# Dividimos y No Conquistamos (D&!C)

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## 1 Template

#### 1.1 C++ Template

```
#include <bits/stdc++.h>
   using namespace std;
   #define LOCAL
   #define L(i, j, n) for (int i = (j); i < (int)n; i ++)
   #define LI(i, j, n) for (int i = (j); i \le (int)n; i \leftrightarrow (int)n
   #define R(i, j, n) for (int i = (j); i > (int)n; i --)
   #define RI(i, j, n) for (int i = (j); i \ge (int)n; i --)
   #define SZ(x) int((x).size())
   #define ALL(x) begin(x),end(x)
   #define IS_IN(x, v) ((x).find(v) != (x).end())
   #define vec vector
   #define pb push_back
14
   using ll = long long;
   using ld = long double;
   using pii = pair<int, int>;
   using pil = pair<int, ll>;
   using pli = pair<ll, int>;
   using pll = pair<ll, 11>;
21
22
   const int N = (int)2e5+5;
   const int MOD = (int)1e9 + 7;
   const int oo = (int)1e9;
26
   void solve()
27
28
29
30
31
   int main()
32
33
       ios::sync_with_stdio(0);cin.tie(0);
34
       int TT = 1:
35
       //cin >> TT:
36
       while (TT--)
37
       {
38
           solve();
39
```

```
}
40
41 }
                           1.2 Fast Python
import os, sys, io
finput = io.BytesIO(os.read(0, os.fstat(0).st_size)).readline
3 fprint = sys.stdout.write
                           1.3 Policy Based
 #include <ext/pb_ds/assoc_container.hpp>
   using namespace __gnu_pbds;
   template<typename Key, typename Val=null_type>
   using indexed_set = tree<Key, Val, less<Key>, rb_tree_tag,
                            tree_order_statistics_node_update>;
   // indexed_set<char> s;
   // char val = *s.find_by_order(0); // access por indice
   // int idx = s.order_of_key('a'); // busca indice del valor
   template<class Key, class Val=null_type>using htable=gp_hash_table<Key,
       Val>:
10 // como unordered_map (o unordered_set si Val es vacio), pero sin metodo
                                    Search
                              2.1 Ternary
1 // Minimo de 'f' en '(l,r)'.
   template<class Fun>11 ternary(Fun f, 11 1, 11 r) {
     for (11 d = r-1; d > 2; d = r-1) {
       11 a = 1 + d/3, b = r - d/3;
       if (f(a) > f(b)) l = a; else r = b;
5
6
     return 1 + 1;
8
   // para error < EPS, usar iters=log((r-1)/EPS)/log(1.618)</pre>
   template<class Fun>double golden(Fun f, double 1, double r, int iters){
     double const ratio = (3-sqrt(5))/2;
11
     double x1=1+(r-1)*ratio, x2=r-(r-1)*ratio, f1=f(x1), f2=f(x2);
12
     while (iters--) {
       if (f1 > f2) 1=x1, x1=x2, f1=f2, x2=r-(r-1)*ratio, f2=f(x2);
14
                    r=x2, x2=x1, f2=f1, x1=1+(r-1)*ratio, f1=f(x1);
15
```

```
return (1+r)/2;
18 }
                              Data structures
                                  3.1 BIT
   #define LSOne(S) (S & -S)
2
   struct BIT {
3
       vector<int> B;
4
       int n;
5
       BIT(int n = 1): B(n + 1), n(n+1){}
6
       BIT(vector<int> &v): B(v.size()+1), n(v.size()+1) {
           for (int i = 1; i \le n; i ++){
8
               B[i] += v[i-1];
9
               if (i + LSOne(i) <= n){</pre>
10
                    B[i + LSOne(i)] += B[i];
11
               }
12
           }
13
       }
14
       void update(int i, int x){
15
           while (i <= n){
16
               B[i] += x;
17
               i += LSOne(i);
18
           }
19
       }
20
       int sum(int i){
21
           int res = 0;
^{22}
           while (i > 0){
23
               res += B[i];
24
               i -= LSOne(i);
25
26
27
           return res;
       }
28
       int range_sum(int 1, int r){
29
           return sum(r) - sum(1 - 1);
30
       }
31
32 };
                                 3.2 DSU
1 | struct DSU {
```

```
vector<int> par, sz;
2
3
       int n;
       DSU(int n = 1): par(n), sz(n, 1), n(n) {
4
           for (int i = 0;i < n; i ++) par[i] = i;</pre>
5
       }
6
       int find(int a){
           return a == par[a] ? a : par[a] = find(par[a]);
8
9
       void join(int a, int b){
           a=find(a);
           b=find(b);
12
           if (a != b){
13
               if (sz[b] > sz[a]) swap(a,b);
               par[b] = a;
               sz[a] += sz[b];
16
           }
17
       }
18
19 };
                            3.3 Sparse Table
int log2_floor(unsigned long long i) {
       return i ? __builtin_clzll(1) - __builtin_clzll(i) : -1;
   }
3
   const int MAXN = 10;
   int K = log2_floor(MAXN);
   int st[K + 1][MAXN];
   // Load Array to st[0][i]
   std::copy(array.begin(), array.end(), st[0]);
11
   // Build
12
  for (int i = 1; (1 << i) <= n; i ++){
       for (int j = 0; j + (1 << (i - 1)) < n; j ++){}
           st[i][j] = min(st[i-1][j], st[i-1][j + (1 << (i - 1))]);
15
       }
16
   }
17
18
   // Query
   int min_range(int 1, int r){
       int C = log2\_floor(r - 1 + 1);
21
       return min(st[C][1], st[C][r - (1 << C) + 1]);
22
```

23 }

```
3.4 Segment tree
1 struct Node {
       long long sum = 0;
2
       long long min_val = LLONG_MAX;
3
       long long max_val = LLONG_MIN;
       long long lazy = 0;
       // Merge function to combine two nodes
7
       void merge(const Node& left, const Node& right) {
           sum = left.sum + right.sum;
9
           min_val = min(left.min_val, right.min_val);
           max_val = max(left.max_val, right.max_val);
11
       }
12
13
       // Update function for lazy propagation
14
       void apply(int 1, int r, long long value) {
15
           sum += (r - 1 + 1) * value:
           min_val += value;
           max_val += value;
18
           lazy += value;
19
       }
20
   };
21
22
   struct SegTree {
23
       int n;
^{24}
       vector<Node> tree;
25
26
       SegTree(int n) : n(n) {
27
           tree.resize(4 * n + 5);
28
       }
29
30
       SegTree(vector<int>& arr) : n(arr.size()) {
31
           tree.resize(4 * n + 5);
32
           build(arr, 0, 0, n-1);
33
       }
34
35
       // Push lazy value to children
36
       void push(int id, int l, int r) {
37
           if (tree[id].lazy == 0) return;
38
39
```

```
int mid = (1 + r) >> 1:
40
           tree[2*id + 1].apply(1, mid, tree[id].lazy);
41
           tree[2*id + 2].apply(mid+1, r, tree[id].lazy);
42
           tree[id].lazy = 0;
43
       }
44
45
       void build(vector<int>& arr, int id, int 1, int r) {
46
           if (1 == r) {
47
                tree[id].sum = arr[1];
48
               tree[id].min_val = arr[l];
49
                tree[id].max_val = arr[l];
50
                return;
51
           }
52
53
           int mid = (1 + r) >> 1;
54
           build(arr, 2*id + 1, 1, mid);
           build(arr, 2*id + 2, mid+1, r);
56
           tree[id].merge(tree[2*id + 1], tree[2*id + 2]);
       }
58
59
       // Range update with lazy propagation
60
       void update(int id, int l, int r, int ql, int qr, long long val) {
61
           if (ql > r \mid | qr < 1) return;
62
63
           if (ql <= l && r <= qr) {
64
                tree[id].apply(l, r, val);
65
                return;
66
           }
67
68
           push(id, 1, r);
69
           int mid = (1 + r) >> 1;
70
           update(2*id + 1, 1, mid, ql, qr, val);
71
           update(2*id + 2, mid+1, r, ql, qr, val);
72
           tree[id].merge(tree[2*id + 1], tree[2*id + 2]);
73
       }
74
75
       // Range query
76
       Node query(int id, int 1, int r, int ql, int qr) {
77
           if (ql > r || qr < 1) return Node();</pre>
78
79
           if (ql <= l && r <= qr) {
80
                return tree[id];
81
           }
82
```

```
83
            push(id, 1, r);
84
            int mid = (1 + r) >> 1;
85
           Node left = query(2*id + 1, 1, mid, ql, qr);
86
           Node right = query(2*id + 2, mid+1, r, ql, qr);
87
88
            Node result;
89
            result.merge(left, right);
90
            return result;
91
       }
92
93
       // Public interface
94
       void update(int 1, int r, long long val) {
95
            update(0, 0, n-1, 1, r, val);
96
       }
97
98
       Node query(int 1, int r) {
99
            return query(0, 0, n-1, 1, r);
100
       }
101
102 };
```

