

Dividimos y No Conquistamos (D&!C)

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1 Template

1.1 C++ Template

```

1 #include <bits/stdc++.h>
2 using namespace std;
3
4 #define L(i, j, n) for (int i = (j); i < (int)n; i++)
5 #define SZ(x) int((x).size())
6 #define ALL(x) begin(x), end(x)
7 #define vec vector
8 #define pb push_back
9 #define eb emplace_back
10
11 using ll = long long;
12 using ld = long double;
13
14 void solve()
15 {
16 }
17
18
19 int main()
20 {
21     ios::sync_with_stdio(0); cin.tie(0);
22     int TT = 1;
23     //cin >> TT;
24     while (TT--)
25     {
26         solve();
27     }
28 }

```

1.2 Fast Python

```

1 import os, sys, io
2 finput = io.BytesIO(os.read(0, os.fstat(0).st_size)).readline
3 fprint = sys.stdout.write

```

1.3 Policy Based

```

1 #include <ext/pb_ds/assoc_container.hpp>
2 using namespace __gnu_pbds;
3 template<typename Key, typename Val=null_type>

```

```

4 using indexed_set = tree<Key, Val, less<Key>, rb_tree_tag,
5                     tree_order_statistics_node_update>;
6 // indexed_set<char> s;
7 // char val = *s.find_by_order(0); // acceso por indice
8 // int idx = s.order_of_key('a'); // busca indice del valor
9 template<class Key, class Val=null_type> using htable=gp_hash_table<Key,
10                          Val>;
11 // como unordered_map (o unordered_set si Val es vacio), pero sin metodo
12 // count

```

2 Search

2.1 Ternary

```

1 // Minimo de 'f' en '(l,r)'.
2 template<class Fun> ll ternary(Fun f, ll l, ll r) {
3     for (ll d = r-l; d > 2; d = r-l) {
4         ll a = l + d/3, b = r - d/3;
5         if (f(a) > f(b)) l = a; else r = b;
6     }
7     return l + 1;
8 }
9 // para error < EPS, usar iters=log((r-l)/EPS)/log(1.618)
10 template<class Fun> double golden(Fun f, double l, double r, int iters){
11     double const ratio = (3-sqrt(5))/2;
12     double x1=l+(r-l)*ratio, x2=r-(r-l)*ratio, f1=f(x1), f2=f(x2);
13     while (iters--){
14         if (f1 > f2) l=x1, x1=x2, f1=f2, x2=r-(r-l)*ratio, f2=f(x2);
15         else r=x2, x2=x1, f2=f1, x1=l+(r-l)*ratio, f1=f(x1);
16     }
17     return (l+r)/2;
18 }

```

2.2 Simulated Annealing

```

1 using my_clock = chrono::steady_clock;
2 struct Random {
3     mt19937_64 engine;
4     Random(): engine(my_clock::now().time_since_epoch().count()) {}
5     template<class Int> Int integer(Int n) {return integer<Int>(0, n);} //
6     '[0,n)'
7     template<class Int> Int integer(Int l, Int r)
8     {return uniform_int_distribution{l, r-1}(engine);} // '[l,r)'

```

```

8  double real() {return uniform_real_distribution{}(engine);} // '[0,1)'
9  } rng;
10 struct Timer {
11     using time = my_clock::time_point;
12     time start = my_clock::now();
13     double elapsed() { // Segundos desde el inicio.
14         time now = my_clock::now();
15         return chrono::duration<double>(now - start).count();
16     }
17 } timer;
18 template<class See, class Upd> struct Annealing {
19     using energy = invoke_result_t<See>;
20     energy curr, low;
21     See see;
22     Upd upd;
23     Annealing(See _see, Upd _upd): see{ _see }, upd{ _upd }
24     { curr = low = see(), upd(); }
25     void simulate(double s, double mult=1) { // Simula por 's' segundos.
26         double t0 = timer.elapsed();
27         for (double t = t0; t-t0 < s; t = timer.elapsed()) {
28             energy near = see();
29             auto delta = double(curr - near);
30             if (delta >= 0) upd(), curr = near, low = min(low, curr);
31             else {
32                 double temp = mult * (1 - (t-t0)/s);
33                 if (exp(delta/temp) > rng.real()) upd(), curr = near;
34             }
35         }
36     }
37 };
38 auto see = [&] -> double {
39     l = rng.integer(gsz); r = rng.integer(gsz);
40     swap(groups[l], groups[r]);
41     int ans = 0, rem = 0;
42     L(i, 0, gsz) {
43         if (groups[i] > rem) {
44             rem = x;
45             ans ++;
46         }
47         rem -= groups[i];
48     }
49     swap(groups[l], groups[r]);
50     return ans;

```

```

51 };
52 auto upd = [&] {swap(groups[l], groups[r]);};

```

3 Data structures

3.1 BIT

```

1  #define LSO(S) (S & -S)
2  struct BIT {
3      vec<int> B;
4      int n;
5      BIT(int n = 1): B(n + 1), n(n+1){}
6      BIT(vec<int> &v): B(SZ(v)+1), n(SZ(v)+1) {
7          LI(i, 1, n){
8              B[i] += v[i-1];
9              if (i + LSO(i) <= n){
10                 B[i + LSO(i)] += B[i];
11             }
12         }
13     }
14     void update(int i, int x){
15         while (i <= n){
16             B[i] += x;
17             i += LSO(i);
18         }
19     }
20     int sum(int i){
21         int res = 0;
22         while (i > 0){
23             res += B[i];
24             i -= LSO(i);
25         }
26         return res;
27     }
28     int range_sum(int l, int r){
29         return sum(r) - sum(l - 1);
30     }
31 };

```

3.2 BIT - 2D

```

1  #define LSO(S) (S & -S)
2  int f[N][N]; // Initial Matrix

```

```

3 struct BIT {
4     vec<vec<int>> B;
5     int n;
6     BIT(int n_ = 1): B(n_+1,vec<int>(n_+1)), sz(n_) { // N * N * log(N)
7         * log(N);
8         for (int i = 1; i <= n_; i++)
9             for (int j = 1; j <= n_; j++)
10                add(i, j, f[i-1][j-1]);
11    }
12    void add(int i, int j, int delta){ // log(N) * log(N)
13        for (int x = i; x <= n; x += LSO(x))
14            for (int y = j; y <= n; y += LSO(y))
15                B[x][y] += delta;
16    }
17    int sum(int i, int j){ // log(N) * log(N)
18        int tot = 0;
19        for (int x = i; x > 0; x -= LSO(x))
20            for (int y = j; y > 0; y -= LSO(y))
21                tot += B[x][y];
22        return tot;
23    }
24 };

```

3.3 DSU

```

1 struct DSU {
2     vec<int> par, sz; int n;
3     DSU(int n = 1): par(n), sz(n, 1), n(n) { iota(ALL(par), 0); }
4     int find(int a){return a == par[a] ? a : par[a] = find(par[a]);}
5     void join(int a, int b){
6         a=find(a);b=find(b);
7         if (a == b) return;
8         if (sz[b] > sz[a]) swap(a,b);
9         par[b] = a;
10        sz[a] += sz[b];
11    }
12 };

```

3.4 Index Compression

```

1 template<class T>
2 struct Index{ // If only 1 use Don't need to copy T type
3     vec<T> d;
4     int sz;

```

```

5     Index(vec<T> &a): d(ALL(a)){
6         sort(ALL(d)); // Sort
7         d.erase(unique(ALL(d)), end(d)); // Erase continuous duplicates
8         sz = SZ(d); }
9     int of(T e){return lower_bound(ALL(d), e) - begin(d);} // get index
10    T at(int i){return d[i];} // get value of index
11 };

