (x ^ (x >> 30)) * 0xbf58476d1ce4e5b9ULL;
    x = (x ^ (x >> 27)) * 0x94d049bb133111ebULL;
    return x ^ (x >> 31);
}
static ll rng_state = 0x1234567890abcdefULL ^ chrono::
    high_resolution_clock::now().time_since_epoch().count();
static inline ll rnd() { return splitmix64(rng_state += 0
    x9e3779b97f4a7c15ULL); }

// trial division pequena para acelerar
static const int SMALL_P_MAX = 1000;
static vector<int> small_primes;

static void sieve_small() {
    vector<bool> is(SMALL_P_MAX + 1, true);
    is[0] = is[1] = false;
    for (int i = 2; i * i <= SMALL_P_MAX; ++i) if (is[i])
        for (int j = i * i; j <= SMALL_P_MAX; j += i) is[j] = false;
    for (int i = 2; i <= SMALL_P_MAX; ++i) if (is[i]) small_primes.
        push_back(i);
}

    bool isPrime(ll n) {
    if (n < 2) return false;
    // divide por primos pequenos
    for (int p : small_primes) {
        if ((ll)p * (ll)p > n) break;
        if (n % p == (ll)0) return n == (ll)p;
    }
    if (n < 4) return true; // 2,3
    // Miller-Rabin deterministico para 64-bit
    ll d = n - 1, s = 0;
    while ((d & 1) == 0) d >>= 1, ++s;
    auto witness = [&](ll a) -> bool {
        if (a % n == 0) return false;
        ll x = powmod(a % n, d, n);
        if (x == 1 || x == n - 1) return false;
        for (int i = 1; i < s; ++i) {
            x = mulmod(x, x, n);
            if (x == n - 1) return false;
        }
        return true; // es testigo: n compuesto
    };
    // Bases correctas para 64-bit
    for (ll a : {2ULL, 3ULL, 5ULL, 7ULL, 11ULL, 13ULL, 17ULL, 19ULL, 23
        ULL,
                 325ULL, 9375ULL, 28178ULL, 450775ULL, 9780504ULL,
                     1795265022ULL}) {
        if (a == 0) continue;
        if (a % n == 0) continue;
        if (witness(a)) return false;
    }
    return true;
}

ll pollard_rho(ll n) {
    if ((n & 1ULL) == 0ULL) return 2ULL;
    while (true) {
        ll c = (rnd() % (n - 1)) + 1; // [1..n-1]
        ll x = (rnd() % (n - 2)) + 2; // [2..n-1]
        ll y = x;
        ll d = 1;
        // limite de iteraciones para evitar lazos raros
        for (int it = 0; it < 1'000'000 && d == 1; ++it) {
            x = (mulmod(x, x, n) + c) % n;
            y = (mulmod(y, y, n) + c) % n;
            y = (mulmod(y, y, n) + c) % n;
            ll diff = x > y ? x - y : y - x;
            d = std::gcd(diff, n);
```

```
        }
        if (d == 1 || d == n) continue;
        return d;
    }
}

void fact(ll n, map<ll,int> &F) {
    if (n == 1) return;
    if (isPrime(n)) { F[n]++; return; }
    for (int p : small_primes) {
        if ((ll)p * (ll)p > n) break;
        while (n % p == 0) { F[p]++; n /= p; }
    }
    if (n == 1) return;
    if (isPrime(n)) { F[n]++; return; }
    ll d = pollard_rho(n);
    fact(d, F);
    fact(n / d, F);
}
```

8.8 Simpson

```
ld simpsonRule(function<ld(ld)> f, ld a, ld b, int n) {
    // Asegurarse de que n sea par
    if (n % 2 != 0) {
        n++;
    }
    ld h = (b - a) / n;
    ld s = f(a) + f(b);

    // Suma de terminos interiores con los factores apropiados
    for (int i = 1; i < n; i++) {
        ld x = a + i * h;
        s += (i % 2 == 1 ? 4.0L : 2.0L) * f(x);
    }
    // Multiplica por h/3
    return (h / 3.0L) * s;
}
// Ejemplo: integrar la funcion x^2 entre 0 y 3
auto f = [&](ld x){ return x * x; };
ld a = 0.0L, b = 3.0L;
int n = 1000; // numero de subintervalos
ld resultado = simpsonRule(f, a, b, n);
```

9 Geometry

9.1 Convex Hull

```
typedef pair<ll, ll> Point;
ll cross_product(Point O, Point A, Point B) {
    return (A.first - O.first) * (B.second - O.second) - (A.second - O.
        second) * (B.first - O.first);
}
vector<Point> convex_hull(vector<Point>& points) {
    sort(points.begin(), points.end());
    points.erase(unique(points.begin(), points.end()), points.end());
    vector<Point> hull;
    // Parte inferior
    for (const auto& p : points) {
        while (hull.size() >= 2 && cross_product(hull[hull.size() - 2],
            hull[hull.size() - 1], p) < 0)
            hull.pop_back();
        if (hull.empty() || hull.back() != p) {
            hull.push_back(p);
        }
    }
    // Parte superior
    int t = hull.size() + 1;
    for (int i = points.size() - 1; i >= 0; --i) {
        while (hull.size() >= t && cross_product(hull[hull.size() - 2],
            hull[hull.size() - 1], points[i]) < 0)
            hull.pop_back();
        if (hull.empty() || hull.back() != points[i]) {
            hull.push_back(points[i]);
        }
    }
    hull.pop_back();
    return hull;
}
```

9.2 Operations

```
ll cross_product(pair<ll, ll> P1, pair<ll, ll> P2, pair<ll, ll> P3) {
    ll x1 = P2.first - P1.first;
    ll y1 = P2.second - P1.second;
    ll x2 = P3.first - P1.first;
    ll y2 = P3.second - P1.second;
```

```
    return x1 * y2 - y1 * x2;
}
double distancia(pair<ll, ll> P1, pair<ll, ll> P2) {
    return sqrt((P2.first - P1.first) * (P2.first - P1.first) +
                (P2.second - P1.second) * (P2.second - P1.second));
}
ll dot_product(pair<ll, ll> P1, pair<ll, ll> P2, pair<ll, ll> P3) {
    ll x1 = P2.first - P1.first;
    ll y1 = P2.second - P1.second;
    ll x2 = P3.first - P1.first;
    ll y2 = P3.second - P1.second;
    return x1 * x2 + y1 * y2;
}
```

9.3 Polygon Area

```
typedef pair<ll, ll> Point;
double polygon_area(const vector<Point>& polygon) {
    ll area = 0;
    int n = polygon.size();
    for (int i = 0; i < n; ++i) {
        ll j = (i + 1) % n;
        area += (polygon[i].first * polygon[j].second - polygon[i].
            second * polygon[j].first);
    }
    return abs(area) / 2.0;
}
```

9.4 Ray Casting

```
typedef pair<ll, ll> Point;
bool is_point_in_polygon(const vector<Point>& polygon, Point p) {
    bool inside = false;
    int n = polygon.size();
    for (int i = 0, j = n - 1; i < n; j = i++) {
        if ((polygon[i].second > p.second) != (polygon[j].second > p.
            second) &&
            p.first < (polygon[j].first - polygon[i].first) * (p.second
                - polygon[i].second) /
                      (polygon[j].second - polygon[i].second) + polygon[
                          i].first) {
            inside = !inside;
        }
    }
    return inside;
}
```