## 4 Dynamic Programming

## 4.1 Knapsack

```
int knapsack(vector<int>& values, vector<int>& weights, int W) {
       int n = values.size():
2
       vector<vector<int>> dp(n + 1, vector<int>(W + 1, 0));
3
       for(int i = 1; i <= n; i++) {
5
           for(int w = 0; w \le W; w++) {
6
               if(weights[i-1] <= w) {</pre>
7
                    dp[i][w] = max(dp[i-1][w],
                                  dp[i-1][w-weights[i-1]] + values[i-1]);
9
               } else {
10
                    dp[i][w] = dp[i-1][w];
11
12
           }
13
       }
14
       return dp[n][W];
15
16 }
```

4.2 LIS

```
vector<int> getLIS(vector<int>& arr) {
       int n = arr.size();
2
       vector<int> dp(n + 1, INT_MAX); // dp[i] = smallest value that ends
3
            an LIS of length i
       vector<int> len(n);
                                         // Length of LIS ending at each
4
           position
       dp[0] = INT_MIN;
5
6
       for(int i = 0; i < n; i++) {</pre>
           int j = upper_bound(dp.begin(), dp.end(), arr[i]) - dp.begin();
8
           dp[j] = arr[i];
9
           len[i] = j;
10
       }
11
12
       // Find maxLen and reconstruct sequence
13
       int maxLen = 0;
       for(int i = n-1; i \ge 0; i--) maxLen = max(maxLen, len[i]);
15
16
       vector<int> lis:
17
       for(int i = n-1, currLen = maxLen; i \ge 0; i--) {
           if(len[i] == currLen) {
19
               lis.push_back(arr[i]);
20
               currLen--;
21
           }
22
       }
23
       reverse(lis.begin(), lis.end());
24
       return lis;
25
26 }
```

#### 4.3 Edit Distance

```
1
   //3. Edit Distance - O(n*m)
   int editDistance(string& s1, string& s2) {
       int n = s1.length(), m = s2.length();
4
       vector<vector<int>> dp(n + 1, vector<math><int>(m + 1));
5
6
       // Base cases
7
       for(int i = 0; i \le n; i++) dp[i][0] = i;
       for(int j = 0; j \le m; j++) dp[0][j] = j;
9
10
       for(int i = 1; i <= n; i++) {
11
12
           for(int j = 1; j <= m; j++) {
```

using namespace std;

3

4

```
if(s1[i-1] == s2[j-1]) {
13
                    dp[i][j] = dp[i-1][j-1];
14
               } else {
15
                   dp[i][j] = 1 + min({dp[i-1][j], // deletion}
16
                                      dp[i][j-1],
                                                     // insertion
17
                                      dp[i-1][j-1]}); // replacement
18
19
20
       }
^{21}
       return dp[n][m];
22
23 | }
                               4.4 Kadane
   pair<int, pair<int,int>> kadane(vector<int>& arr) {
       int maxSoFar = arr[0], maxEndingHere = arr[0];
2
       int start = 0, end = 0, s = 0;
3
4
       for(int i = 1; i < arr.size(); i++) {</pre>
5
           if(maxEndingHere + arr[i] < arr[i]) {</pre>
6
               maxEndingHere = arr[i];
               s = i;
8
           } else {
9
               maxEndingHere += arr[i];
10
           }
11
12
           if(maxEndingHere > maxSoFar) {
13
               maxSoFar = maxEndingHere;
14
               start = s;
15
               end = i;
16
           }
17
18
       return {maxSoFar, {start, end}}; // max, 1, r
19
20 }
                                    Strings
                             5.1 Prefix Trie
  #include <bits/stdc++.h>
```

```
5 | struct TrieNodeStruct {
       TrieNodeStruct* children[26];
6
       bool isEndOfWord;
8
       TrieNodeStruct() {
9
           isEndOfWord = false;
10
           for(int i = 0; i < 26; i++) {
11
               children[i] = nullptr;
12
           }
13
14
   };
15
16
   struct TrieStruct {
       TrieNodeStruct* root;
19
       TrieStruct() {
20
           root = new TrieNodeStruct();
21
       }
22
23
       void insert(string word) {
24
           TrieNodeStruct* current = root;
25
           for(char c : word) {
26
               int index = c - 'a';
27
               if(current->children[index] == nullptr) {
28
                    current->children[index] = new TrieNodeStruct();
29
30
               current = current->children[index];
31
32
           current->isEndOfWord = true;
33
       }
34
35 };
                               5.2 Hashing
static constexpr ll ms[] = {1'000'000'007, 1'000'000'403};
   static constexpr 11 b = 500'000'000;
   struct StrHash { // Hash polinomial con exponentes decrecientes.
     vector<11> hs[2], bs[2];
     StrHash(string const& s) {
       int n = SZ(s);
6
       L(k, 0, 2) {
         hs[k].resize(n+1), bs[k].resize(n+1, 1);
8
         L(i, 1, n) {
9
```

```
hs[k][i+1] = (hs[k][i] * b + s[i]) % ms[k];
10
           bs[k][i+1] = bs[k][i] * b
                                                % ms[k];
11
         }
12
       }
13
14
     ll get(int idx, int len) const { // Hashes en 's[idx, idx+len)'.
15
       ll h[2];
16
       L(k, 0, 2) {
17
         h[k] = hs[k][idx+len] - hs[k][idx] * bs[k][len] % ms[k];
18
         if (h[k] < 0) h[k] += ms[k];
19
20
       return (h[0] << 32) | h[1];
21
22
23
24
   pll union_hash(vector<pll> hs, vector<ll> lens){ //use arrays makes it
       slower
     11 len = 0;
26
     for(int i = hs.size()-1; i > 0; i--){
27
       len += lens[i];
28
       pll& [11, 12] = hs[i];
29
       pll& [r1, r2] = hs[i-1];
30
       11 = ((11 * binpow(b, len, ms[0])) \% ms[0] + r1) \% ms[0];
31
       12 = ((12 * binpow(b, len, ms[1])) \% ms[1] + r2) \% ms[1];
32
     }
33
34
     return hs[0];
35
36 }
```