```

3.5 Sparse Table

```

1 struct SPT {
2     vec<vec<int>> st;
3     int K;
4     SPT(vec<int> &a): K(0) {
5         int n = SZ(a);
6         while((1<<K)<=n) K++;
7         st = vec<vec<int>>(K, vec<int>(n));
8         L(i,0,n) st[0][i] = a[i];
9         for (int i = 1; (1 << i) <= n; i++) {
10             int jmp = (1 << (i - 1));
11             for (int j = 0; j + (1 << i) <= n; j++) {
12                 st[i][j] = min(st[i-1][j], st[i - 1][j + jmp]);
13             }
14         }
15     }
16     int get(int l, int r) {
17         int bit = log2(r - l + 1);
18         return min(st[bit][l], st[bit][r - (1<<bit) + 1]);
19     }
20 };

```

3.6 Segment tree

```

1 #define LC(v) (v<<1)
2 #define RC(v) ((v<<1)|1)
3 #define MD(L, R) (L+((R-L)>>1))
4 struct node {
5     ll mx;
6     ll cant; };
7 struct ST {
8     vec<node> st;
9     vec<ll> lz; int n;
10    ST(int n = 1): st(4 * n + 10, {oo, oo}), lz(4 * n + 10, 0), n(n) {
11        build(1, 0, n - 1);}

```

```

11 node merge(node a, node b){
12     if (a.mx == oo) return b;
13     if (b.mx == oo) return a;
14     if (a.mx == b.mx) return {a.mx, a.cant + b.cant};
15     return {max(a.mx, b.mx), a.mx > b.mx ? a.cant : b.cant};
16 }
17 void build(int v, int L, int R){
18     if (L == R){
19         st[v] = {0, 1};
20     } else {
21         int m = MD(L, R);
22         build(LC(v), L, m);
23         build(RC(v), m + 1, R);
24         st[v] = merge(st[LC(v)], st[RC(v)]);
25     }
26 }
27 void push(int v, int L, int R){
28     if (lz[v]){
29         if (L != R){
30             st[LC(v)].mx += lz[v];
31             st[RC(v)].mx += lz[v];
32             lz[LC(v)] += lz[v];
33             lz[RC(v)] += lz[v];
34         }
35         lz[v] = 0;
36     }
37 }
38 void update(int v, int L, int R, int ql, int qr, ll w){
39     if (ql > R || qr < L) return;
40     push(v, L, R);
41     if (ql == L && qr == R){
42         st[v].mx += w;
43         lz[v] += w;
44         push(v, L, R);
45         return;
46     }
47     int m = MD(L, R);
48     update(LC(v), L, m, ql, min(qr, m), w);
49     update(RC(v), m + 1, R, max(m + 1, ql), qr, w);
50     st[v] = merge(st[LC(v)], st[RC(v)]);
51 }
52 node query(int v, int L, int R, int ql, int qr){
53     if (ql > R || qr < L) return {oo, oo};

```

```

54     push(v, L, R);
55     if (ql == L && qr == R) return st[v];
56     int m = MD(L, R);
57     return merge(query(LC(v), L, m, ql, min(m, qr)), query(RC(v), m
        + 1, R, max(m + 1, ql), qr));
58 }
59 node query(int l, int r){return query(1, 0, n - 1, l, r);}
60 void update(int l, int r, ll w){update(1, 0, n - 1, l, r, w);}
61 };

```

3.7 Segment Tree Iterativo

```

1 struct STI {
2     vec<ll> st;
3     int n, K;
4     STI(vec<ll> &a): n(SZ(a)), K(1) {
5         while(K<=n) K<<=1;
6         st.assign(2*K, 0); // 0 default
7         L(1,0,n) st[K+1] = a[1];
8         for (int i = K - 1; i > 0; i --) st[i] = st[i*2] + st[i*2+1];
9     }
10    void upd(int pos, ll w) {
11        pos += K;
12        st[pos] += w;
13        while((pos>=1) > 0) st[pos] = st[pos * 2] + st[pos * 2 + 1];
14    }
15    ll query(int l, int r) {
16        ll res = 0;
17        l += K; r += K;
18        while (l < r) {
19            if (l & 1) res += st[l++];
20            if (r & 1) res += st[--r];
21            l>>=1;r>>=1;
22        }
23        return res;
24    }
25 };

```

4 Graph

4.1 Bellman Ford

```

1 struct Edge {int a, b, cost;};

```

```

2 vector<Edge> edges;
3 int solve(int s) // Source
4 {
5     vector<int> d(n, INF);
6     d[s] = 0;
7     for (int i = 0; i < n - 1; ++i)
8         for (Edge e : edges)
9             if (d[e.a] < INF)
10                 d[e.b] = min(d[e.b], d[e.a] + e.cost);
11 }

```

4.2 SCC

```

1 vector<int> dfs_num(N, -1), dfs_low(N, -1), visited(N);
2 int dfs_count = 0;
3 int numSCC = 0;
4 stack<int> st;
5 void dfs(int u){
6     dfs_low[u]=dfs_num[u]=dfs_count++;
7     st.push(u);
8     visited[u] = 1;
9     for(int v: G[u]) {
10         if (dfs_num[v] == -1) dfs(v);
11         if (visited[v]) dfs_low[u] = min(dfs_low[u], dfs_low[v]);
12     }
13     if (dfs_num[u] == dfs_low[u]){
14         numSCC++;
15         while(1){
16             int v = st.top(); st.pop();
17             visited[v] = 0;
18             if (u == v) break;
19         }
20     }
21 }

```

4.3 Bipartite Matching Hopcroft-Karp - With Konig

```

1 mt19937 rng((int) chrono::steady_clock::now().time_since_epoch().count()
2 );
3 struct hopcroft_karp {
4     int n, m; // n is Left Partition Size, m is Right Partition Size
5     vector<vector<int>> g;
6     vector<int> dist, nxt, ma, mb;
7     hopcroft_karp(int n_, int m_) : n(n_), m(m_), g(n),

```

```

7     dist(n), nxt(n), ma(n, -1), mb(m, -1) {}
8     void add(int a, int b) { g[a].pb(b); }
9     bool dfs(int i) {
10         for (int &id = nxt[i]; id < g[i].size(); id++) {
11             int j = g[i][id];
12             if (mb[j] == -1 or (dist[mb[j]] == dist[i]+1 and dfs(mb[j]))) {
13                 ma[i] = j, mb[j] = i;
14                 return true;
15             }
16         }
17         return false;
18     }
19     bool bfs() {
20         for (int i = 0; i < n; i++) dist[i] = n;
21         queue<int> q;
22         for (int i = 0; i < n; i++) if (ma[i] == -1) {
23             dist[i] = 0;
24             q.push(i);
25         }
26         bool rep = 0;
27         while (q.size()) {
28             int i = q.front(); q.pop();
29             for (int j : g[i]) {
30                 if (mb[j] == -1) rep = 1;
31                 else if (dist[mb[j]] > dist[i] + 1) {
32                     dist[mb[j]] = dist[i] + 1;
33                     q.push(mb[j]);
34                 }
35             }
36         }
37         return rep;
38     }
39     int matching() {
40         int ret = 0;
41         for (auto& i : g) shuffle(ALL(i), rng);
42         while (bfs()) {
43             for (int i = 0; i < n; i++) nxt[i] = 0;
44             for (int i = 0; i < n; i++)
45                 if (ma[i] == -1 and dfs(i)) ret++;
46         }
47         return ret;
48     }
49     vec<int> cover[2]; // if cover[i][j] = 1 -> node i, j is part of cover

