

Dividimos y No Conquistamos (D&!C)

Contents

1 Template	2
1.1 C++ Template	2
1.2 Bash CMD	2
1.3 C++ Int128	2
1.4 C++ RNG	2
1.5 Python Template	2
2 Search	2
2.1 Ternary	2
3 Data structures	3
3.1 Fenwick	3
3.2 Fenwick - 2D	3
3.3 DSU	3
3.4 Sparse Table	3
3.5 Segment tree	4
3.6 Segment Tree Iterativo	4
3.7 Segment Tree Persistente	4
3.8 Policy Based	5
3.9 Chull Trick	5
4 Graph	6
4.1 Bellman Ford	6
4.2 SCC	6
4.3 Bipartite Matching Hopcroft-Karp - With Konig	6
4.4 Hungarian	7
4.5 Flow - Dinics	8
4.6 Flow - MinCostMaxFlow	8
4.7 2-Sat	10
4.8 Euler Tour	10
4.9 Directed MST/Arborescence	10
5 Trees	11
5.1 Heavy Light Decomposition	11
5.2 Centroid	12
5.3 LCA - Const Time	12

6 Dynamic Programming	13
6.1 LIS	13
6.2 Divide and Conquer Opt	13
6.3 Knuth Opt	13
7 Strings	14
7.1 Hashing	14
7.2 KMP	14
7.3 Z-Function	14
7.4 Manacher	14
7.5 Aho-Corasick	14
7.6 Suffix-Array	15
7.7 Suffix-Automaton	15
8 Math	16
8.1 Euclidean Extended	16
8.2 Euler Totient	16
8.3 Josephus	17
8.4 Mobius	17
8.5 NTT	17
8.6 FFT	18
8.7 Rho	19
8.8 Get Divisors	19
8.9 Simpson	20
8.10 Simplex	20
8.11 EXP MAT	21
8.12 GAUSS	21
9 Geometry	23
9.1 Convex Hull	23
9.2 Operations	23
9.3 Polygon Area	23
9.4 Ray Casting	24
10 Other	24
10.1 Mo's algorithm	24
11 Ecuations	24
11.1 Combinatorics	24
11.2 Catalan Numbers	25

1 Template

1.1 C++ Template

```

1 #pragma GCC target ("avx2")
2 #pragma GCC optimize ("O3")
3 #pragma GCC optimize ("unroll-loops")
4 #include <bits/stdc++.h>
5 using namespace std;
6 #define L(i, j, n) for (int i = (j); i < (int)n; i++)
7 #define SZ(x) int((x).size())
8 #define ALL(x) begin(x),end(x)
9 #define vec vector
10 #define pb push_back
11 #define eb emplace_back
12 using ll = long long;
13 using ld = long double;
14 void solve(){}
15 int main(){
16     ios::sync_with_stdio(0);cin.tie(0);
17     int TT = 1;
18     //cin >> TT;
19     while (TT--) {solve();}
20 }
21 // IF NEEDED FOR FILE READ
22 // freopen("in.txt", "r", stdin);
23 // freopen("out.txt", "w", stdout);

```

1.2 Bash CMD

```

1 co(){g++ $1/$1.cpp -o $1/$1 --std=c++20 -Wall -Wshadow -Wextra}
2 run(){for f in `ls ./$1/*.txt`;do echo $f ;./$1/$1 < $f; done}
3 #Build, template.cpp must exist!
4 for x in {A..Z}; do mkdir $x; cp template.cpp $x/$x.cpp; touch $x/in.txt;done

```

1.3 C++ Int128

```

1 __int128 read() {
2     __int128 x=0,f=1;
3     char ch=getchar();
4     while (ch<'0'||ch>'9') {
5         if(ch == '-')f=-1;
6         ch=getchar();
7     }

```

```

8     while (ch >= '0' && ch <= '9') {
9         x=x*10+ch-'0';
10        ch=getchar();
11    }
12    return x * f;
13 }
14 void print(__int128 x) {
15     if (x < 0) {
16         putchar('‐');
17         x = -x;
18     }
19     if (x > 9) print(x / 10);
20     putchar(x % 10 + '0');
21 }

```

1.4 C++ RNG

```

1 using my_clock = chrono::steady_clock;
2 struct Random {
3     mt19937_64 engine;
4     Random(): engine(my_clock::now().time_since_epoch().count()) {}
5     template<class Int>Int integer(Int n) {return integer<Int>(0, n);} // '[0,n)'
6     template<class Int>Int integer(Int l, Int r)
7         {return uniform_int_distribution{l, r-1}(engine);} // '[l,r)'
8     double real() {return uniform_real_distribution{}(engine);} // '[0,1)'
9 } rng;

```

1.5 Python Template

```

1 import os, sys, io
2 finput = io.BytesIO(os.read(0, os.fstat(0).st_size)).readline
3 fprint = sys.stdout.write

```

2 Search

2.1 Ternary

```

1 // Minimo de 'f' en '(l,r)'.
2 template<class Fun>ll ternary(Fun f, ll l, ll r) {
3     for (ll d = r-1; d > 2; d = r-1) {
4         ll a = l + d/3, b = r - d/3;
5         if (f(a) > f(b)) l = a; else r = b;
6     }

```

```

7   return l + 1;
8 }
9 // para error < EPS, usar iters=log((r-l)/EPS)/log(1.618)
10 template<class Fun>double golden(Fun f, double l, double r, int iters){
11   double const ratio = (3-sqrt(5))/2;
12   double x1=l+(r-l)*ratio, x2=r-(r-l)*ratio, f1=f(x1), f2=f(x2);
13   while (iters--) {
14     if (f1 > f2) l=x1, x1=x2, f1=f2, x2=r-(r-l)*ratio, f2=f(x2);
15     else         r=x2, x2=x1, f2=f1, x1=l+(r-l)*ratio, f1=f(x1);
16   }
17   return (l+r)/2;
18 }
```

3 Data structures

3.1 Fenwick

```

1 #define LSO(S) (S & -S) //LeastsignificantOne
2 struct FT { // 1-Index
3   vec<int> ft; int n;
4   FT(vec<int> &v): ft(SZ(v)+1), n(SZ(v)+1) { // O(n)
5     L(i, 1, n){
6       ft[i] += v[i-1];
7       if (i + LSO(i) <= n) ft[i + LSO(i)]+=ft[i];
8     }
9   }
10  void update(int pos, int x){ for (int it=pos;it<=n;it+=LSO(it))ft[it]
11    ]+=x; }
12  int sum(int pos){
13    int res = 0;
14    for (int it=pos;it>0;it-=LSO(it))res+=ft[it];
15    return res;
16  }
17  int getSum(int l, int r){return sum(r) - sum(l - 1);}
18 };
```

3.2 Fenwick - 2D

```

1 #define LSO(S) (S & -S)
2 struct BIT { // 1-Index
3   vec<vec<int>> B;
4   int n; // BUILD: N * N * log(N) * log(N)
5   BIT(int n_= 1): B(n_+1,vec<int>(n_+1)), sz(n_) {}
```

```

6   void add(int i, int j, int delta){ // log(N) * log(N)
7     for(int x = i; x <= n; x += LSO(x))
8       for(int y = j; y <= n; y += LSO(y))
9         B[x][y] += delta;
10  }
11  int sum(int i, int j){ // log(N) * log(N)
12    int tot = 0;
13    for(int x = i; x > 0; x -= LSO(x))
14      for(int y = j; y > 0; y -= LSO(y))
15        tot += B[x][y];
16    return tot;
17  }
18  int getSum(int x1, int y1, int x2, int y2) {return sum(x2, y2) - sum
19    (x2, y1) - sum(x1, y2) + sum(x1-1,y1-1);}
};
```

3.3 DSU

```

1 struct DSU {
2   vec<int> par, sz; int n;
3   DSU(int n = 1): par(n), sz(n, 1), n(n) { iota(ALL(par), 0); }
4   int find(int a){return a == par[a] ? a : par[a] = find(par[a]);}
5   void join(int a, int b){
6     a=find(a);b=find(b);
7     if (a == b) return;
8     if (sz[b] > sz[a]) swap(a,b);
9     par[b] = a; sz[a] += sz[b];
10  }
11 };
```

3.4 Sparse Table

```

1 struct SPT {
2   vec<vec<int>> st;
3   SPT(vec<int> &a) {
4     int n = SZ(a), K = 0; while((1<<K)<=n) K++;
5     st = vec<vec<int>>(K, vec<int>(n));
6     L(i,0,n) st[0][i] = a[i];
7     L(i,1,K) for (int j = 0; j + (1 << i) <= n; j++)
8       st[i][j] = min(st[i-1][j], st[i - 1][j + (1 << (i - 1))]);
9       // change op
10  }
11  int get(int l, int r) {
12    int bit = log2(r - l + 1);
```

```

12     return min(st[bit][l], st[bit][r - (1<<bit) + 1]); // change op
13 }
14 };

```

3.5 Segment tree

```

1 #define LC(v) (v<<1)
2 #define RC(v) ((v<<1)|1)
3 #define MD(L, R) (L+((R-L)>>1))
4 struct node { ll mx;ll cant; };
5 struct ST {
6     vec<node> st; vec<ll> lz; int n;
7     ST(int n = 1): st(4 * n + 10, {oo, oo}), lz(4 * n + 10, 0), n(n) {
8         build(1, 0, n - 1); }
9     node merge(node a, node b){
10         if (a.mx == oo) return b; if (b.mx == oo) return a;
11         if (a.mx == b.mx) return {a.mx, a.cant + b.cant};
12         return {max(a.mx, b.mx), a.mx > b.mx ? a.cant : b.cant}; }
13     void build(int v, int L, int R){
14         if (L == R){ st[v] = {0, 1}; return ;}
15         int m = MD(L, R);
16         build(LC(v), L, m); build(RC(v), m + 1, R);
17         st[v] = merge(st[LC(v)], st[RC(v)]);
18     }
19     void push(int v, int L, int R){
20         if (lz[v]){
21             if (L != R){
22                 st[LC(v)].mx += lz[v]; // Apply to left
23                 st[RC(v)].mx += lz[v]; // And right
24                 lz[LC(v)] += lz[v];
25                 lz[RC(v)] += lz[v];
26             }
27             lz[v] = 0;
28         }
29     }
30     void update(int v, int L, int R, int ql, int qr, ll w){
31         if (ql > R || qr < L) return;
32         push(v, L, R);
33         if (ql == L && qr == R){
34             st[v].mx += w; // Update acutal node
35             lz[v] += w; // Add lazy
36             push(v, L, R); // Initial spread