#### 5.3 KMP

```
int i = 0:
12
     int j = 0;
13
14
     while((n - i) >= (m - j)){
15
       if(pat[j] == sec[i]){
16
         i++;
17
          j++;
18
       }
19
       if(j == m){
20
         res.push_back(i - j);
21
          j = lps[j - 1];
22
       }
23
       else{
24
         if(i < n && pat[j] != sec[i]){</pre>
25
            if(j != 0) j = lps[j-1];
26
            else i = i + 1;
27
         }
28
       }
29
     }
30
31
32
     return res;
33 }
```

#### 5.4 LPS

```
#include <bits/stdc++.h>
   using namespace std;
   vector<int> getLps(string pat){ //geek4geeks implementatio with some
       changes
     vector<int> lps(pat.length(), 0);
     int len = 0;
6
     int i = 1;
7
     lps[0] = 0;
     while(i < pat.length()){</pre>
9
       if(pat[i] == pat[len]){
10
         len++;
11
         lps[i] = len;
12
         i++;
13
14
       else //pat[i] != pat[len]
15
16
         lps[i] = 0;
17
```

dfs(v, u);

dfs\_low[u] = min(dfs\_low[u], dfs\_low[v]);

if (dfs\_low[v] > dfs\_num[u]){

// Bridge from u -> v

15

16

17

18

dfs\_low[u]=dfs\_num[u]=dfs\_count++;

st.push(u);

8

visited[u] = 1;

for(int v: G[u]) {

```
cout << "Bridge," << u << ",->," << v << "\n";
         i++;
18
                                                                                  19
       }
                                                                                  20
19
                                                                                                  if (dfs_low[v] >= dfs_num[u]) {
     }
20
                                                                                  21
                                                                                                      // u is AP
21
                                                                                  22
                                                                                                      ap[u] = 1;
     return lps;
22
                                                                                  23
23
                                                                                  24
                                                                                              } else dfs_low[u] = min(dfs_low[u], dfs_num[v]);
                                                                                  25
                                Z-FUNCTION
                                                                                         if (u == root){
                                                                                  27
   template<class Char=char>vector<int> zfun(const basic_string<Char>& w) {
                                                                                              ap[u] = child > 1;
     int n = SZ(w), l = 0, r = 0; vector<int> z(n);
2
                                                                                  29
     z[0] = w.length();
                                                                                  30 }
3
     L(i, 1, n) {
                                                                                                                    Bellman Ford
      if (i \le r) \{z[i] = min(r - i + 1, z[i - 1]);\}
5
       while (i + z[i] < n \&\& w[z[i]] == w[i + z[i]]) \{++z[i];\}
                                                                                   1 struct Edge {
       if (i + z[i] - 1 > r) \{l = i, r = i + z[i] - 1;\}
7
                                                                                          int a, b, cost;
8
                                                                                     };
                                                                                   3
     return z;
9
10 }
                                                                                     int n, m, v;
                                                                                     vector<Edge> edges;
                                    Graph
                                                                                     const int INF = 1000000000;
                                6.1 Tarjan
                                                                                     void solve()
                                                                                   9
                                                                                     {
                                                                                  10
   const int N = 10;
                                                                                         vector<int> d(n, INF);
                                                                                  11
2
                                                                                          d[v] = 0;
                                                                                  12
   vector<int> G[N];
                                                                                         for (int i = 0; i < n - 1; ++i)
                                                                                  13
   vector\langle int \rangle dfs_low(N, -1), dfs_num(N, -1), ap(N, 0); // ap for
                                                                                              for (Edge e : edges)
                                                                                  14
       Articulation Points
                                                                                                  if (d[e.a] < INF)</pre>
                                                                                  15
   int dfs_count = 0;
                                                                                                      d[e.b] = min(d[e.b], d[e.a] + e.cost);
                                                                                  16
   int root = -1; // For AP
                                                                                  17 }
   void dfs(int u, int p = -1){
                                                                                                                    6.3 SCC
       dfs_low[u] = dfs_num[u] = dfs_count++;
9
       int child = 0;
                                                                                   vector<int> dfs_num(N, -1), dfs_low(N, -1), visited(N);
10
       for (int v: G[u]){
                                                                                     int dfs_count = 0;
11
           if (v == p) continue;
                                                                                     int numSCC = 0;
12
           if (dfs_num[v] == -1){
                                                                                     stack<int> st:
13
                                                                                     void dfs(int u){
               child ++;
14
```

```
if (dfs_num[v] == -1) dfs(v);
10
       if (visited[v]) dfs_low[u] = min(dfs_low[u], dfs_low[v]);
11
     }
12
     if (dfs_num[u] == dfs_low[u]){
13
       numSCC ++;
14
       while(1){
15
         int v = st.top(); st.pop();
16
         visited[v] = 0;
17
         if (u == v) break;
18
19
     }
20
  |}
21
```

## 6.4 Bipartite Matching Hopcroft-Karp

```
int L_S, R_S;
   vec<int> G[S_MX]; // S_MX (Maxima cantidad de nodos de un lado)
   int mat[S_MX]; // matching [0,L_S) \rightarrow [0,R_S)
   int inv[S_MX]; // matching [0,R_S) \rightarrow [0,L_S)
   int hopkarp() {
       fill(mat,mat+L_S,-1);
6
       fill(inv,inv+R_S,-1);
7
       int size = 0;
8
       vector<int> d(L S):
9
       auto bfs = \lceil k \rceil {
10
            bool aug = false;
11
            queue<int> q;
12
            L(u, 0, L_S) if (mat[u] < 0) q.push(u); else d[u] = -1;
13
            while (!q.empty()) {
14
                int u = q.front();
15
                q.pop();
16
                for (auto v : G[u]) {
17
                     if (inv[v] < 0) aug = true;</pre>
18
                     else if (d[inv[v]] < 0) d[inv[v]] = d[u] + 1, q.push(inv
19
                         [v]);
                }
20
            }
^{21}
            return aug;
22
       };
23
       auto dfs = [\&] (auto&& me. int u) -> bool {
24
            for (auto v : G[u]) if (inv[v] < 0) {
25
                mat[u] = v, inv[v] = u;
26
                return true;
27
```

```
}
28
           for (auto v : G[u]) if (d[inv[v]] > d[u] \&\& me(me,inv[v])) {
29
               mat[u] = v, inv[v] = u;
30
               return true;
31
           }
32
           d[u] = 0;
33
           return false;
34
       };
35
       while (bfs()) L(u, 0, L_S) if (mat[u] < 0) size += dfs(dfs,u);
36
       return size;
37
38 }
                 6.5 Konig Theorem Min V.Cover
```

```
vec<int> cover[2]; // if cover[i][j] = 1 -> node i, j is part of cover
  int konig() {
       cover[0].assign(L_S,true); // L_S left size
       cover[1].assign(R_S,false); // R_S right size
4
       int size = hopkarp(); // alternativamente, tambien funciona con
5
           Kııhn
       auto dfs = [&](auto&& me, int u) -> void {
6
           cover[0][u] = false:
7
           for (auto v : g[u]) if (!cover[1][v]) {
8
               cover[1][v] = true;
9
               me(me,inv[v]);
10
           }
11
       };
12
       L(u,0,L_S) if (mat[u] < 0) dfs(dfs,u);
13
       return size;
14
15 }
```