```

```

50 int konig() {
51     cover[0].assign(n,true); // n left size
52     cover[1].assign(m,false); // m right size
53     auto go = [&](auto&& me, int u) -> void {
54         cover[0][u] = false;
55         for (auto v : g[u]) if (!cover[1][v]) {
56             cover[1][v] = true;
57             me(me,mb[v]);
58         }
59     };
60     L(u,0,n) if (ma[u] < 0) go(go,u);
61     return size;
62 }
63 };

```

4.4 Hungarian

```

1 using vi = vec<int>;
2 using vd = vec<ld>;
3 const ld INF = 1e100; // Para max asignacion, INF = 0, y negar costos
4 bool zero(ld x) {return fabs(x) < 1e-9;} // Para int/ll: return x==0;
5 vec<pii> ans; // Guarda las aristas usadas en el matching: [0..n)x[0..m)
6 struct Hungarian{
7     int n; vec<vd> cs; vi vL, vR;
8     Hungarian(int N, int M) : n(max(N,M)), cs(n,vd(n)), vL(n), vR(n){
9         L(x, 0, N) L(y, 0, M) cs[x][y] = INF;
10    }
11    void set(int x, int y, ld c) { cs[x][y] = c; }
12    ld assign(){
13        int mat = 0; vd ds(n), u(n), v(n); vi dad(n), sn(n);
14        L(i, 0, n) u[i] = *min_element(ALL(cs[i]));
15        L(j, 0, n){
16            v[j] = cs[0][j]-u[0];
17            L(i, 1, n) v[j] = min(v[j], cs[i][j] - u[i]);
18        }
19        vL = vR = vi(n, -1);
20        L(i,0, n) L(j, 0, n) if(vR[j] == -1 and zero(cs[i][j] - u[i] - v[j])){
21            vL[i] = j; vR[j] = i; mat++; break;
22        }
23        for(; mat < n; mat++){
24            int s = 0, j = 0, i;
25            while(vL[s] != -1) s++;

```

```

26        fill(ALL(dad), -1); fill(ALL(sn), 0);
27        L(k, 0, n) ds[k] = cs[s][k]-u[s]-v[k];
28        while(true){
29            j = -1;
30            L(k, 0, n) if(!sn[k] and (j == -1 or ds[k] < ds[j])) j = k;
31            sn[j] = 1; i = vR[j];
32            if(i == -1) break;
33            L(k, 0, n) if(!sn[k]){
34                auto new_ds = ds[j] + cs[i][k] - u[i]-v[k];
35                if(ds[k] > new_ds) ds[k]=new_ds, dad[k]=j;
36            }
37        }
38        L(k, 0, n) if(k!=j and sn[k]){
39            auto w = ds[k]-ds[j]; v[k] += w, u[vR[k]] -= w;
40        }
41        u[s] += ds[j];
42        while(dad[j] >= 0){ int d = dad[j]; vR[j] = vR[d]; vL[vR[j]] = j;
43            j = d; }
44        vR[j] = s; vL[s] = j;
45        ld value = 0; L(i, 0, n) value += cs[i][vL[i]], ans.pb({i, vL[i]});
46        return value;
47    }
48 };

```

4.5 Flow - Dinics

```

1 struct Dinic {
2     bool scaling = false; // com scaling -> O(nm log(MAXCAP)),
3     int lim; // com constante alta
4     struct edge {
5         int to, cap, rev, flow;
6         bool res;
7         edge(int to_, int cap_, int rev_, bool res_)
8             : to(to_), cap(cap_), rev(rev_), flow(0), res(res_) {}
9     };
10    vec<vec<edge>> g;
11    vec<int> lev, beg;
12    ll F;
13    Dinic(int n) : g(n), F(0) {}
14    void add(int a, int b, int c) {
15        g[a].emplace_back(b, c, g[b].size(), false);
16        g[b].emplace_back(a, 0, g[a].size()-1, true);

```

```

17 }
18 bool bfs(int s, int t) {
19     lev = vector<int>(g.size(), -1); lev[s] = 0;
20     beg = vector<int>(g.size(), 0);
21     queue<int> q; q.push(s);
22     while (q.size()) {
23         int u = q.front(); q.pop();
24         for (auto& i : g[u]) {
25             if (lev[i.to] != -1 or (i.flow == i.cap)) continue;
26             if (scaling and i.cap - i.flow < lim) continue;
27             lev[i.to] = lev[u] + 1;
28             q.push(i.to);
29         }
30     }
31     return lev[t] != -1;
32 }
33 int dfs(int v, int s, int f = oo) {
34     if (!f or v == s) return f;
35     for (int& i = beg[v]; i < g[v].size(); i++) {
36         auto& e = g[v][i];
37         if (lev[e.to] != lev[v] + 1) continue;
38         int foi = dfs(e.to, s, min(f, e.cap - e.flow));
39         if (!foi) continue;
40         e.flow += foi, g[e.to][e.rev].flow -= foi;
41         return foi;
42     }
43     return 0;
44 }
45 ll max_flow(int s, int t) {
46     for (lim = scaling ? (1<<30) : 1; lim; lim /= 2)
47         while (bfs(s, t)) while (int ff = dfs(s, t)) F += ff;
48     return F;
49 }
50 };
51 vector<pair<int, int>> get_cut(Dinic& g, int s, int t) {
52     g.max_flow(s, t);
53     vector<pair<int, int>> cut;
54     vector<int> vis(g.g.size(), 0), st = {s};
55     vis[s] = 1;
56     while (st.size()) {
57         int u = st.back(); st.pop_back();
58         for (auto e : g.g[u]) if (!vis[e.to] and e.flow < e.cap)
59             vis[e.to] = 1, st.push_back(e.to);

```

```

60 }
61 for (int i = 0; i < g.g.size(); i++) for (auto e : g.g[i])
62     if (vis[i] and !vis[e.to] and !e.res) cut.emplace_back(i, e.to);
63 return cut;
64 }

```

4.6 Flow - MinCostMaxFlow

```

1 // O(nm + f * m log n)
2 // const ll oo = (1ll)1e18;
3 template<typename T> struct mcmf {
4     struct edge {
5         int to, rev, flow, cap; // para, id da reversa, fluxo, capacidade
6         bool res; // se eh reversa
7         T cost; // custo da unidade de fluxo
8         edge() : to(0), rev(0), flow(0), cap(0), cost(0), res(false) {}
9         edge(int to_, int rev_, int flow_, int cap_, T cost_, bool res_)
10             : to(to_), rev(rev_), flow(flow_), cap(cap_), res(res_), cost(
11                 cost_) {}
12 };
13 vector<vector<edge>> g;
14 vector<int> par_idx, par;
15 T inf;
16 vector<T> dist;
17 mcmf(int n) : g(n), par_idx(n), par(n), inf(numeric_limits<T>::max()
18     /3) {}
19 void add(int u, int v, int w, T cost) { // de u pra v com cap w e
20     custo cost
21     edge a = edge(v, g[v].size(), 0, w, cost, false);
22     edge b = edge(u, g[u].size(), 0, 0, -cost, true);
23     g[u].push_back(a);
24     g[v].push_back(b);
25 }
26 vector<T> spfa(int s) { // nao precisa se nao tiver custo negativo
27     deque<int> q;
28     vector<bool> is_inside(g.size(), 0);
29     dist = vector<T>(g.size(), inf);
30     dist[s] = 0;
31     q.push_back(s);
32     is_inside[s] = true;
33     while (!q.empty()) {
34         int v = q.front();
35         q.pop_front();