```

```

37         return;
38     }
39     int m = MD(L, R);
40     update(LC(v), L, m, ql, min(qr, m), w);
41     update(RC(v), m + 1, R, max(m + 1, ql), qr, w);
42     st[v] = merge(st[LC(v)], st[RC(v)]);
43 }
44 node query(int v, int L, int R, int ql, int qr){
45     if (ql > R || qr < L) return {oo, oo};
46     push(v, L, R);
47     if (ql == L && qr == R) return st[v];
48     int m = MD(L, R);
49     return merge(query(LC(v), L, m, ql, min(m, qr)), query(RC(v), m
50         + 1, R, max(m + 1, ql), qr));
51 }
52 node query(int l, int r){return query(1, 0, n - 1, l, r);}
53 void update(int l, int r, ll w){update(1, 0, n - 1, l, r, w);}

```

3.6 Segment Tree Iterativo

```

1 struct STI {
2     vec<ll> st; int n, K;
3     STI(vec<ll> &a): n(SZ(a)), K(1) {
4         while(K<=n) K<<1;
5         st.assign(2*K, 0); // 0 default
6         L(i,0,n) st[K+i] = a[i];
7         for (int i = K - 1; i > 0; i --) st[i] = st[i*2] + st[i*2+1]; }
8     void upd(int i, ll w) {
9         i += K; st[i] += w;
10        while(i>>=1)st[i]=st[i*2]+st[i*2+1];}
11     ll query(int l, int r) { // [l, r]
12         ll res = 0;
13         for (l += K, r += K; l < r; l>>=1, r>>=1){
14             if (l & 1) res += st[l++];
15             if (r & 1) res += st[--r];
16         }
17         return res;
18     }
19 };

```

3.7 Segment Tree Persistente

```

1 struct Vertex{Vertex * l, *r;int sum;};

```

```

2 const int MVertex = 6000000; // ~= N * logN * 2
3 Vertex pool[MVertex]; // the idea is to keep versions on vec<Vertex*>
4     roots; roots.pb(build(ST_L, ST_R));
5 int p_num = 0;        //
6 Vertex * init_leaf(int x) {
7     pool[p_num].sum = x;
8     pool[p_num].l = pool[p_num].r = NULL;
9     return &pool[p_num++];
10}
11 Vertex * init_node(Vertex * l, Vertex * r) {
12     int sum = 0;
13     if (l) sum += l->sum;
14     if (r) sum += r->sum;
15     pool[p_num].sum = sum; pool[p_num].l = l; pool[p_num].r = r;
16     return &pool[p_num++];
17}
18 Vertex * build(int L, int R){
19     if (L == R){return init_leaf(0);}
20     int m = MD(L, R); return init_node(build(L, m), build(m + 1, R));
21}
22 Vertex * update(Vertex * v, int L, int R, int pos, int w){
23     if (L == R) return init_leaf(v->sum + w);
24     int m = MD(L, R);
25     if (pos <= m) return init_node(update(v->l, L, m, pos, w), v->r);
26     return init_node(v->l, update(v->r, m + 1, R, pos, w));}
27 int query(Vertex * vl, Vertex * vr, int L, int R, int ql, int qr) {
28     if (!vl || !vr) return 0;
29     if (ql > R || qr < L) return 0;
30     if (ql == L && qr == R) {return vr->sum - vl->sum;}
31     int m = MD(L, R);
32     return query(vl->l, vr->l, L, m, ql, min(m, qr)) +
33             query(vl->r, vr->r, m + 1, R, max(m + 1, ql), qr);}

```

3.8 Policy Based

```

1 #include <ext/pb_ds/assoc_container.hpp>
2 using namespace __gnu_pbds;
3 template<typename Key, typename Val=null_type>
4 using indexed_set = tree<Key, Val, less<Key>, rb_tree_tag,
5                         tree_order_statistics_node_update>;
6 // indexed_set<char> s;
7 // char val = *s.find_by_order(0); // acceso por indice
8 // int idx = s.order_of_key('a'); // busca indice del valor
9 template<class Key, class Val=null_type>using htable=gp_hash_table<Key,
10          Val>;

```

```

10 // como unordered_map (o unordered_set si Val es vacio), pero sin metodo
11 count

```

3.9 Chull Trick

```

1 typedef ll tc;
2 const tc is_query=-(1LL<<62); // special value for query
3 struct Line {
4     tc m,b;
5     mutable multiset<Line>::iterator it,end;
6     const Line* succ(multiset<Line>::iterator it) const {
7         return (++it==end? NULL : &*it);}
8     bool operator<(const Line& rhs) const {
9         if(rhs.b!=is_query) return m<rhs.m;
10        const Line *s=succ(it);
11        if(!s) return 0;
12        return b-s->b<(s->m-m)*rhs.m;
13    }
14};
15 struct HullDynamic : public multiset<Line> { // for maximum
16     bool bad(iterator y){
17         iterator z=next(y);
18         if(y==begin()){
19             if(z==end())return false;
20             return y->m==z->m&&y->b<=z->b;
21         }
22         iterator x=prev(y);
23         if(z==end())return y->m==x->m&&y->b<=x->b;
24         return 1.0*(x->b-y->b)*(z->m-y->m)>=1.0*(y->b-z->b)*(y->m-x->m);
25     } //Take care of overflow!
26     iterator next(iterator y){return ++y;}
27     iterator prev(iterator y){return --y;}
28     void add(tc m, tc b){
29         iterator y=insert((Line){m,b});
30         y->it=y;y->end=end();
31         if(bad(y)){erase(y);return;}
32         while(next(y)!=end()&&bad(next(y)))erase(next(y));
33         while(y!=begin()&&bad(prev(y)))erase(prev(y));
34     }
35     tc eval(tc x){
36         Line l=*lower_bound((Line){x,is_query});
37         return l.m*x+l.b;
38     }

```

39 | };

4 Graph

4.1 Bellman Ford

```

1 bool spfa(int s, vec<int>& d) {
2     int n = SZ(g); // needs g graph
3     d.assign(n, INF);
4     vec<int> cnt(n, 0), inqueue(n, 0);
5     queue<int> q; q.push(s);
6     d[s] = 0; inqueue[s] = true;
7     while (SZ(q)) {
8         int u = q.front(); q.pop();
9         inqueue[u] = false;
10        for (auto [v, w] : g[u]) {
11            if (d[u] + w < d[v]) {
12                d[v] = d[u] + w;
13                if (!inqueue[v]) {
14                    q.push(v); inqueue[v] = true;
15                    cnt[v]++;
16                    if (cnt[v] > n) return false; // negative cycle
17                }
18            }
19        }
20    }
21    return true;
22 }
```

4.2 SCC

```

1 vec<int> dfs_num(N, -1), dfs_low(N, -1), in_stack(N);
2 int dfs_count = 0;
3 int numSCC = 0;
4 stack<int> st;
5 void dfs(int u){
6     dfs_low[u]=dfs_num[u]=dfs_count++;
7     st.push(u);
8     in_stack[u] = 1;
9     for(int v: G[u]) {
10        if (dfs_num[v] == -1) dfs(v);
11        if (in_stack[v]) dfs_low[u] = min(dfs_low[u], dfs_low[v]);
12    }
}
```

```

13     if (dfs_num[u] == dfs_low[u]){
14         numSCC++;
15         while(1){
16             int v = st.top(); st.pop();
17             in_stack[v] = 0;
18             if (u == v) break;
19         }
20     }
21 }
```

4.3 Bipartite Matching Hopcroft-Karp - With Konig

```

1 mt19937 rng((int) chrono::steady_clock::now().time_since_epoch().count()
2 );
3 struct hopcroft_karp {
4     int n, m; // n is Left Partition Size, m is Right Partition Size
5     vec<vec<int>> g;
6     vec<int> dist, nxt, ma, mb;
7     hopcroft_karp(int n_, int m_) : n(n_), m(m_), g(n),
8                     dist(n), nxt(n), ma(n, -1), mb(m, -1) {}
9     void add(int a, int b) { g[a].pb(b); }
10    bool dfs(int i) {
11        for (int &id = nxt[i]; id < g[i].size(); id++) {
12            int j = g[i][id];
13            if (mb[j] == -1 or (dist[mb[j]] == dist[i]+1 and dfs(mb[j]))) {
14                ma[i] = j, mb[j] = i;
15                return true;
16            }
17        }
18        return false;
19    }
20    bool bfs() {
21        for (int i = 0; i < n; i++) dist[i] = n;
22        queue<int> q;
23        for (int i = 0; i < n; i++) if (ma[i] == -1) {
24            dist[i] = 0;
25            q.push(i);
26        }
27        bool rep = 0;
28        while (q.size()) {
29            int i = q.front(); q.pop();
30            for (int j : g[i]) {
31                if (mb[j] == -1) rep = 1;
```

```

31     else if (dist[mb[j]] > dist[i] + 1) {
32         dist[mb[j]] = dist[i] + 1;
33         q.push(mb[j]);
34     }
35 }
36 return rep;
37 }
38 int matching() {
39     int ret = 0;
40     for (auto& i : g) shuffle(ALL(i), rng);
41     while (bfs()) {
42         for (int i = 0; i < n; i++) nxt[i] = 0;
43         for (int i = 0; i < n; i++)
44             if (ma[i] == -1 and dfs(i)) ret++;
45     }
46     return ret;
47 }
48 vec<int> cover[2]; // if cover[i][j] = 1 -> node i, j is part of cover
49 int konig() {
50     cover[0].assign(n,1); // n left size
51     cover[1].assign(m,0); // m right size
52     auto go = [&](auto&& me, int u) -> void {
53         cover[0][u] = false;
54         for (auto v : g[u]) if (!cover[1][v]) {
55             cover[1][v] = true;
56             me(me, mb[v]);
57         }
58     };
59     L(u,0,n) if (ma[u] < 0) go(go,u);
60     return size;
61 }
62 };