## 6.6 Hungarian

```
using vi = vec<int>;
using vd = vec<ld>;
const ld INF = 1e100;  // Para max asignacion, INF = 0, y negar costos
bool zero(ld x) {return fabs(x) < 1e-9;}  // Para int/ll: return x==0;
vec<pii> ans; // Guarda las aristas usadas en el matching: [0..n)x[0..m)
struct Hungarian{
  int n; vec<vd> cs; vi vL, vR;
  Hungarian(int N, int M) : n(max(N,M)), cs(n,vd(n)), vL(n), vR(n){
    L(x, 0, N) L(y, 0, M) cs[x][y] = INF;
}
void set(int x, int y, ld c) { cs[x][y] = c; }
```

1 // Complexity (V \* V \* E);

```
2 | struct Dinic {
     ld assign(){
12
       int mat = 0; vd ds(n), u(n), v(n); vi dad(n), sn(n);
                                                                                            struct Edge {
                                                                                    3
13
       L(i, 0, n) u[i] = *min_element(ALL(cs[i]));
                                                                                                int to, rev;
                                                                                    4
14
       L(j, 0, n){
                                                                                                long long cap, flow;
                                                                                     5
15
         v[i] = cs[0][i]-u[0];
                                                                                                Edge(int to, int rev, long long cap) :
                                                                                     6
16
         L(i, 1, n) v[j] = min(v[j], cs[i][j] - u[i]);
                                                                                                    to(to), rev(rev), cap(cap), flow(0) {}
                                                                                    7
17
                                                                                           };
                                                                                     8
18
       vL = vR = vi(n, -1);
19
       L(i,0, n) L(j, 0, n) if (vR[j] == -1 \text{ and } zero(cs[i][j] - u[i] - v[j])
                                                                                            vector<vector<Edge>> g;
20
                                                                                    10
           ){
                                                                                            vector<int> level, ptr;
                                                                                    11
         vL[i] = j; vR[j] = i; mat++; break;
                                                                                            queue<int> q;
                                                                                    12
21
                                                                                            int n, source, sink;
22
                                                                                    13
       for(: mat < n: mat ++){
                                                                                            const long long INF = 1e18;
23
                                                                                    14
         int s = 0, j = 0, i;
                                                                                    15
24
         while(vL[s] != -1) s++;
                                                                                            Dinic(int n, int s, int t) : n(n), source(s), sink(t) {
                                                                                    16
25
         fill(ALL(dad), -1); fill(ALL(sn), 0);
                                                                                                g.resize(n);
                                                                                    17
26
         L(k, 0, n) ds[k] = cs[s][k]-u[s]-v[k];
                                                                                                level.resize(n);
27
                                                                                    18
         while(true){
                                                                                                ptr.resize(n);
28
                                                                                    19
                                                                                           }
           i = -1:
                                                                                    20
29
           L(k, 0, n) if (!sn[k] and (j == -1 \text{ or } ds[k] < ds[j])) <math>j = k;
                                                                                    21
30
           sn[j] = 1; i = vR[j];
                                                                                            void add_edge(int from, int to, long long cap) {
                                                                                    22
31
           if(i == -1) break;
                                                                                                g[from].emplace_back(to, g[to].size(), cap);
32
                                                                                    23
           L(k, 0, n) if(!sn[k]){
                                                                                                g[to].emplace_back(from, g[from].size()-1, 0); // Reverse edge
                                                                                    24
33
              auto new_ds = ds[j] + cs[i][k] - u[i] - v[k];
                                                                                            }
                                                                                    25
34
             if(ds[k] > new_ds) ds[k]=new_ds, dad[k]=j;
                                                                                    26
35
                                                                                            bool bfs() {
                                                                                    27
36
         }
                                                                                                while(!q.empty()) {
                                                                                    28
37
         L(k, 0, n) if(k!=j and sn[k]){
                                                                                                    q.pop();
                                                                                    29
38
           auto w = ds[k]-ds[j]; v[k] += w, u[vR[k]] -= w;
                                                                                                }
                                                                                    30
39
         }
                                                                                                fill(level.begin(), level.end(), -1);
40
                                                                                    31
         u[s] += ds[j];
41
                                                                                    32
         while (dad[j] \ge 0) { int d = dad[j]; vR[j] = vR[d]; vL[vR[j]] = j;
                                                                                                q.push(source);
                                                                                    33
42
             i = d: }
                                                                                                level[source] = 0:
                                                                                    34
         vR[j] = s; vL[s] = j;
                                                                                    35
43
       }
                                                                                                while(!q.empty() && level[sink] == -1) {
                                                                                    36
44
       ld value = 0; L(i, 0, n) value += cs[i][vL[i]], ans.pb({i, vL[i]});
                                                                                                    int v = q.front();
                                                                                    37
45
       return value:
                                                                                                    q.pop();
46
                                                                                    38
     }
47
                                                                                    39
48 };
                                                                                                    for(const Edge& e : g[v]) {
                                                                                    40
                                                                                                        if(level[e.to] == -1 \&\& e.flow < e.cap) {
                                                                                    41
                                  6.7 Flow
                                                                                                            level[e.to] = level[v] + 1;
                                                                                    42
                                                                                                            q.push(e.to);
                                                                                    43
                                                                                                        }
```

