Structural Estimation of Capital Adjustment Costs *

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Abstract

This paper structurally estimates the adjustment cost parameters in firm's investment decision. We adopt convex and non-convex models, and the simulated method of moments is used for our estimation to capture the notable characteristics of investment rate distributions (serial correlation and fat tails). We find that the hybrid model incorporating both convex and non-convex adjustment costs fits the data best.

keywords: capital adjustment cost, investment, convex cost, and profitability.

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*All the estimation is conducted using Python. For the detail of the code, please see https://github.com/NobuyukiFuruta/StructEst_W18/blob/fa471ff39e0a4e60d4d447c9a8f475dd19b8a248/Project/Project.ipynb.

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1 Introduction

This paper studies the nature of capital adjustment costs. Adjustment costs heavily affect firms' decision-making in investment, which is a crucial yet volatile part of the aggregate economy and worth more throughly studied. Thus, the investigation of adjustment costs and investment decisions leads us to examine aggregate behavior of firms and policy suggestions.

Costs associated with adjusting the stock of capital would shed some light on the dynamics of investment not only due to their close connection, but also because these two components of firms' expenditure involve various interrelated factors that are difficult to measure directly or precisely. For example, changing the stock of capital could incur costs during installation of new and replacement of old capital. In addition, the installation of a new machine in a production line could further cause costs (in terms of time and financial) in order for the employees to learn and to be familiar with its use. The reorganization of human resources and the irreversibility of ongoing projects are other examples that demonstrate the necessity for firms to take adjustment costs into consideration.

In the main part of our following analysis of adjustment costs, we adopt the convex cost model. We choose this model because of time limitation and the fact that it is more tractable than non-convex specifications. Our specification follows from the standard neoclassical convex cost model. Previous literature focuses on the industry specific adjustment cost (e.g., Holt et al. (1960), Peck (1974), Ito et al. (1990), and suggests that several forms of adjustment cost models should be considered to encompass various aspects of firm's investment process. Nevertheless, Holt at al. (1960) found a quadratic specification of adjustment costs to be a good approximation of hiring and layoff costs, overtime costs, inventory costs and machine setup costs in selected manufacturing industries. Hence, we expect the convex component of adjustment costs to account for the larger part. Without any adjustment cost, investment will immediately respond to shocks and is negatively correlated. Adding convex capital adjustment cost will temper the response, introduce some positive correlation, and provide some incentive for firms to demonstrate the sticky capital stock.

Alternatively, in the study of Cooper, Haltiwanger and Power (1999) and Caballero, Engel and Haltiwanger (1995), non-convexities and irreversibilities are essential factors in the investment process. Their study suggests that investment of bursts and periods of inaction are two key features of firm's investment decision and provides evidence that they are non-linearly related to investment.

These studies leave several questions to be answered. Our study is to answer 1. what is the nature of capital adjustment costs? 2. what are the structural estimates of these costs. In the subsequent analysis, we follow the method taken by Cooper and Haltiwanger (2005). We use simulated method of moments (SMM) and attempt to match our models with the key features of investment rate data including the serial correlation of investment rate, the correlation between profitability shock and investment rate, and both positive and negative spike rates. Our model is, in turn, specified with convex capital adjustment cost (quadratic form), non-convex capital adjustment cost (fixed cost), and the price of investment. We find that our convex cost model captures the concentration feature of the investment rate data. However, negative investment rates are rarely observed in our simulated data through the convex model. The hybrid model including both of the convex and non-convex components records the best result among all of the three models used in our analysis, capturing the negative spike rate and the correlation between investment and profitability shocks better than the other models. In what follows, we provide a more meticulous explanation of our models, data moments, and the estimation outcomes.

2 Model

A general specification of the plant-level adjustment cost model is assumed to have both components of convex and non-convex components. Therefore, we consider variations of the following stationary dynamic programming problem:

$$V(A,K) = \max_{I} \Pi(A,K) - C(I,A,K) - p(I)I + \beta \mathbb{E}_{A'|A}[V(A',K')] \quad \forall (A,K) \tag{1}$$

where $\Pi(A,K)$ represents the profits attained by a plant with capital level K, a profitability shock given by A, I is the level of investment and $K' = K(1-\delta) + I$. $p(I)$ denotes the price of investment. Here, the primed values denote future values. In this problem, the firm manager chooses the level of investment, I, to maximize the firm value. The cost of adjustment is encompassed in the specification $C(I,A,K)$.