```



```

33     is_inside[v] = false;
34     for (int i = 0; i < g[v].size(); i++) {
35         auto [to, rev, flow, cap, res, cost] = g[v][i];
36         if (flow < cap and dist[v] + cost < dist[to]) {
37             dist[to] = dist[v] + cost;
38
39             if (is_inside[to]) continue;
40             if (!q.empty() and dist[to] > dist[q.front()]) q.push_back(to)
41                 ;
42             else q.push_front(to);
43             is_inside[to] = true;
44         }
45     }
46     return dist;
47 }
48 bool dijkstra(int s, int t, vector<T>& pot) {
49     priority_queue<pair<T, int>, vector<pair<T, int>>, greater<>> q;
50     dist = vector<T>(g.size(), inf);
51     dist[s] = 0;
52     q.emplace(0, s);
53     while (q.size()) {
54         auto [d, v] = q.top();
55         q.pop();
56         if (dist[v] < d) continue;
57         for (int i = 0; i < g[v].size(); i++) {
58             auto [to, rev, flow, cap, res, cost] = g[v][i];
59             cost += pot[v] - pot[to];
60             if (flow < cap and dist[v] + cost < dist[to]) {
61                 dist[to] = dist[v] + cost;
62                 q.emplace(dist[to], to);
63                 par_idx[to] = i, par[to] = v;
64             }
65         }
66     }
67     return dist[t] < inf;
68 }
69 pair<int, T> min_cost_flow(int s, int t, int flow = (int)1e9) {
70     vector<T> pot(g.size(), 0);
71     pot = spfa(s); // mudar algoritmo de caminho minimo aqui
72     int f = 0;
73     T ret = 0;
74     while (f < flow and dijkstra(s, t, pot)) {

```

```

75         for (int i = 0; i < g.size(); i++)
76             if (dist[i] < inf) pot[i] += dist[i];
77         int mn_flow = flow - f, u = t;
78         while (u != s){
79             mn_flow = min(mn_flow,
80                 g[par[u]][par_idx[u]].cap - g[par[u]][par_idx[u]].flow);
81             u = par[u];
82         }
83         ret += pot[t] * mn_flow;
84         u = t;
85         while (u != s) {
86             g[par[u]][par_idx[u]].flow += mn_flow;
87             g[u][g[par[u]][par_idx[u]].rev].flow -= mn_flow;
88             u = par[u];
89         }
90         f += mn_flow;
91     }
92     return make_pair(f, ret);
93 }
94 // Opcional: retorna as arestas originais por onde passa flow = cap
95 vector<pair<int,int>> recover() {
96     vector<pair<int,int>> used;
97     for (int i = 0; i < g.size(); i++) for (edge e : g[i])
98         if (e.flow == e.cap && !e.res) used.push_back({i, e.to});
99     return used;
100 }
101 };

```

4.7 2 Sat

```

1 struct TwoSat {
2     int n, v_n;
3     vec<bool> vis, assign;
4     vec<int> order, comp;
5     vec<vec<int>> g, g_t;
6     TwoSat(int n_): n(n_), v_n(2 * n_), vis(v_n) , assign(n_), comp(v_n
7         , - 1), g(v_n), g_t(v_n) {
8         order.reserve(v_n);
9     }
10    void add_disj(int a, bool na, int b, bool nb) { // negated_a,
11        negated_b
12        a = 2 * a ^ na;
13        b = 2 * b ^ nb;

```

```

12     int neg_a = a ^ 1;
13     int neg_b = b ^ 1;
14     g[neg_a].pb(b);
15     g[neg_b].pb(a);
16     g_t[a].pb(neg_b);
17     g_t[b].pb(neg_a);
18 }
19 void dfs1(int u){
20     vis[u] = 1;
21     for (int v: g[u]) if (!vis[v]) dfs1(v);
22     order.pb(u);
23 }
24 void dfs2(int u, int cc) {
25     comp[u] = cc;
26     for (int v: g_t[u]) if (comp[v] == -1) dfs2(v, cc);
27 }
28 bool solve() {
29     order.clear();
30     vis.assign(v_n, 0);
31     L(i,0, v_n) if (!vis[i]) dfs1(i);
32     comp.assign(v_n, -1);
33     int cc = 0;
34     L(i, 0, v_n) {
35         int v = order[v_n - 1 - i];
36         if (comp[v] == -1) dfs2(v, cc ++);
37     }
38     assign.assign(n, false);
39     for (int i = 0; i < v_n; i += 2) {
40         if (comp[i] == comp[i+1]) return false;
41         assign[i / 2] = comp[i] > comp[i + 1];
42     }
43     return true;
44 }
45 };

```

4.8 Euler Tour

```

1 // Directed version (uncomment commented code for undirected)
2 struct edge {
3     int y;
4     // list<edge>::iterator rev;
5     edge(int y):y(y){}
6 };

```

```

7 list<edge> g[N];
8 void add_edge(int a, int b){
9     g[a].push_front(edge(b)); //auto ia=g[a].begin();
10    // g[b].push_front(edge(a)); auto ib=g[b].begin();
11    // ia->rev=ib; ib->rev=ia;
12 }
13 vector<int> p;
14 void go(int x){
15     while(g[x].size()){
16         int y=g[x].front().y;
17         //g[y].erase(g[x].front().rev);
18         g[x].pop_front();
19         go(y);
20     }
21     p.push_back(x);
22 }
23 vector<int> get_path(int x){ // get a path that begins in x
24    // check that a path exists from x before calling to get_path!
25     p.clear(); go(x); reverse(p.begin(), p.end());
26     return p;
27 }

```

5 Trees

5.1 Heavy Light Decomposition

```

1 int ans[N], par[N], depth[N], head[N], pos[N];
2 vec<int> heavy(N, -1);
3 int t = 0;
4 vec<int> g[N];
5 int dfs(int u) {
6     int size = 1;
7     int max_size = 0;
8     for (int v: g[u]) if (v != par[u]) {
9         par[v] = u;
10        depth[v] = depth[u] + 1;
11        int cur_size = dfs(v);
12        size += cur_size;
13        if (cur_size > max_size) {
14            max_size = cur_size;
15            heavy[u] = v;
16        }
17    }

```

```

18     return size;
19 }
20 void decompose(int u, int h){
21     head[u] = h;
22     pos[u] = t++;
23     if (heavy[u] != -1){ decompose(heavy[u], h); }
24     for (int v: G[u]) if (v != par[u] && v != heavy[u]) {
25         decompose(v, v);
26     }
27 }
28 int query(int a, int b) {
29     int resp = -1;
30     for (; head[a] != head[b]; b = par[head[b]]){ // Subi todo el heavy
31         // path y a su padre // Next
32         if (depth[head[a]] > depth[head[b]]) swap(a, b);
33         resp = max(resp, st.query(pos[head[b]], pos[b])); // pos[head[b]] < pos[b]
34     }
35     if (depth[a] > depth[b]) swap(a, b); // Una vez misma path(head)
36     // entonces es una query [a,b]
37     resp = max(resp, st.query(pos[a], pos[b]));
38     return resp;
39 }
40 dfs(root);
41 decompose(root, root);