```

4.4 Hungarian

```

1 using vi = vec<int>;
2 using vd = vec<lld>;
3 const lld INF = 1e100; // Para max asignacion, INF = 0, y negar costos
4 bool zero(lld x) {return fabs(x) < 1e-9;} // Para int/ll: return x==0;
5 vec<pii> ans; // Guarda las aristas usadas en el matching: [0..n)x[0..m]
6 struct Hungarian{
7     int n; vec<vd> cs; vi vL, vR;

```

```

8     Hungarian(int N, int M) : n(max(N,M)), cs(n, vd(n)), vL(n), vR(n){
9         L(x, 0, N) L(y, 0, M) cs[x][y] = INF;
10    }
11    void set(int x, int y, ld c) { cs[x][y] = c; }
12    ld assign(){
13        int mat = 0; vd ds(n), u(n), v(n); vi dad(n), sn(n);
14        L(i, 0, n) u[i] = *min_element(ALL(cs[i]));
15        L(j, 0, n){
16            v[j] = cs[0][j]-u[0];
17            L(i, 1, n) v[j] = min(v[j], cs[i][j] - u[i]);
18        }
19        vL = vR = vi(n, -1);
20        L(i, 0, n) L(j, 0, n) if(vR[j] == -1 and zero(cs[i][j] - u[i] - v[j]))
21            ){
22            vL[i] = j; vR[j] = i; mat++; break;
23        }
24        for(; mat < n; mat ++){
25            int s = 0, j = 0, i;
26            while(vL[s] != -1) s++;
27            fill(ALL(dad), -1); fill(ALL(sn), 0);
28            L(k, 0, n) ds[k] = cs[s][k]-u[s]-v[k];
29            while(true){
30                j = -1;
31                L(k, 0, n) if(!sn[k] and (j == -1 or ds[k] < ds[j])) j = k;
32                sn[j] = 1; i = vR[j];
33                if(i == -1) break;
34                L(k, 0, n) if(!sn[k]){
35                    auto new_ds = ds[j] + cs[i][k] - u[i]-v[k];
36                    if(ds[k] > new_ds) ds[k]=new_ds, dad[k]=j;
37                }
38                L(k, 0, n) if(k!=j and sn[k]){
39                    auto w = ds[k]-ds[j]; v[k] += w, u[vR[k]] -= w;
40                }
41                u[s] += ds[j];
42                while(dad[j] >= 0){ int d = dad[j]; vR[j] = vR[d]; vL[vR[j]] = j;
43                    j = d; }
44                vR[j] = s; vL[s] = j;
45            }
46            ld value = 0; L(i, 0, n) value += cs[i][vL[i]], ans.pb({i, vL[i]});
47        }
48    };

```

4.5 Flow - Dinics

```

1 const int oo = (int)1e9;
2 struct Dinic {
3     bool scaling = false; // com scaling -> O(nm log(MAXCAP)),
4     int lim;           // com constante alta
5     struct edge {
6         int to, cap, rev, flow;
7         bool res;
8         edge(int to_, int cap_, int rev_, bool res_)
9             : to(to_), cap(cap_), rev(rev_), flow(0), res(res_) {}
10    };
11    vec<vec<edge>> g;
12    vec<int> lev, beg;
13    ll F;
14    Dinic(int n) : g(n), F(0) {}
15    void add(int a, int b, int c) {
16        g[a].emplace_back(b, c, g[b].size(), false);
17        g[b].emplace_back(a, 0, g[a].size()-1, true);
18    }
19    bool bfs(int s, int t) {
20        lev = vector<int>(g.size(), -1); lev[s] = 0;
21        beg = vector<int>(g.size(), 0);
22        queue<int> q; q.push(s);
23        while (q.size()) {
24            int u = q.front(); q.pop();
25            for (auto& i : g[u]) {
26                if (lev[i.to] != -1 or (i.flow == i.cap)) continue;
27                if (scaling and i.cap - i.flow < lim) continue;
28                lev[i.to] = lev[u] + 1;
29                q.push(i.to);
30            }
31        }
32        return lev[t] != -1;
33    }
34    int dfs(int v, int s, int f = oo) {
35        if (!f or v == s) return f;
36        for (int& i = beg[v]; i < g[v].size(); i++) {
37            auto& e = g[v][i];
38            if (lev[e.to] != lev[v] + 1) continue;
39            int foi = dfs(e.to, s, min(f, e.cap - e.flow));
40            if (!foi) continue;
41            e.flow += foi, g[e.to][e.rev].flow -= foi;

```

```

42        return foi;
43    }
44    return 0;
45}
46 ll max_flow(int s, int t) {
47    for (lim = scaling ? (1<<30) : 1; lim; lim /= 2)
48        while (bfs(s, t)) while (int ff = dfs(s, t)) F += ff;
49    return F;
50}
51};
52 vec<pair<int, int>> get_cut(Dinic& g, int s, int t) {
53    g.max_flow(s, t);
54    vec<pair<int, int>> cut;
55    vec<int> vis(g.g.size(), 0), st = {s};
56    vis[s] = 1;
57    while (st.size()) {
58        int u = st.back(); st.pop_back();
59        for (auto e : g.g[u]) if (!vis[e.to] and e.flow < e.cap)
60            vis[e.to] = 1, st.push_back(e.to);
61    }
62    for (int i = 0; i < g.g.size(); i++) for (auto e : g.g[i])
63        if (vis[i] and !vis[e.to] and !e.res) cut.emplace_back(i, e.to);
64    return cut;
65}

```

4.6 Flow - MinCostMaxFlow

```

1 // O(nm + f * m log n)
2 // const ll oo = (ll)1e18;
3 template<typename T> struct mcmf {
4     struct edge {
5         int to, rev, flow, cap; // para, id da reversa, fluxo, capacidade
6         bool res; // se eh reversa
7         T cost; // custo da unidade de fluxo
8         edge() : to(0), rev(0), flow(0), cap(0), cost(0), res(false) {}
9         edge(int to_, int rev_, int flow_, int cap_, T cost_, bool res_)
10            : to(to_), rev(rev_), flow(flow_), cap(cap_), res(res_), cost(cost_) {}
11    };
12    vec<vec<edge>> g;
13    vec<int> par_idx, par;
14    T inf;
15    vec<T> dist;

```

```

16     mcmf(int n) : g(n), par_idx(n), par(n), inf(numeric_limits<T>::max()
17         /3) {}
18
19     void add(int u, int v, int w, T cost) { // de u pra v com cap w e
20         custo cost
21         edge a = edge(v, g[v].size(), 0, w, cost, false);
22         edge b = edge(u, g[u].size(), 0, 0, -cost, true);
23         g[u].push_back(a);
24         g[v].push_back(b);
25     }
26
27     vec<T> spfa(int s) { // nao precisa se nao tiver custo negativo
28         deque<int> q;
29         vec<bool> is_inside(g.size(), 0);
30         dist = vec<T>(g.size(), inf);
31         dist[s] = 0;
32         q.push_back(s);
33         is_inside[s] = true;
34         while (!q.empty()) {
35             int v = q.front();
36             q.pop_front();
37             is_inside[v] = false;
38             for (int i = 0; i < g[v].size(); i++) {
39                 auto [to, rev, flow, cap, res, cost] = g[v][i];
40                 if (flow < cap and dist[v] + cost < dist[to]) {
41                     dist[to] = dist[v] + cost;
42
43                     if (is_inside[to]) continue;
44                     if (!q.empty() and dist[to] > dist[q.front()]) q.push_back(to)
45                         ;
46                     else q.push_front(to);
47                     is_inside[to] = true;
48                 }
49             }
50         }
51         return dist;
52     }
53
54     bool dijkstra(int s, int t, vec<T>& pot) {
55         priority_queue<pair<T, int>, vec<pair<T, int>>, greater<>> q;
56         dist = vec<T>(g.size(), inf);
57         dist[s] = 0;
58         q.emplace(0, s);
59         while (q.size()) {
60             auto [d, v] = q.top();
61             q.pop();
62
63             if (dist[v] < d) continue;
64             for (int i = 0; i < g[v].size(); i++) {
65                 auto [to, rev, flow, cap, res, cost] = g[v][i];
66                 cost += pot[v] - pot[to];
67                 if (flow < cap and dist[v] + cost < dist[to]) {
68                     dist[to] = dist[v] + cost;
69                     q.emplace(dist[to], to);
70                     par_idx[to] = i, par[to] = v;
71                 }
72             }
73         }
74         return dist[t] < inf;
75     }
76
77     pair<int, T> min_cost_flow(int s, int t, int flow = (int)1e9) {
78         vec<T> pot(g.size(), 0);
79         pot = spfa(s); // mudar algoritmo de caminho minimo aqui
80         int f = 0;
81         T ret = 0;
82         while (f < flow and dijkstra(s, t, pot)) {
83             for (int i = 0; i < g.size(); i++)
84                 if (dist[i] < inf) pot[i] += dist[i];
85             int mn_flow = flow - f, u = t;
86             while (u != s){
87                 mn_flow = min(mn_flow,
88                               g[par[u]][par_idx[u]].cap - g[par[u]][par_idx[u]].flow);
89                 u = par[u];
90             }
91             ret += pot[t] * mn_flow;
92             u = t;
93             while (u != s) {
94                 g[par[u]][par_idx[u]].flow += mn_flow;
95                 g[u][g[par[u]][par_idx[u]].rev].flow -= mn_flow;
96                 u = par[u];
97             }
98             f += mn_flow;
99         }
100        return make_pair(f, ret);
101    }
102
103    // Opcional: retorna as arestas originais por onde passa flow = cap
104    vec<pair<int,int>> recover() {
105        vec<pair<int,int>> used;
106        for (int i = 0; i < g.size(); i++) for (edge e : g[i])
107            if(e.flow == e.cap && !e.res) used.push_back({i, e.to});
108    }