44

```
}
                                                                                                     }
45
                                                                                     88
                                                                                                 }
                                                                                     89
46
           return level[sink] != -1;
                                                                                                 return flow;
47
                                                                                     90
       }
                                                                                             }
48
                                                                                     91
                                                                                     92
49
       long long dfs(int v, long long pushed) {
                                                                                             // Find minimum cut
50
                                                                                     93
           if(v == sink || pushed == 0) return pushed;
                                                                                             vector<bool> min_cut() {
51
                                                                                     94
                                                                                                 vector<bool> reachable(n, false);
52
                                                                                     95
           for(int& i = ptr[v]; i < (int)g[v].size(); i++) {</pre>
                                                                                                 queue<int> q;
53
                                                                                     96
                Edge& e = g[v][i];
                                                                                                 q.push(source);
54
                                                                                                 reachable[source] = true;
55
                                                                                     98
                if(level[e.to] != level[v] + 1 || e.flow >= e.cap) continue;
56
                                                                                     99
                                                                                                 while(!q.empty()) {
57
                                                                                     100
                long long flow = dfs(e.to, min(pushed, e.cap - e.flow));
                                                                                                     int v = q.front();
                                                                                     101
58
                if(flow == 0) continue:
                                                                                                     q.pop();
                                                                                     102
59
60
                                                                                     103
                e.flow += flow;
                                                                                                     for(const Edge& e : g[v]) {
61
                                                                                     104
                                                                                                         if(!reachable[e.to] && e.flow < e.cap) {</pre>
                g[e.to][e.rev].flow -= flow;
62
                                                                                     105
                return flow:
                                                                                                              reachable[e.to] = true:
                                                                                     106
63
                                                                                                              q.push(e.to);
                                                                                     107
64
           return 0;
                                                                                                         }
                                                                                     108
65
       }
                                                                                                     }
66
                                                                                     109
                                                                                                 }
                                                                                    110
67
       long long max_flow() {
                                                                                                 return reachable;
                                                                                    111
68
           long long flow = 0;
                                                                                            }
                                                                                    112
69
                                                                                        };
                                                                                    113
70
           while(bfs()) {
                                                                                    114
71
                fill(ptr.begin(), ptr.end(), 0);
                                                                                         // Example usage:
                                                                                    115
72
                while(long long pushed = dfs(source, INF)) {
                                                                                    116
73
                    flow += pushed;
                                                                                        int main() {
74
                }
                                                                                            // Example: 6 vertices, source = 0, sink = 5
                                                                                    118
75
                                                                                            int n = 6;
                                                                                    119
76
           return flow;
                                                                                            Dinic flow(n, 0, 5);
                                                                                    120
77
       }
                                                                                    121
78
                                                                                            // Add edges: (from, to, capacity)
                                                                                    122
79
       // Get the actual flow passing through each edge
                                                                                            flow.add_edge(0, 1, 16);
                                                                                    123
80
       vector<vector<long long>> get_flow() {
                                                                                             flow.add_edge(0, 2, 13);
                                                                                    124
81
           vector<vector<long long>> flow(n, vector<long long>(n, 0));
                                                                                             flow.add_edge(1, 2, 10);
                                                                                    125
82
           for(int v = 0; v < n; v++) {
                                                                                             flow.add_edge(1, 3, 12);
                                                                                    126
83
                for(const Edge& e : g[v]) {
                                                                                    127
                                                                                             flow.add_edge(2, 1, 4);
84
                    if(e.cap > 0) { // Only original edges, not residual
                                                                                            flow.add_edge(2, 4, 14);
                                                                                    128
85
                        flow[v][e.to] = e.flow;
                                                                                            flow.add_edge(3, 2, 9);
                                                                                    129
86
                                                                                            flow.add_edge(3, 5, 20);
                    }
                                                                                    130
87
```

```
flow.add_edge(4, 3, 7);
                                                                                             vector<Edge> edges;
131
                                                                                     10
        flow.add_edge(4, 5, 4);
                                                                                             vector<vector<int>> adj;
                                                                                     11
132
                                                                                             vector<int> level, ptr;
                                                                                     12
133
        // Calculate maximum flow
                                                                                             int n;
134
                                                                                     13
        long long max_flow = flow.max_flow();
                                                                                             queue<int> q;
135
                                                                                     14
        cout << "Maximum flow: " << max_flow << "\n";</pre>
136
                                                                                     15
                                                                                             MaxFlow(int n) : n(n) {
                                                                                     16
137
        // Get minimum cut
                                                                                                 adj.resize(n);
138
                                                                                     17
        vector<bool> cut = flow.min_cut();
                                                                                                 level.resize(n);
139
                                                                                     18
        cout << "Vertices on source side of min cut: ";</pre>
                                                                                                 ptr.resize(n);
140
                                                                                     19
                                                                                             }
        for(int i = 0; i < n; i++) {
141
                                                                                     20
            if(cut[i]) cout << i << " ";
142
                                                                                     21
        }
                                                                                             void add_edge(int from, int to, ll cap) {
143
                                                                                     22
        cout << "\n";
                                                                                                 edges.emplace_back(from, to, cap);
144
                                                                                     23
                                                                                                 edges.emplace_back(to, from, 0);
                                                                                     24
145
        // Get flow through each edge
                                                                                                 adj[from].push_back(edges.size() - 2);
146
                                                                                     25
        auto flow_matrix = flow.get_flow();
                                                                                                 adj[to].push_back(edges.size() - 1);
147
                                                                                     26
        cout << "Flow matrix:\n";</pre>
                                                                                             }
148
                                                                                     27
        for(int i = 0: i < n: i++) {
                                                                                     28
149
            for(int j = 0; j < n; j++) {
                                                                                             bool bfs(int s, int t) {
150
                if(flow_matrix[i][j] > 0) {
                                                                                                 while(!q.empty()) q.pop();
                                                                                     30
151
                     cout << i << " -> " << j << ": " << flow_matrix[i][j] <</pre>
                                                                                                 fill(level.begin(), level.end(), -1);
152
                                                                                     31
                          "\n";
                                                                                     32
                                                                                                 q.push(s);
                                                                                     33
153
                                                                                                 level[s] = 0;
                                                                                     34
154
                                                                                     35
155
                                                                                                 while(!q.empty() && level[t] == -1) {
                                                                                     36
156
                                                                                                      int v = q.front();
157
        return 0;
                                                                                     37
                                                                                                     q.pop();
158
                                                                                     38
159
                                                                                     39
                                                                                                      for(int id : adj[v]) {
                                                                                     40
                                 Ford Fulkerson
                                                                                                          if(level[edges[id].to] == -1 && edges[id].cap - edges[id
                                                                                     41
                                                                                                              1.flow > 0) {
                                                                                                              level[edges[id].to] = level[v] + 1;
                                                                                     42
   #define ll long long
                                                                                                              q.push(edges[id].to);
    const 11 INF = (11)4e18;
                                                                                     43
                                                                                                          }
   struct Edge {
                                                                                     44
 3
                                                                                                     }
        int from, to;
                                                                                     45
 4
        ll cap, flow;
                                                                                     46
 5
                                                                                                 return level[t] != -1;
        Edge(int from, int to, 11 cap): from(from), to(to), cap(cap), flow
                                                                                     47
                                                                                             }
            (0) {}
                                                                                     48
   };
                                                                                     49
 7
                                                                                             11 dfs(int v, int t, ll pushed) {
                                                                                     50
                                                                                                 if(v == t || pushed == 0)
   struct MaxFlow {
                                                                                     51
```

```
return pushed;
52
53
           for(; ptr[v] < (int)adj[v].size(); ptr[v]++) {</pre>
54
                int id = adj[v][ptr[v]];
55
                int u = edges[id].to;
56
57
                if(level[u] != level[v] + 1) continue;
58
59
                11 tr = dfs(u, t, min(pushed, edges[id].cap - edges[id].flow
60
                    ));
                if(tr > 0) {
61
                     edges[id].flow += tr;
62
                     edges[id ^ 1].flow -= tr;
                    return tr;
                }
65
66
            return 0;
67
       }
68
69
       11 max_flow(int s, int t) {
70
            11 \text{ flow} = 0;
71
            while(bfs(s, t)) {
72
                fill(ptr.begin(), ptr.end(), 0);
73
                while(ll pushed = dfs(s, t, LLONG_MAX)) {
74
                    flow += pushed;
75
76
            }
77
            return flow;
78
       }
79
80
       vector<ll> get_flows() {
81
            vector<ll> flows;
82
           for(int i = 0; i < edges.size(); i += 2) {</pre>
83
                flows.push_back(edges[i].flow);
84
            }
85
            return flows;
86
       }
87
88 | };
```

## 7 Math

#### 7.1 Euclidean Extended

```
1 | 11 extendedGCD(11 a, 11 b, 11 &x, 11 &y) {
       if (b == 0) {
2
3
           x = 1;
           y = 0;
5
           return a;
       }
6
       ll x1, y1;
       11 gcd = extendedGCD(b, a % b, x1, y1);
       x = v1;
       v = x1 - (a / b) * v1;
       return gcd;
11
12 }
13
   bool findSolutionWithConstraints(ll a, ll b, ll c, ll x_min, ll y_min,
       11 &x, 11 &y) {
       ll g = extendedGCD(a, b, x, y);
       if (c % g != 0) return false;
18
       x *= c / g;
       y *= c / g;
21
       // Ajustamos las variables a/g y b/g para mover las soluciones
22
       a /= g;
23
       b /= g;
24
25
       if (x < x_min) {</pre>
26
           ll k = (x_min - x + b - 1) / b; // Redondeo hacia arriba
27
           x += k * b;
28
           y -= k * a;
       } else if (x > x_min) {
           11 k = (x - x_min) / b;
31
           x -= k * b:
32
           y += k * a;
33
       }
34
35
       if (y < y_min) {
36
           ll k = (y_min - y + a - 1) / a; // Redondeo hacia arriba
           x += k * b;
38
           v -= k * a;
39
       } else if (y > y_min) {
40
           ll k = (y - y_min) / a;
41
           x -= k * b;
42
```

```
y += k * a;
43
       }
44
45
       return x >= x_min && y >= y_min;
46
47 }
                           7.2 Euler Totient
   #include <bits/stdc++.h>
   using namespace std;
   typedef long long 11;
   vector<ll> compute_totients(ll n) {
6
       vector<ll> phi(n + 1);
7
       for (ll i = 0; i <= n; i++) {
8
           phi[i] = i;
9
       }
10
11
       for (ll i = 2; i <= n; i++) {
12
           if (phi[i] == i) { // i es primo
13
               for (ll j = i; j <= n; j += i) {
14
                   phi[j] = phi[j] * (i - 1) / i;
15
               }
16
17
       }
18
19
       return phi;
20
21
                              7.3 Josephus
   #include <iostream>
   using namespace std;
2
   typedef long long 11;
5
   ll josephus_iterative(ll n, ll k) {
6
       ll result = 0:
7
       for (ll i = 2; i <= n; ++i) {
8
```

result = (result + k) % i;