```

5.2 Centroid

```

1 int sz[N];
2 bool removed[N];
3 int getSize(int u, int p){
4     sz[u] = 1;
5     for(int v: G[u]) if (v != p && !removed[v]){
6         sz[u] += getSize(v, u);
7     }
8     return sz[u];
9 }
10 int centroid(int u, int p, int tz){
11     for (int v: g[u])
12         if (v != p && !removed[v] && sz[v] * 2 > tz) return centroid(v, u, tz);
13     return u;
14 }

```

```

15 int build(int u){
16     int c = centroid(u, -1, getSize(u, -1));
17     removed[c] = 1;
18     for (int v: G[c]) if (!removed[v]) { build(v); }
19     return c;
20 }

```

5.3 LCA - Binary exponentiation

```

1 vector<int> g[N];
2 int K; // K should be (1<K) > n
3 int jump[20][N];
4 int depth[N];
5
6 void dfs(int u, int p){
7     for (int v: g[u]) if (v != p) {
8         jump[0][v] = u;
9         L(i, 1, K + 1) {
10             jump[i][v] = -1;
11             if (jump[i - 1][v] != -1) {
12                 jump[i][v] = jump[i - 1][jump[i - 1][v]];
13             }
14         }
15         depth[v] = depth[u] + 1;
16         dfs(v, u);
17     }
18 }
19
20 int LCA(int u, int v){
21     if (depth[u] < depth[v]) swap(u, v); // Make u the deepest
22     for (int i = K; i >= 0; i --){ // make them same depth
23         if (jump[i][u] != -1 && depth[jump[i][u]] >= depth[v]){
24             u = jump[i][u];
25         }
26     }
27     if (u == v) return u; // u is parent of v
28     for (int i = K; i >= 0; i --){
29         if (jump[i][u] != jump[i][v] && jump[i][u] != -1 && jump[i][v] != -1){
30             u = jump[i][u];
31             v = jump[i][v];
32         }
33     }
34 }

```

```

34     return jump[0][u];
35 }

```

5.4 LCA - Const Time

```

1 struct LCA {
2     vec<int> depth, in, euler;
3     vec<vec<int>> g, st;
4     int K, n;
5     inline int Min(int i, int j) {return depth[i] <= depth[j] ? i : j;}
6     void dfs(int u, int p) {
7         in[u] = SZ(euler);
8         euler.pb(u);
9         for (int v: g[u]) if (v != p){
10             depth[v] = depth[u] + 1;
11             dfs(v, u);
12             euler.pb(u);
13         }
14     }
15     LCA(int n_): depth(n_), g(vec<vec<int>>(n_)), K(0), n(n_), in(n_) {
16         euler.reserve(2 * n); }
17     void add_edge(int u, int v) {g[u].pb(v);}
18     void build(int root){
19         dfs(root, -1);
20         int ln = SZ(euler);
21         while((1<<K)<=ln)K++;
22         st = vec<vec<int>> (K, vec<int>(ln));
23         L(i,0,ln) st[0][i] = euler[i];
24         for (int i = 1; (1 << i) <= ln; i++) {
25             for (int j = 0; j + (1<<i) <= ln; j++) {
26                 st[i][j] = Min(st[i-1][j], st[i-1][j + (1<<(i-1))]);
27             }
28         }
29         int get(int u, int v) {
30             int su = in[u];
31             int sv = in[v];
32             if (sv < su) swap(sv, su);
33             int bit = log2(sv - su + 1);
34             return Min(st[bit][su], st[bit][sv - (1<<bit) + 1]);
35         }
36 };

```

6 Dynamic Programming

6.1 Knapsack

```

1 int knapsack(vector<int>& values, vector<int>& weights, int W) {
2     int n = values.size();
3     vector<vector<int>> dp(n + 1, vector<int>(W + 1, 0));
4
5     for(int i = 1; i <= n; i++) {
6         for(int w = 0; w <= W; w++) {
7             if(weights[i-1] <= w) {
8                 dp[i][w] = max(dp[i-1][w],
9                               dp[i-1][w-weights[i-1]] + values[i-1]);
10            } else {
11                dp[i][w] = dp[i-1][w];
12            }
13        }
14    }
15    return dp[n][W];
16 }

```

6.2 LIS

```

1 vector<int> getLIS(vector<int>& arr) {
2     int n = arr.size();
3     vector<int> dp(n + 1, INT_MAX); // dp[i] = smallest value that ends
4                                     // an LIS of length i
5     vector<int> len(n); // Length of LIS ending at each
6                           // position
7     dp[0] = INT_MIN;
8     for(int i = 0; i < n; i++) {
9         int j = upper_bound(dp.begin(), dp.end(), arr[i]) - dp.begin();
10        dp[j] = arr[i];
11        len[i] = j;
12    }
13    // Find maxLen and reconstruct sequence
14    int maxLen = 0;
15    for(int i = n-1; i >= 0; i--) maxLen = max(maxLen, len[i]);
16    vector<int> lis;
17    for(int i = n-1, currLen = maxLen; i >= 0; i--) {
18        if(len[i] == currLen) {
19            lis.push_back(arr[i]);
20            currLen--;
21        }
22    }
23    reverse(lis.begin(), lis.end());
24    return lis;
25 }

```

```

19     }
20 }
21 reverse(lis.begin(), lis.end());
22 return lis;
23 }

```

6.3 Edit Distance

```

1 int editDistance(string& s1, string& s2) {
2     int n = s1.length(), m = s2.length();
3     vector<vector<int>> dp(n + 1, vector<int>(m + 1));
4     for(int i = 0; i <= n; i++) dp[i][0] = i;
5     for(int j = 0; j <= m; j++) dp[0][j] = j;
6     for(int i = 1; i <= n; i++) {
7         for(int j = 1; j <= m; j++) {
8             if(s1[i-1] == s2[j-1]) {
9                 dp[i][j] = dp[i-1][j-1];
10            } else {
11                dp[i][j] = 1 + min({dp[i-1][j],      // deletion
12                                   dp[i][j-1],      // insertion
13                                   dp[i-1][j-1]}); // replacement
14            }
15        }
16    }
17    return dp[n][m];
18 }

```

6.4 Kadane

```

1 pair<int, pair<int,int>> kadane(vector<int>& arr) {
2     int maxSoFar = arr[0], maxEndingHere = arr[0];
3     int start = 0, end = 0, s = 0;
4
5     for(int i = 1; i < arr.size(); i++) {
6         if(maxEndingHere + arr[i] < arr[i]) {
7             maxEndingHere = arr[i];
8             s = i;
9         } else {
10            maxEndingHere += arr[i];
11        }
12
13        if(maxEndingHere > maxSoFar) {
14            maxSoFar = maxEndingHere;
15            start = s;
16        }
17    }
18 }

```

```

16         end = i;
17     }
18 }
19 return {maxSoFar, {start, end}}; // max, l, r
20 }

```

7 Strings

7.1 Prefix Trie

```

1 struct Trie {
2     map<char, int> ch;
3     bool eee;
4     Trie(): eee(0) {}
5 };
6 vec<Trie> t;
7 void initTrie(){t.clear();t.pb(Trie());}
8 void insert(string &word) {
9     int v = 0;
10    for(char c : word) {
11        if(!t[v].ch[c]) {
12            t[v].ch[c] = SZ(t);
13            t.pb(Trie());
14        }
15        v = t[v].ch[c];
16    }
17    t[v].eee = 1;
18 }