We assume that firms maximize their profits through the variable input, labor denoted by L. Therefore,

$$\Pi(A,K) = \max_{L} R(\hat{A},K,L) - Lw(L) \tag{2}$$

where $R(\hat{A},K,L)$ denotes the revenues given capital K, labor L, and a shock to revenues $\hat{A}$. w is the wage rate of labor.Once we specify a revenue function, we can use this optimization problem to determine L and to derive the profit function $\Pi(A,K)$, where A reflects both the shocks to the revenue function and variations in costs of L. Throughout the analysis, the plant level profit function is specified as

$$\Pi(A,K) = AK^{\theta} \tag{3}$$

where θ is a parameter.

The rest of this section provides two specifications of the model, convex and non-convex adjustment cost model.

2.1 Convex Adjustment Cost Model

In this specification, we assume the convex adjustment cost to have a quadratic form and consider the following function,

$$C(I,A,K) = \frac{\gamma}{2}(\frac{I}{K})^2 K \tag{4}$$

where γ is a parameter. The first-order condition of the equation (1) relates the investment rate to the derivative of the value function with respect to capital and the cost of capital, p. That is, the solution to (1) implies

$$i = \frac{I}{K} = \frac{\beta \mathbb{E}[V_K(A', K')] - p}{\gamma} \tag{5}$$

where i is the investment rate, and $\mathbb{E}[V_K(A', K')]$ is the expectation of the partial derivative of the value function with respect to K in the subsequent period. This condition allows investment to respond to predictable variations in profitability.

2.2 Non-Convex Adjustment Cost Model

Non-convex components of adjustment costs are considered to capture indivisibilities in capital, increasing returns to the installation of new capital and increasing returns to retraining and restructuring of production activity. This formulation specifies the dynamic programming problem as:

$$V(A, K) = \max\{V^i(A, K), V^a(A, K)\} \quad \forall (A, K) \tag{6}$$

where the superscripts refer to the active investment "a" and inactivity "i". These options are respectively defined by:

$$V^i(A, K) = \Pi(A, K) + \beta \mathbb{E}_{A'|A}[V(A', K(1 - \delta))] \tag{7}$$

and

$$V^a(A, K) = \max_I \Pi(A, K)\lambda - FK - pI + \beta \mathbb{E}_{A'|A}[V(A', K(1 - \delta))] \tag{8}$$

The second optimization problem includes two types of fixed costs of adjustment. The first adjustment cost $\lambda < 1$ represents an opportunity cost of investment. If there is any capital adjustment, then plant productivity falls by a factor of $(1 - \lambda)$ during the adjustment period.

The second adjustment cost is a fixed cost denoted by F, which is independent of the level of activity at the plant. It is proportional to the level of capital at the plant to eliminate any size effects.

Moreover, we consider the possibility that there is a gap between the buying and selling price of capital. This is incorporated in the model by assuming $p(I) = p_b$ if $I > 0$ and $p(I) = p_s$ if $I < 0$ where $p_b \geq p_s$. This assumption enables us to consider the price of new and old capital, which creates a region of inaction. This formulation specifies the dynamic programming problem as:

$$V(A, K) = \max\{V^b(A, K), V^s(A, K), V^i(A, K)\} \quad \forall(A, K) \tag{9}$$

where the superscripts refer to the act of buying capital "b", selling capital "s", and inaction "i". These options are respectively defined by:

$$V^b(A, K) = \max_I \Pi(A, K) - p_b I + \beta \mathbb{E}_{A'|A}[V(A', K(1-\delta) + I)], \tag{10}$$

$$V^s(A, K) = \max_R \Pi(A, K) - p_s R + \beta \mathbb{E}_{A'|A}[V(A', K(1-\delta) - R)], \tag{11}$$

and

$$V^i(A, K) = \Pi(A, K) + \beta \mathbb{E}_{A'|A}[V(A', K(1-\delta))] \tag{12}$$

By this specification, we distinguish the purchase of new capital I and retirements of existing capital R.

3 Data

Cooper and Haltiwanger (2005) used a balanced panel from the Longitudinal Research Database (LRD) consisting of approximately 7,000 manufacturing plants that were continually in operation between 1972 and 1988. Although we also attempted to gain the firm-level microdata, we were unable to obtain the access. Therefore, as an alternative way, we use the data moments provided by Cooper and Haltiwanger (2005) for our estimation. In what follows, we focus on the two key moments related to the investment rate I_t/K_t.

3.1 Moments

Figure 1 reports the histogram of investment rates provided by Cooper and Haltiwanger (2005). A large part of the observations is concentrated around zero percent, and the distribution is highly skewed to the right.

