Cognitive Skills learned through AdvGeo Tutor

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This document shows instructional objectives of AdvGeo tutor to discuss a topic of interests in AdvGeo tutor project. We then mention methods to assess students' achievement in learning proof writing. Finally, we discuss research questions for the dissertation study.

1. Target Cognitive Skills

AdvGeo tutor provides scaffolding on the proof steps along with the elaborated forward and backward procedures. These procedures have several cognitive skills required to complete geometry proofs as shown in Table 1. It is those cognitive skills that the students are supposed to learn through AdvGeo tutor.

Table 1: Cognitive Skills taught in AdvGeo Tutor

Task general	**Content Specific**
Write a proof	Transform postulate into operational rules
Make a backward inference	Identify consequence of the rule
Select a goal	Identify premises of the rule
Recognize goals that have yet to prove	
Select a goal to prove next	
Apply a rule	
Select a rule	
Instantiate consequence	
Unify the consequence with the problem	
Execute the rule	
Instantiate premises	
Unify the premises with the problem	
Check goal iteration	
Assert the premises as subgoals	
Make a forward inference	
…	

We expect that acquisition of cognitive skills fits in the power curve of learning when the students have enough chance to practice these skills (at least 8?). More interestingly, we may observe students learning to learn, namely, *the performance on the first use of a rule would improve* (*or, the reaction time decrease, if you will*) as learning proceeds (Anderson, Bellezza, & Boyle, 1993). With our elaborated cognitive model, this phenomenon can be explained as follows. The target skills can be divided into two classes: the task general skills and the content specific skills. As learning progress, students practice on different rules on different problems. Even though practices on different rules look different from each other, practices on the task general skills actually do not differ. Hence it might be those task general skills that even give familiarity to an application of a novel rule.

2. Topics of Interest

The primary interest is to see if providing finer scaffolding upon the elaborated cognitive model enhances performance for proof-writing. Hence we would build two versions of tutors: a *full* tutor fully provides finer scaffolding upon the fully elaborated cognitive model, whereas a *restricted* tutor suppresses scaffolding that is not a direct sub-step of backward and forward inference. So, the restricted tutor only provides scaffolding on select-a-goal and apply-a-rule, the only two immediate sub-skills for backward inference. The restricted tutor roughly corresponds to the model tracing tutors with coarser cognitive model that only realizes postulate application as a single production rule (i.e., GPT).

> Koedinger conducted empirical evaluation for Angle (1990) that is claimed to have a finer cognitive model (diagrammatic configuration model) than GPT. He showed that Angle is as good as Anderson's GPT for the students with low scores on the pre-test. Angle is known to be less effective than GPT for the students with high scores on the pre-test. Since Koedinger only provides the ratio of the correct answers to the incorrect ones, we will not compare Angle and AdvGeo, but we would still be able to feel a flavor of effectiveness in AdvGeo tutor against Angle.
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> *Memo: GTP experiment showed that it is as effective as one grade level higher (or one standard deviation) than classroom instruction. In the best case, GTP group reached the same level of mastery as classroom instruction in one third of learning time. Regression analysis showed that the final performance = 35 + 7.5*(letter grade in preceding algebra class) + [14: if monopolized a tutor, or 4: if two shared a tutor]*

It is preferable to have a control condition that learn proof writing without using ITS. However, it is not plausible to spare a time for students to learn proof writing as a regular classroom activity. Furthermore, it would not be great surprise to know that the AdvGeo group shows superior performance to the non-tutored group (namely, students would do nothing but taking pre and post-test). Hence instead of conducting a comparison against a classroom instruction, we compare the performance of AdvGeo group to empirical results in the literature.

For a classroom instruction, (Senk, 1985) conducted a large scale evaluation on high school students' geometry proof-writing skills. According to her study, after a full-year geometry course, 25% (N=1520) of the students have virtually no competence in writing proofs, 25% can do only trivial proofs, 20% can do some proofs of greater complexity, and 30% master proofs similar to the theorems in textbook. We can use this result as a standard of effectiveness in the ordinal classroom instruction.