9

10

11 12 } }

return result;

```
13
14
   11 josephus_recursive(ll n, ll k) {
15
16
       if (n == 1)
17
           return 0;
18
19
       return (josephus_recursive(n - 1, k) + k) % n;
20
^{21}
22
23
   11 josephus_power_of_2(11 n) {
25
       11 power = 1;
26
       while (power <= n) {</pre>
27
           power <<= 1;
       }
29
       power >>= 1;
31
32
       return 2 * (n - power);
33
34 }
                                7.4 Mobius
#include <bits/stdc++.h>
   using namespace std;
   typedef long long 11;
   vector<ll> compute_mobius(ll n) {
       vector<ll> mu(n + 1, 1);
7
       vector<bool> is_prime(n + 1, true);
8
9
       for (11 i = 2; i <= n; i++) {
10
           if (is_prime[i]) { // i es un primo
11
                for (ll j = i; j <= n; j += i) {
12
                   mu[j] *= -1; // Multiplicamos por -1 para cada primo
13
                    is_prime[j] = false;
14
15
               for (ll j = i * i; j <= n; j += i * i) {
16
                    mu[j] = 0; // Si tiene un cuadrado de un primo, se pone
17
                        en 0
```

```
}
18
            }
19
        }
20
21
        return mu;
^{22}
23
^{24}
25
   11 mobius(ll x) {
26
        11 count = 0;
27
        for (ll i = 2; i * i <= x; i++) {
28
            if (x \% (i * i) == 0)
29
                return 0:
30
            if (x \% i == 0) {
31
                 count++;
32
                 x /= i;
33
            }
34
        }
35
36
        if (x > 1) count++;
37
38
       return (count % 2 == 0) ? 1 : -1;
39
40 }
```

#### 7.5 NTT

```
#include <bits/stdc++.h>
   using namespace std;
   using cd = complex<double>;
   typedef long long 11;
   const 11 mod = 998244353;
   const ll root = 31;
   const ll root_1 = inverse(root, mod);
   const 11 root_pw = 1 << 23;</pre>
9
   ll inverse(ll a, ll m) {
10
       11 \text{ res} = 1, \exp = m - 2;
11
       while (exp) {
12
           if (exp % 2 == 1) res = (1LL * res * a) % m;
13
           a = (1LL * a * a) % m;
14
           exp /= 2;
15
       }
16
       return res;
17
```

```
18 }
19
   void ntt(vector<ll> & a, bool invert) {
20
       int n = a.size();
21
22
       for (int i = 1, j = 0; i < n; i++) {
23
           int bit = n \gg 1;
24
           for (; j & bit; bit >>= 1)
                j ^= bit;
           i ^= bit;
28
           if (i < j)
29
                swap(a[i], a[j]);
30
       }
31
32
       for (int len = 2; len <= n; len <<= 1) {
33
           int wlen = invert ? root_1 : root;
34
           for (int i = len; i < root_pw; i <<= 1)</pre>
                wlen = (int)(1LL * wlen * wlen % mod);
36
37
           for (int i = 0; i < n; i += len) {
38
                int w = 1;
39
                for (int j = 0; j < len / 2; j++) {
40
                    int u = a[i+j], v = (int)(1LL * a[i+j+len/2] * w % mod);
41
                    a[i+j] = u + v < mod ? u + v : u + v - mod;
42
                    a[i+j+len/2] = u - v >= 0 ? u - v : u - v + mod;
43
                    w = (int)(1LL * w * wlen % mod);
44
45
           }
46
       }
47
48
       if (invert) {
           int n_1 = inverse(n, mod);
           for (auto & x : a)
51
                x = (int)(1LL * x * n_1 \% mod);
52
       }
53
   }
54
   vector<ll> multiply(vector<ll> const &a, vector<ll> const &b) {
       vector<ll> fa(a.begin(), a.end()), fb(b.begin(), b.end());
57
       11 n = 1;
58
       while (n < a.size() + b.size())</pre>
59
           n <<= 1;
60
```