```

7.2 Hashing

```

1 static constexpr ll ms[] = {1'000'000'007, 1'000'000'403};
2 static constexpr ll b = 500'000'000;
3 struct StrHash { // Hash polinomial con exponentes decrecientes.
4     vector<ll> hs[2], bs[2];
5     StrHash(string const& s) {
6         int n = SZ(s);
7         L(k, 0, 2) {
8             hs[k].resize(n+1), bs[k].resize(n+1, 1);
9             L(i, 0, n) {
10                hs[k][i+1] = (hs[k][i] * b + s[i]) % ms[k];
11                bs[k][i+1] = bs[k][i] * b % ms[k];
12            }
13        }
14    }
15 }

```

```

13     }
14 }
15 ll get(int idx, int len) const { // Hashes en 's[idx, idx+len)'.
16     ll h[2];
17     L(k, 0, 2) {
18         h[k] = hs[k][idx+len] - hs[k][idx] * bs[k][len] % ms[k];
19         if (h[k] < 0) h[k] += ms[k];
20     }
21     return (h[0] << 32) | h[1];
22 }
23 };
24
25 pll union_hash(vector<pll> hs, vector<ll> lens){ //use arrays makes it
26     slower
27     ll len = 0;
28     for(int i = hs.size()-1; i > 0; i--){
29         len += lens[i];
30         pll& [l1, l2] = hs[i];
31         pll& [r1, r2] = hs[i-1];
32         l1 = ((l1 * binpow(b, len, ms[0])) % ms[0] + r1) % ms[0];
33         l2 = ((l2 * binpow(b, len, ms[1])) % ms[1] + r2) % ms[1];
34     }
35     return hs[0];
36 }

```

7.3 KMP

```

1  vec<int> kmp(string pat, string sec){ //geeks4geeks implementation with
2      some changes
3      int m = pat.length();
4      int n = sec.length();
5      cout << m << " " << n << endl;
6
7      vec<int> lps = getLps(pat);
8      vec<int> res;
9
10     int i = 0;
11     int j = 0;
12
13     while((n - i) >= (m - j)){
14         if(pat[j] == sec[i]){
15             i++;

```

```

15         j++;
16     }
17     if(j == m){
18         res.push_back(i - j);
19         j = lps[j - 1];
20     }
21     else{
22         if(i < n && pat[j] != sec[i]){
23             if(j != 0) j = lps[j - 1];
24             else i = i + 1;
25         }
26     }
27 }
28
29 return res;
30 }

```

7.4 LPS

```

1  vec<int> getLps(string pat){ //geek4geeks implementatio with some
2      changes
3      vec<int> lps(pat.length(), 0);
4      int len = 0;
5      int i = 1;
6      lps[0] = 0;
7      while(i < pat.length()){
8          if(pat[i] == pat[len]){
9              len++;
10             lps[i] = len;
11             i++;
12         }
13         else //pat[i] != pat[len]
14         {
15             lps[i] = 0;
16             i++;
17         }
18     }
19     return lps;

```

7.5 Z-FUNCTION

```

1  template<class Char=char>vector<int> zfun(const basic_string<Char>& w) {
2      int n = SZ(w), l = 0, r = 0; vector<int> z(n);

```

```

3  z[0] = w.length();
4  L(i, 1, n) {
5      if (i <= r) {z[i] = min(r - i + 1, z[i - 1]);}
6      while (i + z[i] < n && w[z[i]] == w[i + z[i]]) {++z[i];}
7      if (i + z[i] - 1 > r) {l = i, r = i + z[i] - 1;}
8  }
9  return z;
10 }
```

7.6 Aho-Corasick

```

1  bool vis[N], r[N];
2  struct ACvertex {
3      map<char,int> next,go;
4      int p,link;
5      char pch;
6      vector<int> leaf;
7      ACACvertex(int p=-1, char pch=-1):p(p),pch(pch),link(-1){}
8  };
9  vector<ACvertex> t;
10 void aho_init(){ //do not forget!!
11     t.clear();t.pb(ACvertex());
12 }
13 void add_string(string s, int id){
14     int v=0;
15     for(char c:s){
16         if(!t[v].next.count(c)){
17             t[v].next[c]=t.size();
18             t.pb(ACvertex(v,c));
19         }
20         v=t[v].next[c];
21     }
22     t[v].leaf.pb(id);
23 }
24 int go(int v, char c);
25 int get_link(int v){ // Failure link
26     if(t[v].link<0)
27         if(!v||!t[v].p)t[v].link=0;
28         else t[v].link=go(get_link(t[v].p),t[v].pch);
29     return t[v].link;
30 }
31 int go(int v, char c){ // state = go(state, ch) this state is ACvertex
    id
```

```

32     if(!t[v].go.count(c))
33         if(t[v].next.count(c))t[v].go[c]=t[v].next[c];
34         else t[v].go[c]=v==0?0:go(get_link(v),c);
35     return t[v].go[c];
36 }
37 void proc(int x){
38     if (x == - 1|| vis[x]) return;
39     vis[x] = 1;
40     L(i,0,SZ(t[x].leaf)) r[t[x].leaf[i]] = 1;
41     proc(get_link(x));
42 }
```

8 Math

8.1 Euclidean Extended

```

1  ll extendedGCD(ll a, ll b, ll &x, ll &y) {
2      if (b == 0) {
3          x = 1;
4          y = 0;
5          return a;
6      }
7      ll x1, y1;
8      ll gcd = extendedGCD(b, a % b, x1, y1);
9      x = y1;
10     y = x1 - (a / b) * y1;
11     return gcd;
12 }
13
14 bool findSolutionWithConstraints(ll a, ll b, ll c, ll x_min, ll y_min,
15     ll &x, ll &y) {
16     ll g = extendedGCD(a, b, x, y);
17
18     if (c % g != 0) return false;
19
20     x *= c / g;
21     y *= c / g;
22
23     // Ajustamos las variables a/g y b/g para mover las soluciones
24     a /= g;
25     b /= g;
26
27     if (x < x_min) {
```

```

27     ll k = (x_min - x + b - 1) / b; // Redondeo hacia arriba
28     x += k * b;
29     y -= k * a;
30 } else if (x > x_min) {
31     ll k = (x - x_min) / b;
32     x -= k * b;
33     y += k * a;
34 }
35
36 if (y < y_min) {
37     ll k = (y_min - y + a - 1) / a; // Redondeo hacia arriba
38     x += k * b;
39     y -= k * a;
40 } else if (y > y_min) {
41     ll k = (y - y_min) / a;
42     x -= k * b;
43     y += k * a;
44 }
45
46 return x >= x_min && y >= y_min;
47 }

```

8.2 Euler Totient

```

1 #include <bits/stdc++.h>
2 using namespace std;
3 typedef long long ll;
4
5
6 vector<ll> compute_totients(ll n) {
7     vector<ll> phi(n + 1);
8     for (ll i = 0; i <= n; i++) {
9         phi[i] = i;
10    }
11
12    for (ll i = 2; i <= n; i++) {
13        if (phi[i] == i) { // i es primo
14            for (ll j = i; j <= n; j += i) {
15                phi[j] = phi[j] * (i - 1) / i;
16            }
17        }
18    }
19 }

```

```

20     return phi;
21 }

```

8.3 Josephus

```

1 #include <iostream>
2 using namespace std;
3
4 typedef long long ll;
5
6 ll josephus_iterative(ll n, ll k) {
7     ll result = 0;
8     for (ll i = 2; i <= n; ++i) {
9         result = (result + k) % i;
10    }
11    return result;
12 }
13
14
15 ll josephus_recursive(ll n, ll k) {
16
17     if (n == 1)
18         return 0;
19
20     return (josephus_recursive(n - 1, k) + k) % n;
21 }
22
23
24 ll josephus_power_of_2(ll n) {
25
26     ll power = 1;
27     while (power <= n) {
28         power <<= 1;
29     }
30     power >>= 1;
31
32     return 2 * (n - power);
33 }
34 }