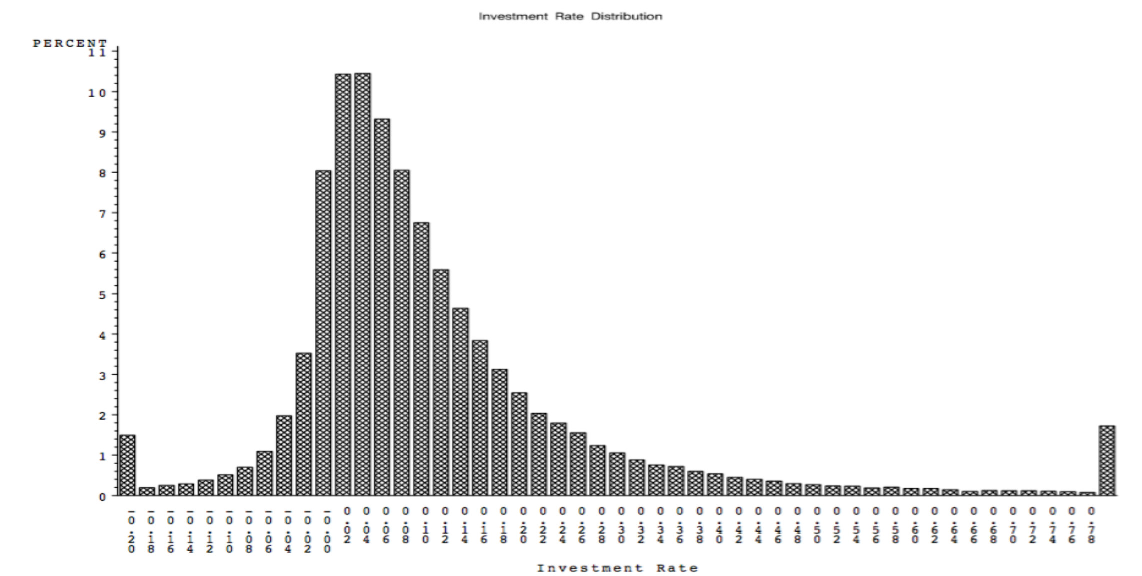

Figure 1: Investment Rate Distribution

Source: Cooper and Haltiwanger (2005)

A striking characteristic of this figure is two fat tails. The following analysis focuses on investment rates exceeding 20% and falling below -20%. In Figure 1, approximately 18% of the entire observations exceeds 20& investment rates, whereas 1.8% of observations record investment rates less than -20%. This asymmetry between positive and negative investment is a key feature of the data that our analysis seeks to match.

Table 1 reports the data moments used in our analysis. The first moment is the serial correlation in investment calculated as the correlation of the plant-level investment rate with the lagged investment rate. According to the past literature on adjustment costs such as Caballero and Engel (2003) and Cooper, Haltiwanger and Power (1999), the serial correlation of investment is sensitive to the structure of

Table 1: Data Moments

Moment	Description	Value
$corr(i, i_{-1})$	Serial correlation in investment	0.058
$corr(i, a)$	Correlation between investment and profit shocks	0.143
$spike^{+}$	Positive spike rates	0.186
$spike^{-}$	Negative spike rates	0.018

adjustment costs. The second moment is the correlation between investment rates and profit shocks. This correlation is positive, which indicates that investment rates are high in periods of high profitability. The last two moments are positive and negative spike rates of investment that are considered to capture the asymmetry and the right fat tail of the distribution.

4 Estimation

We consider the case in which the adjustment cost parameters are $F > 0$ and $\lambda = 1$. We assume that the dynamic programming problem for a plant is given by:

$$V(A, K) = \max\{V^b(A, K), V^s(A, K), V^i(A, K)\} \quad \forall (A, K) \tag{13}$$

We define these options as:

$$V^b(A, K) = \max_I \Pi(A, K) - FK - I - \frac{\gamma}{2}(\frac{I}{K})^2 K + \beta \mathbb{E}_{A'|A}[V(A', K(1-\delta) + I)], \tag{14}$$

$$V^s(A, K) = \max_R \Pi(A, K) - FK - p_s R - \frac{\gamma}{2}(\frac{I}{K})^2 K + \beta \mathbb{E}_{A'|A}[V(A', K(1-\delta) - R)], \tag{15}$$

and

$$V^i(A, K) = \Pi(A, K) + \beta \mathbb{E}_{A'|A}[V(A', K(1-\delta))]. \tag{16}$$

In this paper, we estimate γ and F using the specification $V^b(A, K)$ and do not take $V^s(A, K)$ and $V^i(A, K)$ into consideration. Nevertheless, we estimate three different cases. The first formulation involves only γ and set $F = 0$ to capture the goodness of fit of the convex adjustment cost model. Conversely, we set $\gamma = 0$ and estimate F

in the second specification. Lastly, we estimate both of γ and F and capture both of convex and non-convex components of adjustment costs. Throughout the analysis, we set $p = 1$ to avoid the price of capital offsetting the changes in our targeted parameters.