25

```
fa.resize(n):
                                                                                             }
61
                                                                                  26
       fb.resize(n);
                                                                                     }
                                                                                  27
62
63
                                                                                  28
       ntt(fa, false);
                                                                                     vll multiply(const vll& a, const vll& b) {
64
       ntt(fb, false);
                                                                                         if (a.empty() || b.empty()) return {};
65
       for (ll i = 0; i < n; i++)
                                                                                         vd fa(a.begin(), a.end()), fb(b.begin(), b.end());
66
                                                                                         int L = 32 - \_builtin\_clz(fa.size() + fb.size() - 1), n = 1 << L;
           fa[i] = (fa[i] * fb[i]) % mod;
67
                                                                                         vector<C> in(n), out(n);
       ntt(fa, true);
68
69
                                                                                  34
       vector<ll> result(n);
                                                                                         for (int i = 0; i < a.size(); i++) in[i] = C(fa[i], 0);
70
       for (ll i = 0; i < n; i++)
                                                                                         for (int i = 0; i < b.size(); i++) in[i].imag(fb[i]);</pre>
71
           result[i] = fa[i];
72
                                                                                  37
       return result:
                                                                                         fft(in):
73
                                                                                  38
74 }
                                                                                         for (C\& x : in) x *= x:
                                                                                         for (int i = 0; i < n; i++) out[i] = in[-i & (n - 1)] - conj(in[i]);
                                 7.6 FFT
                                                                                               // Corregido aqui
                                                                                         fft(out);
   typedef long long 11;
                                                                                         vll res(a.size() + b.size() - 1):
   typedef complex<double> C;
                                                                                         for (int i = 0; i < res.size(); i++) {</pre>
   typedef vector<double> vd;
                                                                                             res[i] = llround(imag(out[i]) / (4 * n));
   typedef vector<ll> vll;
                                                                                  45
                                                                                         }
   const double PI = acos(-1);
                                                                                  46
                                                                                  47
                                                                                         return res;
                                                                                  48 }
   void fft(vector<C>& a) {
       int n = a.size(), L = 31 - __builtin_clz(n);
8
                                                                                                                   7.7 Rho
       static vector<C> R(2, 1);
9
       static vector<C> rt(2, 1);
10
       for (static int k = 2; k < n; k *= 2) {
                                                                                   1 //RECOMENDADO USAR UNSIGNED LONG LONG
11
           R.resize(n); rt.resize(n);
                                                                                     11 mulmod(l1 a, l1 b, l1 m) { return l1(__int128(a) * b % m); }
12
           auto x = polar(1.0, PI / k);
13
           for (int i = k; i < 2 * k; i++)
                                                                                     ll expmod(ll b, ll e, ll m) {
14
               rt[i] = R[i] = i & 1 ? R[i / 2] * x : R[i / 2];
                                                                                      ll ret = 1;
15
       }
                                                                                       while (e) {
                                                                                  6
16
       vector<int> rev(n);
                                                                                         if (e%2) ret = mulmod(ret, b, m);
17
       for (int i = 0; i < n; i++) rev[i] = (rev[i / 2] | (i & 1) << L) /
                                                                                         b = mulmod(b, b, m);
18
           2;
                                                                                         e /= 2;
                                                                                  9
                                                                                      }
       for (int i = 0; i < n; i++) if (i < rev[i]) swap(a[i], a[rev[i]]);
                                                                                  10
19
       for (int k = 1: k < n: k *= 2)
                                                                                  11
                                                                                       return ret;
20
           for (int i = 0; i < n; i += 2 * k) for (int j = 0; j < k; j++) {
                                                                                  12 }
21
               auto x = (double*) &rt[j + k], y = (double*) &a[i + j + k];
                                                                                  13 | bool miller(ll n) {
22
               C z(x[0] * y[0] - x[1] * y[1], x[0] * y[1] + x[1] * y[0]);
                                                                                         if (n < 2) return false;
                                                                                  14
23
               a[i + j + k] = a[i + j] - z;
                                                                                  15
                                                                                         for (ll p : {2, 3, 5, 7, 11, 13, 17, 19}) {
24
               a[i + j] += z;
                                                                                 16
                                                                                             if (n \% p == 0)
```

```
return (n == p);
17
       }
18
       if (n < 529) return true; // 23^2 = 529
19
       int s = 0;
20
       11 d = n - 1;
21
       while ((d \& 1) == 0) {
22
           d >>= 1;
23
           s++;
24
       }
25
       auto witness = [&](ll a) {
26
           11 x = expmod(a \% n, d, n);
27
           if (x == 1 \mid | x == n - 1) return false;
28
           for (int i = 1; i < s; i++) {
29
                x = mulmod(x, x, n);
30
                if (x == n - 1) return false;
31
32
           return true; // "true" => 'a' es testigo => n es compuesto
33
       };
34
       // Bases para 64 bits
35
       for (11 b: {2, 325, 9375, 28178, 450775, 9780504, 1795265022}) {
36
           if (b % n == 0) return true;
37
           if (witness(b)) return false;
38
39
       return true; // "probable primo"
40
41
   11 rho(11 n) {
     if(n % 2 == 0) return 2;
43
     11 x = 2, y = 2, d = 1;
44
     ll c = rand() % n + 1;
45
     while(d == 1) {
46
       x = (mulmod(x, x, n) + c) \% n;
47
       y = (mulmod(y, y, n) + c) \% n;
48
       y = (mulmod(y, y, n) + c) \% n;
49
       d = \_gcd(x - y, n);
50
51
     return d == n ? rho(n) : d;
52
53
   void fact(ll n, map<ll, int>& F) { // Agrega los factores de n en F
54
     if (n == 1) return;
55
     if (miller(n)) {F[n]++; return;}
     ll q = rho(n); fact(q, F); fact(n / q, F);
57
58 }
```

#### 7.8 Simpson

```
1 | ld simpsonRule(function<ld(ld)> f, ld a, ld b, int n) {
       // Asegurarse de que n sea par
       if (n % 2 != 0) {
           n++;
5
       1d h = (b - a) / n;
6
       1d s = f(a) + f(b);
8
       // Suma de terminos interiores con los factores apropiados
       for (int i = 1; i < n; i++) {
10
           1d x = a + i * h;
11
           s += (i \% 2 == 1 ? 4.0L : 2.0L) * f(x);
12
13
       // Multiplica por h/3
14
       return (h / 3.0L) * s;
15
16
17 // Ejemplo: integrar la funcion x^2 entre 0 y 3
auto f = [\&](ld x)\{ return x * x; \};
_{19} | 1d a = 0.0L, b = 3.0L;
20 | int n = 1000; // numero de subintervalos
ld resultado = simpsonRule(f, a, b, n);
```

## 8 Geometry

#### 8.1 Convex Hull

```
#include <iostream>
#include <vector>
#include <algorithm>
using namespace std;

typedef long long ll;
typedef pair<ll, ll> Point;

ll cross_product(Point 0, Point A, Point B) {
    return (A.first - 0.first) * (B.second - 0.second) - (A.second - 0.second) * (B.first - 0.first);
}

vector<Point> convex_hull(vector<Point>& points) {
    sort(points.begin(), points.end());
```

```
points.erase(unique(points.begin(), points.end());
15
       vector<Point> hull;
16
17
       // Parte inferior
18
       for (const auto& p : points) {
19
           while (hull.size() >= 2 && cross_product(hull[hull.size() - 2],
20
               hull[hull.size() - 1], p) < 0)
               hull.pop_back();
21
           if (hull.empty() || hull.back() != p) {
^{22}
               hull.push_back(p);
23
           }
24
       }
25
26
       // Parte superior
27
       int t = hull.size() + 1;
28
       for (int i = points.size() - 1; i \ge 0; --i) {
29
           while (hull.size() >= t && cross_product(hull[hull.size() - 2],
30
               hull[hull.size() - 1], points[i]) < 0)</pre>
               hull.pop_back();
31
           if (hull.empty() || hull.back() != points[i]) {
32
               hull.push_back(points[i]);
33
           }
34
       }
35
36
       hull.pop_back();
37
       return hull;
38
39 }
                             8.2 Operations
```

```
#include <bits/stdc++.h>
   using namespace std;
2
   typedef long long 11;
4
6
   11 cross_product(pair<11, 11> P1, pair<11, 11> P2, pair<11, 11> P3) {
       ll x1 = P2.first - P1.first:
8
       11 y1 = P2.second - P1.second;
9
       11 x2 = P3.first - P1.first;
10
       11 y2 = P3.second - P1.second;
11
       return x1 * y2 - y1 * x2;
12
13 }
```

```
14
15
   double distancia(pair<11, 11> P1, pair<11, 11> P2) {
16
        return sqrt((P2.first - P1.first) * (P2.first - P1.first) +
                     (P2.second - P1.second) * (P2.second - P1.second));
18
19
20
21
   11 dot_product(pair<11, 11> P1, pair<11, 11> P2, pair<11, 11> P3) {
22
       11 x1 = P2.first - P1.first;
23
       11 y1 = P2.second - P1.second;
24
       11 \times 2 = P3.first - P1.first;
       11 \text{ y2} = P3.\text{second} - P1.\text{second};
       return x1 * x2 + y1 * y2;
27
28 }
```

## 8.3 Polygon Area

```
1 #include <iostream>
  #include <vector>
   #include <cmath>
   using namespace std;
   typedef long long 11;
   typedef pair<11, 11> Point;
   double polygon_area(const vector<Point>& polygon) {
       11 \text{ area} = 0;
       int n = polygon.size();
       for (int i = 0; i < n; ++i) {
           11 j = (i + 1) \% n;
           area += (polygon[i].first * polygon[j].second - polygon[i].
15
               second * polygon[j].first);
16
       return abs(area) / 2.0;
17
18 }
```