```

```

99     return used;
100    }
101 };

```

4.7 2-Sat

```

1 struct TwoSat {
2     int n, v_n;
3     vec<bool> vis, assign;
4     vec<int> order, comp;
5     vec<vec<int>> g, g_t;
6     TwoSat(int n_): n(n_), v_n(2 * n_), vis(v_n) , assign(n_), comp(v_n
7         , - 1), g(v_n), g_t(v_n) {
8         order.reserve(v_n);
9     }
10    void add_disj(int a, bool na, int b, bool nb) { // negated_a,
11        negated_b
12        a = 2 * a ^ na;
13        b = 2 * b ^ nb;
14        int neg_a = a ^ 1;
15        int neg_b = b ^ 1;
16        g[neg_a].pb(b);
17        g[neg_b].pb(a);
18        g_t[a].pb(neg_b);
19        g_t[b].pb(neg_a);
20    }
21    void dfs1(int u){
22        vis[u] = 1;
23        for (int v: g[u]) if (!vis[v]) dfs1(v);
24        order.pb(u);
25    }
26    void dfs2(int u, int cc) {
27        comp[u] = cc;
28        for (int v: g_t[u]) if (comp[v] == -1) dfs2(v, cc);
29    }
30    bool solve() {
31        order.clear();
32        vis.assign(v_n, 0);
33        L(i, 0, v_n) if (!vis[i]) dfs1(i);
34        comp.assign(v_n, - 1);
35        int cc = 0;
36        L(i, 0, v_n) {
37            int v = order[v_n - 1 - i];

```

```

38                if (comp[v] == -1) dfs2(v, cc++);
39            }
40            assign.assign(n, false);
41            for (int i = 0;i < v_n; i += 2) {
42                if (comp[i] == comp[i+1]) return false;
43                assign[i / 2] = comp[i] > comp[i + 1];
44            }
45        }

```

4.8 Euler Tour

```

1 // Directed version (uncomment commented code for undirected)
2 struct edge {
3     int y;
4     // list<edge>::iterator rev;
5     edge(int y):y(y){}
6 };
7 list<edge> g[N];
8 void add_edge(int a, int b){
9     g[a].push_front(edge(b));//auto ia=g[a].begin();
10    // g[b].push_front(edge(a));auto ib=g[b].begin();
11    // ia->rev=ib;ib->rev=ia;
12 }
13 vec<int> p;
14 void go(int x){
15     while(g[x].size()){
16         int y=g[x].front().y;
17         //g[y].erase(g[x].front().rev);
18         g[x].pop_front();
19         go(y);
20     }
21     p.push_back(x);
22 }
23 vec<int> get_path(int x){ // get a path that begins in x
24     // check that a path exists from x before calling to get_path!
25     p.clear();go(x);reverse(p.begin(),p.end());
26     return p;
27 }

```

4.9 Directed MST/Arborescence

```

1 | struct RollbackUF { // Required

```

```

2   vi e; vector<pii> st;
3   RollbackUF<int> uf(n) : e(n, -1) {}
4   int size(int x) { return -e[find(x)]; }
5   int find(int x) { return e[x] < 0 ? x : find(e[x]); }
6   int time() { return sz(st); }
7   void rollback(int t) {
8     for (int i = time(); i --> t;) {
9       e[st[i].first] = st[i].second;
10      st.resize(t);
11    }
12    bool join(int a, int b) {
13      a = find(a), b = find(b);
14      if (a == b) return false;
15      if (e[a] > e[b]) swap(a, b);
16      st.push_back({a, e[a]});
17      st.push_back({b, e[b]});
18      e[a] += e[b]; e[b] = a;
19      return true;
20    }
21  };
22
23 struct Edge { int a, b; ll w; };
24 struct Node { /// lazy skew heap node
25   Edge key;
26   Node *l, *r;
27   ll delta;
28   void prop() {
29     key.w += delta;
30     if (l) l->delta += delta;
31     if (r) r->delta += delta;
32     delta = 0;
33   }
34   Edge top() { prop(); return key; }
35 };
36 Node *merge(Node *a, Node *b) {
37   if (!a || !b) return a ?: b;
38   a->prop(), b->prop();
39   if (a->key.w > b->key.w) swap(a, b);
40   swap(a->l, (a->r = merge(b, a->r)));
41   return a;
42 }
43 void pop(Node*& a) { a->prop(); a = merge(a->l, a->r); }
44 pair<ll, vi> dmst(int n, int r, vector<Edge>& g) {
45   RollbackUF uf(n);
46   vector<Node*> heap(n);
47   for (Edge e : g) heap[e.b] = merge(heap[e.b], new Node{e});
48   ll res = 0;
49   vi seen(n, -1), path(n), par(n);
50   seen[r] = r;
51   vector<Edge> Q(n), in(n, {-1,-1}), comp;
52   deque<tuple<int, int, vector<Edge>>> cycs;
53   rep(s,0,n) {
54     int u = s, qi = 0, w;
55     while (seen[u] < 0) {
56       if (!heap[u]) return {-1,{}};
57       Edge e = heap[u]->top();
58       heap[u]->delta -= e.w, pop(heap[u]);
59       Q[qi] = e, path[qi++] = u, seen[u] = s;
60       res += e.w, u = uf.find(e.a);
61       if (seen[u] == s) { /// found cycle, contract
62         Node* cyc = 0;
63         int end = qi, time = uf.time();
64         do cyc = merge(cyc, heap[w = path[--qi]]);
65         while (uf.join(u, w));
66         u = uf.find(u), heap[u] = cyc, seen[u] = -1;
67         cycs.push_front({u, time, {&Q[qi], &Q[end]}});
68       }
69     }
70     rep(i,0,qi) in[uf.find(Q[i].b)] = Q[i];
71   }
72   for (auto& [u,t,comp] : cycs) { // restore sol (optional)
73     uf.rollback(t);
74     Edge inEdge = in[u];
75     for (auto& e : comp) in[uf.find(e.b)] = e;
76     in[uf.find(inEdge.b)] = inEdge;
77   }
78   rep(i,0,n) par[i] = in[i].a;
79   return {res, par};
80 }

```

5 Trees

5.1 Heavy Light Decomposition

```

1 | int ans[N], par[N], depth[N], head[N], pos[N];
2 | vec<int> heavy(N, -1);

```

```

3 int t = 0;
4 vec<int> g[N];
5 int dfs(int u) {
6     int size = 1;
7     int max_size = 0;
8     for (int v: g[u]) if (v != par[u]) {
9         par[v] = u;
10        depth[v] = depth[u] + 1;
11        int cur_size = dfs(v);
12        size += cur_size;
13        if (cur_size > max_size) {
14            max_size = cur_size;
15            heavy[u] = v;
16        }
17    }
18    return size;
19}
20 void decompose(int u, int h){
21    head[u] = h;
22    pos[u] = t++;
23    if (heavy[u] != -1){ decompose(heavy[u], h); }
24    for (int v: G[u]) if (v != par[u] && v != heavy[u]) {
25        decompose(v, v);
26    }
27}
28 int query(int a, int b) {
29    int resp = -1;
30    for (; head[a] != head[b]; b = par[head[b]]){ // Subi todo el heavy
31        path y a su padre // Next
32        if (depth[head[a]] > depth[head[b]]) swap(a, b);
33        resp = max(resp, st.query(pos[head[b]], pos[b])); // pos[head[b]
34        ]] < pos[b]
35    }
36    if (depth[a] > depth[b]) swap(a, b); // Una vez misma path(head)
37    entonces es una query [a,b]
38    resp = max(resp, st.query(pos[a], pos[b]));
39    return resp;
40}
41 dfs(root);
42 decompose(root, root);

```

5.2 Centroid

```

1 int sz[N];
2 bool removed[N];
3 int getSize(int u, int p){
4     sz[u] = 1;
5     for(int v: G[u]) if (v != p && !removed[v]){
6         sz[u] += getSize(v, u);
7     }
8     return sz[u];
9 }
10 int centroid(int u, int p, int tz){
11     for (int v: g[u])
12         if (v != p && !removed[v] && sz[v] * 2 > tz) return centroid(v,
13             u, tz);
14     return u;
15 }
16 int build(int u){
17     int c = centroid(u, -1, getSize(u, -1));
18     removed[c] = 1;
19     for (int v: G[c]) if (!removed[v]) { build(v); }
20 }

```

5.3 LCA - Const Time

```

1 struct LCA {
2     vec<int> depth, in, euler;
3     vec<vec<int>> g, st;
4     int K, n;
5     inline int Min(int i, int j) {return depth[i] <= depth[j] ? i : j;}
6     void dfs(int u, int p) {
7         in[u] = SZ(euler);
8         euler.pb(u);
9         for (int v: g[u]) if (v != p){
10             depth[v] = depth[u] + 1;
11             dfs(v, u);
12             euler.pb(v);
13         }
14     }
15     LCA(int n_): depth(n_), g(vec<vec<int>>(n_)), K(0), n(n_), in(n_) {
16         euler.reserve(2 * n); }
17     void add_edge(int u, int v) {g[u].pb(v);}
18     void build(int root){
19         dfs(root, -1);