As a predictor of students' achievement on proof writing, (Senk, 1989) shows that the performance on van Hiele test has a high correlation with the performance on geometry proof writing. We can also use this result as a standard on the amount of achievement in proof writing within the ordinal classroom instruction.

The third topic of interest is on any kind of interaction between the competence level before the tutoring session (i.e., VHL and the score on pretest) and the improvement (i.e, the difference between pre and post-test scores) or achievement (i.e., the post-test score) in proof writing.

3. Assessment

Prior to the tutoring session, students' prior knowledge is assessed with the following tests:

Van Hiele Geometry Test: 25 items on 5 sub-skills (Usiskin, 1982): This test is known as a good predictor for student's performance on proof writing at the end of geometry course.

Koedinger's proof test (Koedinger, 1990): 4 proof problems $\times$ 2 sets. Set A and B are used for both pre- and post-test to make a counter balance among the groups.

During the tutoring session, the tutor measures *time* to complete each step and *accuracy* of the step. The time to complete a step is the duration from the point when the step is available to the point when it is done. When a step can be further divided into substeps, the time to complete the step is an aggregate of the time to complete all the substeps. The accuracy and the response time on individual rule are used to control the amount of fading scaffolding away. They would be also plotted to see if they fit in the power curve of learning.

After the tutoring session, the target cognitive skills would be tested in the post-test. Table 2 shows some examples of the test items for sub-skills in backward inference. The scores on these tests must have high correlation with the accuracy of the corresponding steps measured at the last time of the tutoring session (validity of the test).

Table 2: Examples of test items to assess individual sub-skills

Target sub-skill	**Test Item**
Write-a-proof	Koedinger's proof test
Make-a-backward-inference	Backward-inference test that shows a proof that is done half way through all by backward inference and asks to make another back ward inference. It does not matter if the inference made is on a correct path or not.
Apply-a-rule	Rule application test that asks what rule can be applied to achieve a certain goal and what subgoals should be further proved
Select-a-rule	Rule selection test that asks what rule can be applied to achieve a certain goal
Execute-a-rule	Rule execution test that asks what subgoals should be stated to prove for an application of a certain rule for a certain goal.
Transform-postulate	Rule transformation test that asks to complete "procedural rule" in the form of "According to the postulate X, to prove Y, it is necessary to prove _______" where the students fill up the blank.
Identify-consequence	Consequence identification test that asks the consequence of a particular rule
Identify-premises	Premise identification test that asks the premises of a particular rule

4. Research Questions

Providing scaffolding along with the elaborated proof procedure enhances students' performance on proof writing compare to the normal classroom instruction or a model tracing tutor with coarser cognitive model that only provides problem solving model at coarser grain size. In other words, we claim that compare to other tutors, AdvGeo tutor leads the student to higher score on proof-writing test within a given amount of time.

If a student fails to achieve satisfactory proof writing level (i.e., performance on write-a-proof skill does not meet a criterion), there would be one of following cases:

> He/she has achieved mastery on only some, but not all, sub-skills. These are the sub-skills that he/she can not achieve by a repetitive practice provided by AdvGeo tutor. Complementing such sub-skills by other learning method may or may not lead he/she to mastery of proof writing.
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> He/she has achieved mastery on all sub-skill. If this is the case, then the elaborated proof procedure does not actually realize all the sub-skills needed to achieve mastery of proof writing.

In sum, we investigate following research questions:

- Does providing scaffolding on the elaborated proof steps that fades as learning proceeds enhance performance on proof-writing?
- If so, is such tutor better than a model tracing tutor that provides scaffolding upon coarser cognitive model?
- If such scaffolding is not effective for proof-writing, then do students still reach mastery on all the sub-skills involved in the elaborated cognitive model?
- If there are sub-skills that are not acquired through AdvGeo tutor, what are they?

Reference:

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