#### 8.4 Ray Casting

```
#include <iostream>
#include <vector>
using namespace std;
```

```
5 typedef long long 11;
   typedef pair<11, 11> Point;
   bool is_point_in_polygon(const vector<Point>& polygon, Point p) {
       bool inside = false;
10
       int n = polygon.size();
11
       for (int i = 0, j = n - 1; i < n; j = i++) {
^{12}
           if ((polygon[i].second > p.second) != (polygon[j].second > p.
13
                second) &&
                p.first < (polygon[j].first - polygon[i].first) * (p.second</pre>
14
                    - polygon[i].second) /
                          (polygon[j].second - polygon[i].second) + polygon[
15
                              il.first) {
                inside = !inside;
16
           }
17
       }
18
       return inside;
19
20 }
```

#### 9 Trees

#### 9.1 Centroid

```
#include <bits/stdc++.h>
  using namespace std;
   #define L(i, j, n) for (int i = (j); i < int(n); i ++)
   #define ii pair<int, int>
   const int inf = 1e9;
   const int N = 1e5;
   vector<int> G[N];
  int ct[N];
   set<ii> dist[N];
  int up[N][18];
12
   int colors[N];
  int depth[N];
  int sz[N];
  bool removed[N]:
   int n, root, L;
18
int getSize(int u, int p){
```

```
int szi = 1:
20
       for(int v: G[u]){
21
           if (p == v || removed[v]) continue;
22
           szi += getSize(v, u);
23
24
       return sz[u] = szi;
25
   }
26
27
   int centroid(int u, int tree_size, int p){
       for (int v: G[u]){
29
           if (v == p || removed[v]) continue;
30
           if (sz[v] * 2 > tree_size) return centroid(v, tree_size, u);
31
       }
32
       return u;
33
   }
34
35
   void build(int node, int tree_size, int p)
   {
37
       getSize(node, - 1);
38
       int cen = centroid(node, tree_size, -1);
       removed[cen] = 1;
40
       ct[cen] = p;
       if (p == -1) root = cen;
42
43
       if (tree_size == 1) return;
44
45
       for (int v: G[cen]){
46
           if (removed[v]) continue;
47
           build(v, sz[v], cen);
48
       }
49
50
51
   void update(int v){
       int u = v;
54
       while(v != -1){
55
           dist[v].insert(distance(u, v), v);
           v = par[v];
57
58
       return res;
59
60
61
62 | int query(int v){
```

```
int u = v:
                                                                                           return u;
63
                                                                                   32
                                                                                      }
       int res = INT_MAX;
                                                                                   33
64
       while(v != -1){
65
                                                                                   34
           res = min(res, distance(u, v), dist[v].begin()->first); //
                                                                                       void build(int node, int tree_size, int p)
66
               Minimun
                                                                                   36
           v = par[v];
                                                                                           getSize(node, - 1);
67
                                                                                   37
       }
                                                                                           int cen = centroid(node, tree_size, -1);
68
                                                                                           removed[cen] = 1;
       return res;
69
70 }
                                                                                           if (p != -1){
                                                                                               ct[cen].push_back(p);
                                 9.2 LCA
                                                                                           } else root = cen;
                                                                                   42
                                                                                   43
                                                                                           if (tree size == 1) return:
   #include <bits/stdc++.h>
                                                                                   44
   using namespace std;
                                                                                           for (int v: G[cen]){
                                                                                               if (removed[v]) continue;
   #define L(i, j, n) for (int i = (j); i < int(n); i ++)
                                                                                               build(v, sz[v], cen);
   #define ii pair<int, int>
                                                                                   48
                                                                                           }
   const int inf = 1e9;
   const int N = 1e5;
                                                                                   50
                                                                                   51
   vector<int> G[N], ct[N];
                                                                                   52
                                                                                       void dfs(int u, int p){
   set<ii> dist[N];
                                                                                           up[u][0] = p;
   int up[N][18];
                                                                                   54
                                                                                           for (int i = 1; i \le L; i ++){
   int colors[N]:
                                                                                               if (up[u][i-1] != -1) up[u][i] = up[up[u][i-1]][i-1];
   int depth[N];
                                                                                   56
                                                                                               else up[u][i] = -1;
   int sz[N];
                                                                                   57
                                                                                           }
   bool removed[N];
                                                                                   58
                                                                                           for (int v: G[u]){
   int n, root, L;
                                                                                   59
16
                                                                                               if (v == p) continue;
                                                                                   60
17
                                                                                               depth[v] = depth[u] + 1;
   int getSize(int u, int p){
                                                                                   61
18
                                                                                               dfs(v, u);
       int szi = 1;
                                                                                   62
19
                                                                                           }
                                                                                   63
       for(int v: G[u]){
20
                                                                                      }
           if (p == v || removed[v]) continue;
                                                                                   64
^{21}
           szi += getSize(v, u);
^{22}
                                                                                       int LCA(int u, int v){
       }
23
                                                                                           if (depth[u] < depth[v]) swap(u, v);</pre>
       return sz[u] = szi;
                                                                                   67
^{24}
                                                                                          for (int i = L; i \ge 0; i --){
                                                                                   68
25
                                                                                               if (up[u][i] != -1 && depth[up[u][i]] >= depth[v]){
                                                                                   69
26
                                                                                                   u = up[u][i];
   int centroid(int u, int tree_size, int p){
                                                                                   70
27
                                                                                               }
       for (int v: G[u]){
                                                                                   71
28
                                                                                           }
           if (v == p || removed[v]) continue;
                                                                                   72
29
                                                                                           if (u == v) return u;
                                                                                   73
           if (sz[v] * 2 > tree_size) return centroid(v, tree_size, u);
30
                                                                                          for (int i = L; i >= 0; i --){
                                                                                   74
       }
31
```

```
if (up[u][i] != up[v][i] && up[u][i] != -1 && up[v][i] != -1){
75
                u = up[u][i];
76
                v = up[v][i];
77
            }
78
        }
79
        return up[u][0];
80
81
82
    int dis(int u, int v){
83
        int cmm = LCA(u, v);
84
       // cout << u << " " << v << " " << cmm << "\n";
85
       return depth[u] + depth[v] - (2 * depth[cmm]);
86
87
88
   void uup(int u, int node){
89
       dist[u].insert({dis(u, node), node});
90
        for (int v: ct[u])
91
            uup(v, node);
92
93
94
    void update(int node){
95
       dist[node].insert({0, node});
96
        for (int v: ct[node])
97
            uup(v, node);
98
99
100
    int qup(int u, int node){
101
        int mn = dis(node, u) + dist[u].begin()->first;
102
       for (int v: ct[u]) mn = min(mn, qup(v, node));
103
        return mn;
104
105
106
   int query(int node){
107
        int mn = dist[node].begin()->first;
108
       for (int v: ct[node]) mn = min(mn, qup(v, node));
109
        return mn;
110
111
112
   int main()
113
114
       ios::sync_with_stdio(0);cin.tie(0);
115
        int m; cin >> n >> m;
116
        L = log2(n);
117
```

```
L(i, 1, n){
118
             int u, v; cin >> u >> v;
119
             u --; v --;
120
             G[u].push_back(v);
121
            G[v].push_back(u);
122
        }
123
        L(i, 0, n){
124
             dist[i].insert({inf, i});
125
        }
126
        build(0, n, -1);
127
        L(i, 0, L + 1) \text{ up[root]}[i] = -1;
128
        run(root, -1);
129
        update(0);
130
        L(_q, 0, m){
131
            int op, node; cin >> op >> node;
132
             if (op == 2){
                 cout << query(node-1) << '\n';</pre>
134
            } else {
                 update(node-1); // Log Log
136
            }
137
        }
138
139 }
```