```

```

19     int ln = SZ(euler);
20     while((1<<K)<=ln)K++;
21     st = vec<vec<int>> (K, vec<int>(ln));
22     L(i,0,ln) st[0][i] = euler[i];
23     for (int i = 1; (1 << i) <= ln; i++) {
24         for (int j = 0; j + (1<<i) <= ln; j++) {
25             st[i][j] = Min(st[i-1][j], st[i-1][j + (1<<(i-1))]);
26         }
27     }
28 }
29 int get(int u, int v) {
30     int su = in[u];
31     int sv = in[v];
32     if (sv < su) swap(sv, su);
33     int bit = log2(sv - su + 1);
34     return Min(st[bit][su], st[bit][sv - (1<<bit) + 1]);
35 }
36 };

```

6 Dynamic Programming

6.1 LIS

```

1 int lis(vec<int> &a) {
2     int n = SZ(a);
3     vector<int> d(n+1, oo);
4     d[0] = -oo;
5     L(i,0,n){
6         int l = upper_bound(d.begin(), d.end(), a[i]) - d.begin();
7         if (d[l-1] < a[i] && a[i] < d[l]) d[l] = a[i];
8     }
9     int ans = 0;
10    for (int l = 0; l <= n; l++) if (d[l] < oo) ans = l;
11    return ans;
12 }

```

6.2 Divide and Conquer Opt

```

1 // dp[k][i] = min(dp[k-1][j] + cost(j,i)), con j <= i
2 // requiere monotonicidad: opt[k][i] <= opt[k][i+1]
3 // uso: definir cost(j,i) y llamar dnc_dp(n, K)
4 // base 1
5 auto calc = [&](int j, int l, int r, int optL, int optR, auto&& self) ->
    void {

```

```

6     if (l > r) return;
7     int m = (l + r) >> 1;
8     ll best = oo;
9     int opt = optL;
10    int hi = min(m, optR);
11    L(i, optL, hi + 1) {
12        ll v = dp[i - 1][j - 1] + cost[i][m];
13        if (v < best) {
14            best = v;
15            opt = i;
16        }
17    }
18    dp[m][j] = best;
19    self(j, l, m - 1, optL, opt, self);
20    self(j, m + 1, r, opt, optR, self);
21 }
22 L(j, 1, k + 1) {
23     calc(j, 1, n, 1, n, calc);
24 }

```

6.3 Knuth Opt

```

1 // dp[i][j] = min(dp[i][k] + dp[k+1][j] + cost(i,j))
2 // requiere: opt[i][j-1] <= opt[i][j] <= opt[i+1][j]
3 // uso: inicializar dp[i][i]=0 y opt[i][i]=i, definir cost(i,j)
4 vec<vec<ll>> dp(n + 1, vec<ll>(n + 1, oo));
5 vec<vec<ll>> opt(n + 1, vec<ll>(n + 1, oo));
6 L(s, 1, n) {
7     L(i, 0, n - s) {
8         ll j = i + s;
9         ll l = opt[i][j - 1];
10        ll r = opt[i + 1][j];
11        l = max(l, i);
12        r = min(r, j - 1);
13        if (l > r) swap(l, r);
14        L(k, l, r+1) {
15            ll val = dp[i][k] + dp[k + 1][j] + cost(i, j);
16            if (val < dp[i][j]) {
17                dp[i][j] = val;
18                opt[i][j] = k;
19            }
20        }
21    }

```

22 | }

7 Strings

7.1 Hashing

```

1 static constexpr ll ms[] = {1'000'000'007, 1'000'000'403};
2 static constexpr ll b = 500'000'000;
3 struct StrHash { // Hash polinomial con exponentes decrecientes.
4     vec<ll> hs[2], bs[2];
5     StrHash(string const& s) {
6         int n = SZ(s);
7         L(k, 0, 2) {
8             hs[k].resize(n+1), bs[k].resize(n+1, 1);
9             L(i, 0, n) {
10                 hs[k][i+1] = (hs[k][i] * b + s[i]) % ms[k];
11                 bs[k][i+1] = bs[k][i] * b % ms[k];
12             }
13         }
14     }
15     ll get(int idx, int len) const { // Hashes en 's[idx, idx+len)'.
16         ll h[2];
17         L(k, 0, 2) {
18             h[k] = hs[k][idx+len] - hs[k][idx] * bs[k][len] % ms[k];
19             if (h[k] < 0) h[k] += ms[k];
20         }
21         return (h[0] << 32) | h[1];
22     }
23 };

```

7.2 KMP

```

1 struct KMP {
2     string s; int n; vec<int> p; vec<vec<int>> dfa;
3     KMP(string &s_): s(s_), n(SZ(s_)), p(SZ(s_) + 1), dfa(SZ(s_)+1, vec<
4         int>(26)) {
5         L(i,1,n) p[i + 1] = nxt(p[i], s[i]); // Calculate phi
6     }
7     int nxt(int i, char c) {for (;i;i=p[i])if(i<n&&c==s[i])return i+1;
8         return s[0]==c;}
9     void build_dfa(){
10         dfa[0][s[0]-'a'] = 1; // WARN: check lower_case vs upper
11         L(i,1,n+1)L(c,0,26) // If complicated char set use map

```

```

10         if (i<n&&s[i]=='a'+c)dfa[i][c]=i+1;
11         else dfa[i][c]=dfa[p[i]][c]; // fallar en i e ir al c
12     }
13     int go(int v, char c){return dfa[v][c-'a'];}
14 };

```

7.3 Z-Function

```

1 vec<int> zfun(const string &w){
2     int n = SZ(w), l = 0, r = 0; vec<int> z(n);
3     z[0] = n;
4     L(i, 1, n) {
5         if (i <= r) {z[i] = min(r - i + 1, z[i - 1]);}
6         while (i + z[i] < n && w[z[i]] == w[i + z[i]]) {++z[i];}
7         if (i + z[i] - 1 > r) {l = i, r = i + z[i] - 1;}
8     }
9     return z;
10 }

```

7.4 Manacher

```

1 struct Manacher {
2     vec<int> p;
3     Manacher(string const& s) {
4         int n = SZ(s), m = 2*n+1, l = -1, r = 1;
5         vec<char> t(m); L(i, 0, n) t[2*i+1] = s[i];
6         p.resize(m); L(i, 1, m) {
7             if (i < r) p[i] = min(r-i, p[l+r-i]);
8             while (p[i] <= i && i < m-p[i] && t[i-p[i]] == t[i+p[i]]) ++p[i];
9             if (i+p[i] > r) l = i-p[i], r = i+p[i];
10        }
11    } // Retorna palindromos de la forma {comienzo, largo}.
12    pii at(int i) const {int k = p[i]-1; return pair{i/2-k/2, k};}
13    pii odd(int i) const {return at(2*i+1);} // Mayor centrado en s[i].
14    pii even(int i) const {return at(2*i);} // Mayor centrado en s[i-1,i].
15 };

```

7.5 Aho-Corasick

```

1 struct node {
2     int ch[26], next[26]; // Full DFA Transitions
3     int link = 0, minx = oo; // Suffix Link
4     vec<int> ixs; // Indices of patterns ending here
5     node() { memset(ch, -1, sizeof(ch));}

```

```

6  };
7  vec<node> t; vec<int> bfs_order; // easy traverse from short to longer
   words
8  void init_aho() {t.clear();t.pb(node());bfs_order.clear();}
9  void add_string(const string &s, const int ix) {
10     int v = 0;
11     for (char c_raw : s) {
12         int c = c_raw - 'a';
13         if (t[v].ch[c] == -1) {
14             t[v].ch[c] = SZ(t);
15             t.pb(node());
16         }
17         v = t[v].ch[c];
18     }
19     t[v].ixs.pb(ix);
20 }
21 void build_aho() {
22     bfs_order.pb(0); // Root is first
23     L(c,0,26){
24         if (t[0].ch[c] != -1) {
25             t[0].next[c] = t[0].ch[c];
26             bfs_order.pb(t[0].ch[c]);
27         } else t[0].next[c] = 0;
28     }
29     L(q, 1, SZ(bfs_order)){ // warn: 1 not 0!
30         int u = bfs_order[q];
31         L(c,0,26){
32             if (t[u].ch[c] != -1) {
33                 int v = t[u].ch[c];
34                 t[u].next[c] = v;
35                 t[v].link = t[t[u].link].next[c];
36                 bfs_order.pb(v);
37             } else t[u].next[c] = t[t[u].link].next[c];
38         }
39     }
40 }

```

7.6 Suffix-Array

```

1 #define RB(x) ((x) < n ? r[x] : 0)
2 void csort(vec<int>& sa, vec<int>& r, int k) {
3     int n = SZ(sa);
4     vec<int> f(max(255, n)), t(n);

```

```

5     L(i,0, n) ++f[RB(i+k)];
6     int sum = 0;
7     L(i,0, max(255, n)) f[i] = (sum += f[i]) - f[i];
8     L(i,0, n) t[f[RB(sa[i]+k)]++] = sa[i];
9     sa = t;
10 }
11 vec<int> compute_sa(string& s){ // O(n*log2(n))
12     int n = SZ(s) + 1, rank;
13     vec<int> sa(n), r(n), t(n);
14     iota(ALL(sa), 0);
15     L(i,0, n) r[i] = s[i];
16     for (int k = 1; k < n; k *= 2) {
17         csort(sa, r, k), csort(sa, r, 0);
18         t[sa[0]] = rank = 0;
19         L(i, 1, n) {
20             if(r[sa[i]] != r[sa[i-1]] || RB(sa[i]+k) != RB(sa[i-1]+k)) ++rank;
21             t[sa[i]] = rank;
22         }
23         r = t;
24         if (r[sa[n-1]] == n-1) break;
25     }
26     return sa; // sa[i] = i-th suffix of s in lexicographical order
27 }
28 vec<int> compute_lcp(string& s, vec<int>& sa){
29     int n = SZ(s) + 1, K = 0;
30     vec<int> lcp(n), plcp(n), phi(n);
31     phi[sa[0]] = -1;
32     L(i, 1, n) phi[sa[i]] = sa[i-1];
33     L(i,0,n) {
34         if (phi[i] < 0) { plcp[i] = 0; continue; }
35         while(s[i+K] == s[phi[i]+K]) ++K;
36         plcp[i] = K;
37         K = max(K - 1, 0);
38     }
39     L(i,0, n) lcp[i] = plcp[sa[i]];
40     return lcp; // lcp[i] = longest common prefix between sa[i-1] and sa[i]
41 }