```

8.4 Mobius

```

1 #include <bits/stdc++.h>
2 using namespace std;

```



```

3 typedef long long ll;
4
5
6 vector<ll> compute_mobius(ll n) {
7     vector<ll> mu(n + 1, 1);
8     vector<bool> is_prime(n + 1, true);
9
10    for (ll i = 2; i <= n; i++) {
11        if (is_prime[i]) { // i es un primo
12            for (ll j = i; j <= n; j += i) {
13                mu[j] *= -1; // Multiplicamos por -1 para cada primo
14                is_prime[j] = false;
15            }
16            for (ll j = i * i; j <= n; j += i * i) {
17                mu[j] = 0; // Si tiene un cuadrado de un primo, se pone
18                           // en 0
19            }
20        }
21    }
22    return mu;
23 }
24
25
26 ll mobius(ll x) {
27     ll count = 0;
28     for (ll i = 2; i * i <= x; i++) {
29         if (x % (i * i) == 0)
30             return 0;
31         if (x % i == 0) {
32             count++;
33             x /= i;
34         }
35     }
36
37     if (x > 1) count++;
38
39     return (count % 2 == 0) ? 1 : -1;
40 }

```

8.5 NTT

```
1 #include <bits/stdc++.h>
```

```

2 using namespace std;
3 using cd = complex<double>;
4 typedef long long ll;
5 const ll mod = 998244353;
6 const ll root = 31;
7 const ll root_1 = inverse(root, mod);
8 const ll root_pw = 1 << 23;
9
10 ll inverse(ll a, ll m) {
11     ll res = 1, exp = m - 2;
12     while (exp) {
13         if (exp % 2 == 1) res = (1LL * res * a) % m;
14         a = (1LL * a * a) % m;
15         exp /= 2;
16     }
17     return res;
18 }
19
20 void ntt(vector<ll> & a, bool invert) {
21     int n = a.size();
22
23     for (int i = 1, j = 0; i < n; i++) {
24         int bit = n >> 1;
25         for (; j & bit; bit >>= 1)
26             j ^= bit;
27         j ^= bit;
28
29         if (i < j)
30             swap(a[i], a[j]);
31     }
32
33     for (int len = 2; len <= n; len <= 1) {
34         int wlen = invert ? root_1 : root;
35         for (int i = len; i < root_pw; i <= 1)
36             wlen = (int)(1LL * wlen * wlen % mod);
37
38         for (int i = 0; i < n; i += len) {
39             int w = 1;
40             for (int j = 0; j < len / 2; j++) {
41                 int u = a[i+j], v = (int)(1LL * a[i+j+len/2] * w % mod);
42                 a[i+j] = u + v < mod ? u + v : u + v - mod;
43                 a[i+j+len/2] = u - v >= 0 ? u - v : u - v + mod;
44                 w = (int)(1LL * w * wlen % mod);

```

```

45     }
46 }
47
48
49 if (invert) {
50     int n_1 = inverse(n, mod);
51     for (auto & x : a)
52         x = (int)(1LL * x * n_1 % mod);
53 }
54 }
55
56 vector<ll> multiply(vector<ll> const &a, vector<ll> const &b) {
57     vector<ll> fa(a.begin(), a.end()), fb(b.begin(), b.end());
58     ll n = 1;
59     while (n < a.size() + b.size())
60         n <<= 1;
61     fa.resize(n);
62     fb.resize(n);
63
64     ntt(fa, false);
65     ntt(fb, false);
66     for (ll i = 0; i < n; i++)
67         fa[i] = (fa[i] * fb[i]) % mod;
68     ntt(fa, true);
69
70     vector<ll> result(n);
71     for (ll i = 0; i < n; i++)
72         result[i] = fa[i];
73     return result;
74 }

```

8.6 FFT

```

1  typedef long long ll;
2  typedef complex<double> C;
3  typedef vector<double> vd;
4  typedef vector<ll> vll;
5  const double PI = acos(-1);
6
7  void fft(vector<C>& a) {
8      int n = a.size(), L = 31 - __builtin_clz(n);
9      static vector<C> R(2, 1);
10     static vector<C> rt(2, 1);

```

```

11     for (static int k = 2; k < n; k *= 2) {
12         R.resize(n); rt.resize(n);
13         auto x = polar(1.0, PI / k);
14         for (int i = k; i < 2 * k; i++)
15             rt[i] = R[i] = i & 1 ? R[i / 2] * x : R[i / 2];
16     }
17     vector<int> rev(n);
18     for (int i = 0; i < n; i++) rev[i] = (rev[i / 2] | (i & 1) << L) /
19         2;
20     for (int i = 0; i < n; i++) if (i < rev[i]) swap(a[i], a[rev[i]]);
21     for (int k = 1; k < n; k *= 2)
22         for (int i = 0; i < n; i += 2 * k) for (int j = 0; j < k; j++) {
23             auto x = (double*)&rt[j + k], y = (double*)&a[i + j + k];
24             C z(x[0] * y[0] - x[1] * y[1], x[0] * y[1] + x[1] * y[0]);
25             a[i + j + k] = a[i + j] - z;
26             a[i + j] += z;
27         }
28
29     vll multiply(const vll& a, const vll& b) {
30         if (a.empty() || b.empty()) return {};
31         vd fa(a.begin(), a.end()), fb(b.begin(), b.end());
32         int L = 32 - __builtin_clz(fa.size() + fb.size() - 1), n = 1 << L;
33         vector<C> in(n), out(n);
34
35         for (int i = 0; i < a.size(); i++) in[i] = C(fa[i], 0);
36         for (int i = 0; i < b.size(); i++) in[i].imag(fb[i]);
37
38         fft(in);
39         for (C& x : in) x *= x;
40         for (int i = 0; i < n; i++) out[i] = in[-i & (n - 1)] - conj(in[i]);
41         // Corregido aqui
42         fft(out);
43
44         vll res(a.size() + b.size() - 1);
45         for (int i = 0; i < res.size(); i++) {
46             res[i] = llround(imag(out[i]) / (4 * n));
47         }
48         return res;
49     }

```

8.7 Rho

```

1 //RECOMENDADO USAR UNSIGNED LONG LONG
2 static inline ll mulmod(ll a, ll b, ll m) {
3     return (ll)((__int128)a * b % m);
4 }
5
6 static inline ll powmod(ll b, ll e, ll m) {
7     ll r = 1;
8     while (e) {
9         if (e & 1) r = mulmod(r, b, m);
10        b = mulmod(b, b, m);
11        e >>= 1;
12    }
13    return r;
14 }
15
16 // RNG rapido
17 static inline ll splitmix64(ll x) {
18     x += 0x9e3779b97f4a7c15ULL;
19     x = (x ^ (x >> 30)) * 0xbf58476d1ce4e5b9ULL;
20     x = (x ^ (x >> 27)) * 0x94d049bb133111ebULL;
21     return x ^ (x >> 31);
22 }
23 static ll rng_state = 0x1234567890abcdefULL ^ chrono::
24     high_resolution_clock::now().time_since_epoch().count();
25 static inline ll rnd() { return splitmix64(rng_state += 0
26     x9e3779b97f4a7c15ULL); }
27
28 // trial division pequena para acelerar
29 static const int SMALL_P_MAX = 1000;
30 static vector<int> small_primes;
31
32 static void sieve_small() {
33     vector<bool> is(SMALL_P_MAX + 1, true);
34     is[0] = is[1] = false;
35     for (int i = 2; i * i <= SMALL_P_MAX; ++i) if (is[i])
36         for (int j = i * i; j <= SMALL_P_MAX; j += i) is[j] = false;
37     for (int i = 2; i <= SMALL_P_MAX; ++i) if (is[i]) small_primes.
38         push_back(i);
39 }
40
41 bool isPrime(ll n) {
42     if (n < 2) return false;
43     // divide por primos pequenos