```

7.7 Suffix-Automaton

```

1 struct state {int len,link;map<char,int> next;}; //clear next!!
2 state st[2 * N]; // Important 2 * n

```

```

3 int sz,last;
4 void sa_init(){
5     last=st[0].len=0;sz=1;
6     st[0].link=-1;
7 }
8 void sa_extend(char c){
9     int k=sz++,p;
10    st[k].len=st[last].len+1;
11    for(p=last;p!=-1&&!st[p].next.count(c);p=st[p].link)st[p].next[c]=k;
12    if(p==-1)st[k].link=0;
13    else {
14        int q=st[p].next[c];
15        if(st[p].len+1==st[q].len)st[k].link=q;
16        else {
17            int w=sz++; // Is clone should not be counted!!!
18            st[w].len=st[p].len+1;
19            st[w].next=st[q].next;st[w].link=st[q].link;
20            for(;p!=-1&&st[p].next[c]==q;p=st[p].link)st[p].next[c]=w;
21            st[q].link=st[k].link=w;
22        }
23    }
24    last=k;
25 }

```

8 Math

8.1 Euclidean Extended

```

1 ll extendedGCD(ll a, ll b, ll &x, ll &y) {
2     if (b == 0) {
3         x = 1;
4         y = 0;
5         return a;
6     }
7     ll x1, y1;
8     ll gcd = extendedGCD(b, a % b, x1, y1);
9     x = y1;
10    y = x1 - (a / b) * y1;
11    return gcd;
12 }
13
14 bool findSolutionWithConstraints(ll a, ll b, ll c, ll x_min, ll y_min,
15     ll &x, ll &y) {

```

```

15     ll g = extendedGCD(a, b, x, y);
16
17     if (c % g != 0) return false;
18
19     x *= c / g;
20     y *= c / g;
21
22     // Ajustamos las variables a/g y b/g para mover las soluciones
23     a /= g;
24     b /= g;
25
26     if (x < x_min) {
27         ll k = (x_min - x + b - 1) / b; // Redondeo hacia arriba
28         x += k * b;
29         y -= k * a;
30     } else if (x > x_min) {
31         ll k = (x - x_min) / b;
32         x -= k * b;
33         y += k * a;
34     }
35
36     if (y < y_min) {
37         ll k = (y_min - y + a - 1) / a; // Redondeo hacia arriba
38         x += k * b;
39         y -= k * a;
40     } else if (y > y_min) {
41         ll k = (y - y_min) / a;
42         x -= k * b;
43         y += k * a;
44     }
45
46     return x >= x_min && y >= y_min;
47 }

```

8.2 Euler Totient

```

1 vector<ll> compute_totients(ll n) {
2     vector<ll> phi(n + 1);
3     for (ll i = 0; i <= n; i++) phi[i] = i;
4     for (ll i = 2; i <= n; i++) {
5         if (phi[i] != i) continue;
6         for (ll j = i; j <= n; j += i)
7             phi[j] = phi[j] * (i - 1) / i;

```

```

8     }
9     return phi;
10    }

```

8.3 Josephus

```

1 ll josephus_iterative(ll n, ll k) {
2     ll result = 0;
3     for (ll i = 2; i <= n; ++i)
4         result = (result + k) % i;
5     return result;
6 }
7 ll josephus_recursive(ll n, ll k) {
8     if (n == 1) return 0;
9     return (josephus_recursive(n - 1, k) + k) % n;
10}
11 ll josephus_power_of_2(ll n) {
12     ll power = 1;
13     while (power <= n) power <<= 1;
14     power >>= 1;
15     return 2 * (n - power);
16 }

```

8.4 Möbius

```

1 vector<ll> compute_mobius(ll n) {
2     vector<ll> mu(n + 1, 1);
3     vector<bool> is_prime(n + 1, true);
4     for (ll i = 2; i <= n; i++) {
5         if (is_prime[i]) { // i es un primo
6             for (ll j = i; j <= n; j += i) {
7                 mu[j] *= -1; // Multiplicamos por -1 para cada primo
8                 is_prime[j] = false;
9             }
10            for (ll j = i * i; j <= n; j += i * i) {
11                mu[j] = 0; // Si tiene un cuadrado de un primo, se pone
12                    en 0
13            }
14        }
15        return mu;
16    }
17    ll mobius(ll x) {
18        ll count = 0;

```

```

19    for (ll i = 2; i * i <= x; i++) {
20        if (x % (i * i) == 0)
21            return 0;
22        if (x % i == 0) {
23            count++;
24            x /= i;
25        }
26    }
27    if (x > 1) count++;
28    return (count % 2 == 0) ? 1 : -1;
29 }

```

8.5 NTT

```

1 #include <bits/stdc++.h>
2 using namespace std;
3 using cd = complex<double>;
4 typedef long long ll;
5 const ll mod = 998244353;
6 const ll root = 31;
7 const ll root_1 = inverse(root, mod);
8 const ll root_pw = 1 << 23;
9
10 ll inverse(ll a, ll m) {
11     ll res = 1, exp = m - 2;
12     while (exp) {
13         if (exp % 2 == 1) res = (1LL * res * a) % m;
14         a = (1LL * a * a) % m;
15         exp /= 2;
16     }
17     return res;
18 }
19
20 void ntt(vector<ll> & a, bool invert) {
21     int n = a.size();
22
23     for (int i = 1, j = 0; i < n; i++) {
24         int bit = n >> 1;
25         for (; j & bit; bit >>= 1)
26             j ^= bit;
27         j ^= bit;
28
29         if (i < j)

```

```

30         swap(a[i], a[j]);
31     }
32
33     for (int len = 2; len <= n; len <= 1) {
34         int wlen = invert ? root_1 : root;
35         for (int i = len; i < root_pw; i <= 1)
36             wlen = (int)(1LL * wlen * wlen % mod);
37
38         for (int i = 0; i < n; i += len) {
39             int w = 1;
40             for (int j = 0; j < len / 2; j++) {
41                 int u = a[i+j], v = (int)(1LL * a[i+j+len/2] * w % mod);
42                 a[i+j] = u + v < mod ? u + v : u + v - mod;
43                 a[i+j+len/2] = u - v >= 0 ? u - v : u - v + mod;
44                 w = (int)(1LL * w * wlen % mod);
45             }
46         }
47     }
48
49     if (invert) {
50         int n_1 = inverse(n, mod);
51         for (auto & x : a)
52             x = (int)(1LL * x * n_1 % mod);
53     }
54 }
55
56 vector<ll> multiply(vector<ll> const &a, vector<ll> const &b) {
57     vector<ll> fa(a.begin(), a.end()), fb(b.begin(), b.end());
58     ll n = 1;
59     while (n < a.size() + b.size())
60         n <= 1;
61     fa.resize(n);
62     fb.resize(n);
63
64     ntt(fa, false);
65     ntt(fb, false);
66     for (ll i = 0; i < n; i++)
67         fa[i] = (fa[i] * fb[i]) % mod;
68     ntt(fa, true);
69
70     vector<ll> result(n);
71     for (ll i = 0; i < n; i++)
72         result[i] = fa[i];

```

```

73         return result;
74     }

```

8.6 FFT

```

1  typedef long long ll;
2  typedef complex<double> C;
3  typedef vector<double> vd;
4  typedef vector<ll> vll;
5  const double PI = acos(-1);
6
7  void fft(vector<C>& a) {
8      int n = a.size(), L = 31 - __builtin_clz(n);
9      static vector<C> R(2, 1);
10     static vector<C> rt(2, 1);
11     for (static int k = 2; k < n; k *= 2) {
12         R.resize(n); rt.resize(n);
13         auto x = polar(1.0, PI / k);
14         for (int i = k; i < 2 * k; i++)
15             rt[i] = R[i] = i & 1 ? R[i / 2] * x : R[i / 2];
16     }
17     vector<int> rev(n);
18     for (int i = 0; i < n; i++) rev[i] = (rev[i / 2] | (i & 1) << L) /
19         2;
20     for (int i = 0; i < n; i++) if (i < rev[i]) swap(a[i], a[rev[i]]);
21     for (int k = 1; k < n; k *= 2)
22         for (int i = 0; i < n; i += 2 * k) for (int j = 0; j < k; j++) {
23             auto x = (double*)&rt[j + k], y = (double*)&a[i + j + k];
24             C z(x[0] * y[0] - x[1] * y[1], x[0] * y[1] + x[1] * y[0]);
25             a[i + j + k] = a[i + j] - z;
26             a[i + j] += z;
27         }
28
29     vll multiply(const vll& a, const vll& b) {
30         if (a.empty() || b.empty()) return {};
31         vd fa(a.begin(), a.end()), fb(b.begin(), b.end());
32         int L = 32 - __builtin_clz(fa.size() + fb.size() - 1), n = 1 << L;
33         vector<C> in(n), out(n);
34
35         for (int i = 0; i < a.size(); i++) in[i] = C(fa[i], 0);
36         for (int i = 0; i < b.size(); i++) in[i].imag(fb[i]);
37

```

```

38     fft(in);
39     for (C& x : in) x *= x;
40     for (int i = 0; i < n; i++) out[i] = in[-i & (n - 1)] - conj(in[i]);
        // Corregido aqui
41     fft(out);
42
43     vll res(a.size() + b.size() - 1);
44     for (int i = 0; i < res.size(); i++) {
45         res[i] = llround(imag(out[i]) / (4 * n));
46     }
47     return res;
48 }
```

8.7 Rho

```

1  ll mul(ll a, ll b, ll mod) {
2      return (_int128)a * b % mod;
3  }
4
5  ll power(ll a, ll b, ll mod) {
6      ll res = 1;
7      while (b) {
8          if (b & 1) res = mul(res, a, mod);
9          a = mul(a, a, mod);
10         b >>= 1;
11     }
12     return res;
13 }
14
15 bool isPrime(ll n) {
16     if (n < 2) return false;
17     for (ll p : {2, 3, 5, 7, 11, 13, 17, 19, 23}) {
18         if (n % p == 0) return n == p;
19     }
20     ll d = n - 1, s = 0;
21     while ((d & 1) == 0) d >>= 1, ++s;
22     for (ll a : {2, 325, 9375, 28178, 450775, 9780504, 1795265022}) {
23         if (a % n == 0) continue;
24         ll x = power(a, d, n);
25         if (x == 1 || x == n - 1) continue;
26         bool ok = false;
27         for (int i = 0; i < s; ++i) {
28             x = mul(x, x, n);
```

```

29             if (x == n - 1) { ok = true; break; }
30         }
31         if (!ok) return false;
32     }
33     return true;
34 }
35
36 ll rho(ll n) {
37     if (n % 2 == 0) return 2;
38     while (true) {
39         ll c = rand() % (n - 1) + 1;
40         ll x = 2, y = 2, d = 1;
41         while (d == 1) {
42             x = (mul(x, x, n) + c) % n;
43             y = (mul(y, y, n) + c) % n;
44             y = (mul(y, y, n) + c) % n;
45             d = std::gcd((x > y ? x - y : y - x), n);
46         }
47         if (d != n) return d;
48     }
49 }
50
51 void fact(ll n, std::map<ll, int>& f) {
52     if (n == 1) return;
53     if (isPrime(n)) { f[n]++; return; }
54     ll d = rho(n);
55     if (d == n) {
56         f[n]++;
57         return;
58     }
59     fact(d, f);
60     fact(n / d, f);
61 }
```

8.8 Get Divisors

```

1  vector<ll> getDivisors(const map<ll, int>& f) {
2      vector<ll> divisors = { 1 };
3      for (auto [p, e] : f) {
4          vector<ll> next;
5          ll pe = 1;
6          for (int i = 0; i <= e; i++) {
7              for (ll d : divisors)next.pb(d * pe);
```

```

8     pe *= p;
9 }
10    divisors.swap(next);
11 }
12 sort(ALL(divisors));
13 return divisors;
14 }
```

8.9 Simpson

```

1 ld simpsonRule(function<ld(ld)> f, ld a, ld b, int n) {
2     // Asegurarse de que n sea par
3     if (n % 2 != 0) {
4         n++;
5     }
6     ld h = (b - a) / n;
7     ld s = f(a) + f(b);
8
9     // Suma de terminos interiores con los factores apropiados
10    for (int i = 1; i < n; i++) {
11        ld x = a + i * h;
12        s += (i % 2 == 1 ? 4.0L : 2.0L) * f(x);
13    }
14    // Multiplica por h/3
15    return (h / 3.0L) * s;
16 }
17 // Ejemplo: integrar la funcion x^2 entre 0 y 3
18 auto f = [&](ld x){ return x * x; };
19 ld a = 0.0L, b = 3.0L;
20 int n = 1000; // numero de subintervalos
21 ld resultado = simpsonRule(f, a, b, n);
```

8.10 Simplex

```

1 pair<ld, vec<ld>> simplex(vec<vec<ld>> A, vec<ld> b, vec<ld> c) {
2     const ld EPS = (ld)1e-9;
3     int n = SZ(b), m = SZ(c);
4
5     vec<int> X(m), Y(n);
6     L(j, 0, m) X[j] = j;
7     L(i, 0, n) Y[i] = m + i;
8
9     ld z = 0;
```

```

11    auto pivot = [&](int x, int y) {
12        swap(X[y], Y[x]);
13
14        ld inv = (ld)1 / A[x][y];
15        b[x] *= inv;
16        L(j, 0, m) if (j != y) A[x][j] *= inv;
17        A[x][y] = inv;
18
19        L(i, 0, n) if (i != x && fabsl(A[i][y]) > EPS) {
20            ld coef = A[i][y];
21            b[i] -= coef * b[x];
22            L(j, 0, m) if (j != y) A[i][j] -= coef * A[x][j];
23            A[i][y] = -coef * A[x][y];
24        }
25
26        z += c[y] * b[x];
27        L(j, 0, m) if (j != y) c[j] -= c[y] * A[x][j];
28        c[y] = -c[y] * A[x][y];
29    };
30
31    while (true) {
32        int x = -1, y = -1;
33        ld mn = -EPS;
34        L(i, 0, n) if (b[i] < mn) { mn = b[i]; x = i; }
35        if (x < 0) break;
36        L(j, 0, m) if (A[x][j] < -EPS) { y = j; break; }
37        if (y < 0) {
38            return { numeric_limits<ld>::quiet_NaN(), {} };
39        }
40        pivot(x, y);
41    }
42
43    while (true) {
44        int y = -1, x = -1;
45        ld mx = EPS;
46        L(j, 0, m) if (c[j] > mx) { mx = c[j]; y = j; }
47        if (y < 0) break;
48
49        ld best = numeric_limits<ld>::infinity();
50        L(i, 0, n) if (A[i][y] > EPS) {
51            ld val = b[i] / A[i][y];
52            if (val < best) { best = val; x = i; }
53        }
54    }
55}
```

```

54     if (x < 0) {
55         return { numeric_limits<ld>::infinity(), {} };
56     }
57     pivot(x, y);
58 }
59
60 vec<ld> sol(m, 0);
61 L(i, 0, n) if (Y[i] < m) sol[Y[i]] = b[i];
62 return { z, sol };
63 }
```

8.11 EXP MAT

```

1 vec<vec<ll>> expbinmat(vec<vec<ll>> mat, ll b) {
2
3     ll n = mat.size();
4
5     vec<vec<ll>> resp(n, vec<ll>(n, 0));
6     L(i, 0, n) {
7         resp[i][i] = 1;
8     }
9     while (b > 0) {
10        if (b % 2 == 1) {
11            mult(resp, mat);
12        }
13        mult(mat, mat);
14        b /= 2;
15    }
16    return resp;
17 }
18
19 // Ejemplo de uso: An = C1*An-1 + C2*An-2 + C3*An-3
20 // con A0 = B0, A1 = B1, A2 = B2
21
22 if (n == 0) cout << B0 << '\n';
23 else if (n == 1) cout << B1 << '\n';
24 else if (n == 2) cout << B2 << '\n';
25 else {
26     vec<vec<ll>> M = {
27         {C1, C2, C3},
28         {1, 0, 0},
29         {0, 1, 0}
30     }
```

```

31     };
32
33     M = expbinmat(M, n - 2);
34
35     ll resp = (B2 * M[0][0] + B1 * M[0][1] + B0 * M[0][2]) % MOD;
36
37     cout << resp << '\n';
38 }
```

8.12 GAUSS

```

1 const double EPS = 1e-9;
2 const int INF = 2; // it doesn't actually have to be infinity or a big
3 number
4
5 int gauss(vec < vec<double> > a, vec<double>& ans) {
6     int n = (int)a.size();
7     int m = (int)a[0].size() - 1;
8     vec<int> where(m, -1);
9     for (int col = 0, row = 0; col < m && row < n; ++col) {
10        int sel = row;
11        L(i, row, n)
12            if (abs(a[i][col]) > abs(a[sel][col]))
13                sel = i;
14            if (abs(a[sel][col]) < EPS)
15                continue;
16            L(i, col, m + 1)
17                swap(a[sel][i], a[row][i]);
18            where[col] = row;
19
20        L(i, 0, n)
21            if (i != row) {
22                double c = a[i][col] / a[row][col];
23                L(j, col, m + 1)
24                    a[i][j] -= a[row][j] * c;
25                }
26            ++row;
27        }
28        ans.assign(m, 0);
29        L(i, 0, m)
30            if (where[i] != -1)
31                ans[i] = a[where[i]][m] / a[where[i]][i];
32        L(i, 0, n) {
```

```

32     double sum = 0;
33     for (int j = 0; j < m; ++j)
34         sum += ans[j] * a[i][j];
35     if (abs(sum - a[i][m]) > EPS)
36         return 0;
37 }
38 L(i, 0, m)
39     if (where[i] == -1)
40         return INF;
41     return 1;
42 }
43
44 int gauss_mod2(vec<bitset<BS>> a, int n, int m, bitset<BS>& ans) {
45     const int INF = 2;
46     vec<int> where(m, -1);
47     int row = 0;
48     for (int col = 0; col < m && row < n; ++col) {
49         int sel = -1;
50         L(i, row, n) {
51             if (a[i][col]) { sel = (int)i; break; }
52         }
53         if (sel == -1) continue;
54
55         swap(a[sel], a[row]);
56         where[col] = row;
57
58         L(i, 0, n) {
59             if ((int)i != row && a[i][col]) a[i] ^= a[row];
60         }
61         ++row;
62     }
63     ans.reset();
64     int free_col = -1;
65     L(j, 0, m) {
66         if (where[j] == -1) { free_col = (int)j; break; }
67     }
68     if (free_col != -1) ans[free_col] = 1;
69     for (int col = 0; col < m; ++col) {
70         if (where[col] == -1) continue;
71         int r = where[col];
72         bool v = a[r][m];
73         L(j, 0, m) {
74             if (where[j] == -1 && a[r][j] && ans[j]) v ^= 1;
75         }
76         ans[col] = v;
77     }
78     L(i, 0, n) {
79         bool lhs = 0;
80         L(j, 0, m) {
81             if (ans[j] && a[i][j]) lhs ^= 1;
82         }
83         if (lhs != a[i][m]) return 0;
84     }
85     L(i, 0, m) if (where[i] == -1) return INF;
86     return 1;
87 }
88
89 int gauss_mod(vec<vec<ll>>& a, vec<ll>& ans) {
90     int n = (int)a.size();
91     int m = (int)a[0].size() - 1;
92     L(i, 0, n) L(j, 0, m + 1) {
93         a[i][j] %= MOD;
94         if (a[i][j] < 0) a[i][j] += MOD;
95     }
96     vec<int> where(m, -1);
97     for (int col = 0, row = 0; col < m && row < n; ++col) {
98         int sel = -1;
99         L(i, row, n) {
100             if (a[i][col] != 0) { sel = (int)i; break; }
101         }
102         if (sel == -1) continue;
103         swap(a[sel], a[row]);
104         where[col] = row;
105         ll inv_pivot = bPow(a[row][col], MOD - 2, MOD);
106         L(i, 0, n) if ((int)i != row && a[i][col] != 0) {
107             ll c = (a[i][col] * inv_pivot) % MOD;
108             L(j, col, m + 1) {
109                 ll sub = (a[row][j] * c) % MOD;
110                 a[i][j] = a[i][j] - sub;
111                 a[i][j] %= MOD;
112                 if (a[i][j] < 0) a[i][j] += MOD;
113             }
114         }
115         ++row;
116     }
117     ans.assign(m, 0);