```

```

41 for (int p : small_primes) {
42     if ((ll)p * (ll)p > n) break;
43     if (n % p == (ll)0) return n == (ll)p;
44 }
45 if (n < 4) return true; // 2,3
46 // Miller-Rabin deterministico para 64-bit
47 ll d = n - 1, s = 0;
48 while ((d & 1) == 0) d >>= 1, ++s;
49 auto witness = [&](ll a) -> bool {
50     if (a % n == 0) return false;
51     ll x = powmod(a % n, d, n);
52     if (x == 1 || x == n - 1) return false;
53     for (int i = 1; i < s; ++i) {
54         x = mulmod(x, x, n);
55         if (x == n - 1) return false;
56     }
57     return true; // es testigo: n compuesto
58 };
59 // Bases correctas para 64-bit
60 for (ll a : {2ULL, 3ULL, 5ULL, 7ULL, 11ULL, 13ULL, 17ULL, 19ULL, 23
61     ULL,
62     325ULL, 9375ULL, 28178ULL, 450775ULL, 9780504ULL,
63     1795265022ULL}) {
64     if (a == 0) continue;
65     if (a % n == 0) continue;
66     if (witness(a)) return false;
67 }
68
69 ll pollard_rho(ll n) {
70     if ((n & 1ULL) == 0ULL) return 2ULL;
71     while (true) {
72         ll c = (rnd() % (n - 1)) + 1; // [1..n-1]
73         ll x = (rnd() % (n - 2)) + 2; // [2..n-1]
74         ll y = x;
75         ll d = 1;
76         // limite de iteraciones para evitar lazos raros
77         for (int it = 0; it < 1'000'000 && d == 1; ++it) {
78             x = (mulmod(x, x, n) + c) % n;
79             y = (mulmod(y, y, n) + c) % n;
80             y = (mulmod(y, y, n) + c) % n;
81             ll diff = x > y ? x - y : y - x;

```

```

82         d = std::gcd(diff, n);
83     }
84     if (d == 1 || d == n) continue;
85     return d;
86 }
87 }
88
89 void fact(ll n, map<ll,int> &F) {
90     if (n == 1) return;
91     if (isPrime(n)) { F[n]++; return; }
92     for (int p : small_primes) {
93         if ((ll)p * (ll)p > n) break;
94         while (n % p == 0) { F[p]++; n /= p; }
95     }
96     if (n == 1) return;
97     if (isPrime(n)) { F[n]++; return; }
98     ll d = pollard_rho(n);
99     fact(d, F);
100    fact(n / d, F);
101 }

```

8.8 Simpson

```

1 ld simpsonRule(function<ld(ld)> f, ld a, ld b, int n) {
2     // Asegurarse de que n sea par
3     if (n % 2 != 0) {
4         n++;
5     }
6     ld h = (b - a) / n;
7     ld s = f(a) + f(b);
8
9     // Suma de terminos interiores con los factores apropiados
10    for (int i = 1; i < n; i++) {
11        ld x = a + i * h;
12        s += (i % 2 == 1 ? 4.0L : 2.0L) * f(x);
13    }
14    // Multiplica por h/3
15    return (h / 3.0L) * s;
16 }
17 // Ejemplo: integrar la funcion x^2 entre 0 y 3
18 auto f = [&](ld x){ return x * x; };
19 ld a = 0.0L, b = 3.0L;
20 int n = 1000; // numero de subintervalos

```

```

21 ld resultado = simpsonRule(f, a, b, n);

```

9 Geometry

9.1 Convex Hull

```

1 typedef pair<ll, ll> Point;
2 ll cross_product(Point O, Point A, Point B) {
3     return (A.first - O.first) * (B.second - O.second) - (A.second - O.
4         second) * (B.first - O.first);
5 }
6 vector<Point> convex_hull(vector<Point>& points) {
7     sort(points.begin(), points.end());
8     points.erase(unique(points.begin(), points.end()), points.end());
9     vector<Point> hull;
10    // Parte inferior
11    for (const auto& p : points) {
12        while (hull.size() >= 2 && cross_product(hull[hull.size() - 2],
13            hull[hull.size() - 1], p) < 0)
14            hull.pop_back();
15        if (hull.empty() || hull.back() != p) {
16            hull.push_back(p);
17        }
18    }
19    // Parte superior
20    int t = hull.size() + 1;
21    for (int i = points.size() - 1; i >= 0; --i) {
22        while (hull.size() >= t && cross_product(hull[hull.size() - 2],
23            hull[hull.size() - 1], points[i]) < 0)
24            hull.pop_back();
25        if (hull.empty() || hull.back() != points[i]) {
26            hull.push_back(points[i]);
27        }
28    }
29    hull.pop_back();
30    return hull;
31 }

```

9.2 Operations

```

1 ll cross_product(pair<ll, ll> P1, pair<ll, ll> P2, pair<ll, ll> P3) {
2     ll x1 = P2.first - P1.first;
3     ll y1 = P2.second - P1.second;

```

```

4     ll x2 = P3.first - P1.first;
5     ll y2 = P3.second - P1.second;
6     return x1 * y2 - y1 * x2;
7 }
8 double distancia(pair<ll, ll> P1, pair<ll, ll> P2) {
9     return sqrt((P2.first - P1.first) * (P2.first - P1.first) +
10                (P2.second - P1.second) * (P2.second - P1.second));
11 }
12 ll dot_product(pair<ll, ll> P1, pair<ll, ll> P2, pair<ll, ll> P3) {
13     ll x1 = P2.first - P1.first;
14     ll y1 = P2.second - P1.second;
15     ll x2 = P3.first - P1.first;
16     ll y2 = P3.second - P1.second;
17     return x1 * x2 + y1 * y2;
18 }

```

```

10     }
11     }
12     return inside;
13 }

```

9.3 Polygon Area

```

1 typedef pair<ll, ll> Point;
2 double polygon_area(const vector<Point>& polygon) {
3     ll area = 0;
4     int n = polygon.size();
5     for (int i = 0; i < n; ++i) {
6         ll j = (i + 1) % n;
7         area += (polygon[i].first * polygon[j].second - polygon[i].
8                 second * polygon[j].first);
9     }
9     return abs(area) / 2.0;
10 }

```

9.4 Ray Casting

```

1 typedef pair<ll, ll> Point;
2 bool is_point_in_polygon(const vector<Point>& polygon, Point p) {
3     bool inside = false;
4     int n = polygon.size();
5     for (int i = 0, j = n - 1; i < n; j = i++) {
6         if ((polygon[i].second > p.second) != (polygon[j].second > p.
7             second) &&
8             p.first < (polygon[j].first - polygon[i].first) * (p.second
9                 - polygon[i].second) /
10                (polygon[j].second - polygon[i].second) + polygon[
11                    i].first) {
12             inside = !inside;
13         }
14     }
15 }

```