```

```

118     L(i, 0, m) if (where[i] != -1) {
119         int r = where[i];
120         ll inv_diag = bPow(a[r][i], MOD - 2, MOD);
121         ans[i] = (a[r][m] * inv_diag) % MOD;
122     }
123     L(i, 0, n) {
124         ll sum = 0;
125         L(j, 0, m) {
126             sum = (sum + (ans[j] * a[i][j]) % MOD) % MOD;
127         }
128         if (sum != a[i][m]) return 0;
129     }
130     L(i, 0, m) if (where[i] == -1) return INF;
131     return 1;
132 }
```

9 Geometry

9.1 Convex Hull

```

1 typedef pair<ll, ll> Point;
2 ll cross_product(Point O, Point A, Point B) {
3     return (A.first - O.first) * (B.second - O.second) - (A.second - O.
4         second) * (B.first - O.first);
5 }
6 vector<Point> convex_hull(vector<Point>& points) {
7     sort(points.begin(), points.end());
8     points.erase(unique(points.begin(), points.end()), points.end());
9     vector<Point> hull;
10    // Parte inferior
11    for (const auto& p : points) {
12        while (hull.size() >= 2 && cross_product(hull[hull.size() - 2],
13            hull[hull.size() - 1], p) < 0)
14            hull.pop_back();
15        if (hull.empty() || hull.back() != p) {
16            hull.push_back(p);
17        }
18    }
19    // Parte superior
20    int t = hull.size() + 1;
21    for (int i = points.size() - 1; i >= 0; --i) {
22        while (hull.size() >= t && cross_product(hull[hull.size() - 2],
23            hull[hull.size() - 1], points[i]) < 0)
```

```

21         hull.pop_back();
22         if (hull.empty() || hull.back() != points[i]) {
23             hull.push_back(points[i]);
24         }
25     }
26     hull.pop_back();
27     return hull;
28 }
```

9.2 Operations

```

1 ll cross_product(pair<ll, ll> P1, pair<ll, ll> P2, pair<ll, ll> P3) {
2     ll x1 = P2.first - P1.first;
3     ll y1 = P2.second - P1.second;
4     ll x2 = P3.first - P1.first;
5     ll y2 = P3.second - P1.second;
6     return x1 * y2 - y1 * x2;
7 }
8 double distancia(pair<ll, ll> P1, pair<ll, ll> P2) {
9     return sqrt((P2.first - P1.first) * (P2.first - P1.first) +
10                 (P2.second - P1.second) * (P2.second - P1.second));
11 }
12 ll dot_product(pair<ll, ll> P1, pair<ll, ll> P2, pair<ll, ll> P3) {
13     ll x1 = P2.first - P1.first;
14     ll y1 = P2.second - P1.second;
15     ll x2 = P3.first - P1.first;
16     ll y2 = P3.second - P1.second;
17     return x1 * x2 + y1 * y2;
18 }
```

9.3 Polygon Area

```

1 typedef pair<ll, ll> Point;
2 double polygon_area(const vector<Point>& polygon) {
3     ll area = 0;
4     int n = polygon.size();
5     for (int i = 0; i < n; ++i) {
6         ll j = (i + 1) % n;
7         area += (polygon[i].first * polygon[j].second - polygon[i].
8             second * polygon[j].first);
9     }
10    return abs(area) / 2.0;
11 }
```

9.4 Ray Casting

```

1 int inPolygon(const vector<pt>& p, pt a) { // 0: Outside, 1: Inside, 2:
2     Boundary
3     int ans = 0; int n = SZ(p);
4     L(i,0,n) {
5         pt p1 = p[i], p2 = p[(i + 1) % n];
6         if ((p2 - p1) % (a - p1) == 0 &&
7             min(p1.x, p2.x) <= a.x && a.x <= max(p1.x, p2.x) &&
8             min(p1.y, p2.y) <= a.y && a.y <= max(p1.y, p2.y)) return 2;
9         if ((p1.y > a.y) != (p2.y > a.y)) {
10            ll cp = (p2 - p1) % (a - p1);
11            if (p1.y < p2.y ? cp > 0 : cp < 0) ans = 1 - ans;
12        }
13    }
14    return ans;
}

```

10 Other

10.1 Mo's algorithm

```

1 const int BLOCK_SIZE = 450; using U64 = uint64_t;
2 struct query {int l, r, id;U64 order;};
3 U64 hilbertorder(U64 x, U64 y) {
4     const U64 logn = __lg(max(x, y) * 2 + 1) | 1;
5     const U64 maxn = (1ull << logn) - 1;
6     U64 res = 0;
7     for (U64 s = 1ull << (logn - 1); s; s >>= 1) {
8         bool rx = x & s, ry = y & s;
9         res = (res << 2) | (rx ? ry ? 2 : 1 : ry ? 3 : 0);
10        if (!rx) {
11            if (ry) x ^= maxn, y ^= maxn;
12            swap(x, y);
13        }
14    }
15    return res;
16 } // sort by this order
17 auto add = [&](int ix) { /* Add A[ix] to state*/};
18 auto rem = [&](int ix) { /* Remove A[ix] from state*/};
19 int c_l = 0, c_r = -1; // Cursors [0,-1] so r add 0 on first q
20 for(const auto &qr: qs){
21     while(c_l > qr.l) add(--c_l);

```

```

22     while(c_r < qr.r) add(++c_r);
23     while(c_l < qr.l) rem(c_l++);
24     while (c_r > qr.r) rem(c_r--);
25     ans[qr.id] = /*State.Answer()*/;
26 }

```

11 Ecuations

11.1 Combinatorics

$$\binom{n}{k} = \binom{n}{n-k}$$

$$\binom{n}{k} = \frac{n}{k} \binom{n-1}{k-1}, \quad (1 \leq k \leq n)$$

$$\sum_{k=0}^n \binom{n}{k} = 2^n$$

$$\sum_{m=0}^n \binom{m}{k} = \binom{n+1}{k+1}, \quad (n \geq k \geq 0)$$

$$\sum_{k=0}^m \binom{n+k}{k} = \binom{n+m+1}{m}$$

$$\binom{n}{0}^2 + \binom{n}{1}^2 + \cdots + \binom{n}{n}^2 = \binom{2n}{n}$$

$$\sum_{k=1}^n k \binom{n}{k} = n2^{n-1}$$

$$F_n = \sum_{k=0}^{\lfloor \frac{n-1}{2} \rfloor} \binom{n-k-1}{k}$$

$$F_{n+1} = \sum_{k=0}^{\lfloor \frac{n}{2} \rfloor} \binom{n-k}{k}$$

11.2 Catalan Numbers

Recursive definition:

$$C_0 = C_1 = 1$$

$$C_n = \sum_{k=0}^{n-1} C_k C_{n-1-k}, \quad n \geq 2$$

Closed form:

$$C_n = \frac{1}{n+1} \binom{2n}{n}$$

Combinatorial equivalent:

$$C_n = \binom{2n}{n} - \binom{2n}{n-1} = \frac{1}{n+1} \binom{2n}{n}, \quad n \geq 0$$

Combinatorial meaning:

Number of ways to: (i) arrange n balanced parenthesis pairs; (ii) full binary trees with $n+1$ leaves;
 (iii) Dyck paths of length $2n$ that never cross the diagonal.

Generalized form (k):

$$C_n^{(k)} = \frac{k+1}{n+k+1} \binom{2n+k}{n}$$

Extended recurrence:

$$C_n^{(k)} = \sum_{a_1+\dots+a_k=n} C_{a_1} C_{a_2} \cdots C_{a_k}, \quad C_0 = 1$$

Efficient recurrence (for computation):

$$C_n = \frac{2(2n-1)}{n+1} C_{n-1}, \quad n \geq 1$$

Generating function:

$$C(x) = \sum_{n=0}^{\infty} C_n x^n = \frac{1 - \sqrt{1 - 4x}}{2x}$$

Asymptotic behavior:

$$C_n \sim \frac{4^n}{n^{3/2} \sqrt{\pi}}$$

Examples:

$$C_0 = 1, C_1 = 1, C_2 = 2, C_3 = 5, C_4 = 14, C_5 = 42$$