4.1 Estimation Method

We estimate models with convex, non-convex, and a hybrid version of capital adjustment cost to match important properties observed in the data. Two moments $corr(i, i_{-1})$ and $corr(i, a)$ directly target the structure of adjustment costs, while the other two moments (positive and negative spike rates of investment) are included to capture the the tails of the distribution. The set of parameters to be estimated is $\Theta = (\gamma, F)$, measuring the convex cost and the non-convex fixed cost respectively. More specifically, below is the estimation strategy of the models.

We discretized the values that K could take, and transformed the exponential AR(1) process of the productivity shock A into a Markov matrix to obtain the expectation of future value in the Bellman equation. Next, we rewrite each term on the right hand side as a matrix with three dimensions: K, K', and A. Each entry in the cubes is determined by a function of K, K', A. For arbitrary values of $\Theta = (\gamma, F)$, the Bellman equation of firms is solved by the value function iteration to generate the policy function $K = \psi(K, A)$. The data is simulated according to the Markov process of A, and the vector of the model moments $\Psi^s(\Theta)$ is generated from the simulated data. The estimate $\hat{\Theta}$ should minimize

$$\mathcal{L}(\Theta) = \min_{\Theta} [\Psi^d - \Psi^s(\Theta)]' W [\Psi^d - \Psi^s(\Theta)], \tag{17}$$

where W is the weighting matrix and Ψ^d denotes the vector of the data moments. This SMM procedure generates a consistent estimation of Θ.

4.2 Calibration

In order to estimate the parameters of convex and non-convex capital adjustment cost, other important parameters such as the curvature of the production function, the AR(1) specification value, the AR(1) standard error, the discount factor, and the capital depreciation rate are calibrated. We use the same values as in Cooper and Haltiwanger(2005). Table 2 summarizes the calibrated parameters and their values.

Table 2: Calibration

Parameter	Description	Value
θ	curvature of production function	0.592
ρ	AR(1) specification	0.085
σ	AR(1) standard error	0.298
β	discount factor	0.95
δ	capital depreciation rate	0.069

We let $a_{it} = ln(A_{it})$. Then, the distribution of A is assumed to follow the exponential AR(1) process. That is:

$$a_{it} = \rho a_{it-1} + (1-\rho)\bar{a} + \eta_{it} \quad \text{where} \quad \eta_{it} \sim N(0, \sigma^2) \tag{18}$$

Using the calibrated parameters and Equation (18), we create the sequence of a_{it}. The shock process not only reflects the nature of capital adjustment but also provides information for the firm level optimization problem through the expectation value. For example, investment bursts are directly associated with shocks on productivity; the persistence of such shocks may affect how capital stock level vary through time.

4.3 Estimation Results

4.3.1 Convex Adjustment Cost

Table 3 reports the estimation results from the model with the convex capital adjustment cost only. We can see that the 2-step estimation slightly improved the match with the data in a sense that the autocorrelation of investment rate is positive in

Table 3: Convex Adjustment Cost

W	γ	$corr(i, i_{-1})$	$corr(i, a)$	$spike^+$	$spike^-$	$\mathcal{L}(\hat{\Theta})$
I	1.456	-0.016	0.616	0.0048	0.00	14.504
W^{2step}	1.735	0.011	0.623	0.0045	0.00	0.943
Data Moment		0.058	0.143	0.186	0.018	

the estimation with W^{2step}. Figure 2 is the graph of the investment rate distribution generated from our simulated data. Our simulated investment rates are concentrated around 5%, which captures the concentration of the investment rate data in Figure 1. However, the 2-step estimation matches the spike rates poorly; especially the negative spike rate. The negative investment rates are rarely observed in Figure 2. Although the 2-step estimation provides a closer value for $corr(i, i_{-1})$, the estimation results for the rest of the moments are still far from satisfactory. This result is expected since the non-convex adjustment costs are not taken into consideration in this formulation. The following section reports the estimation outcomes of the non-convex adjustment cost specification.

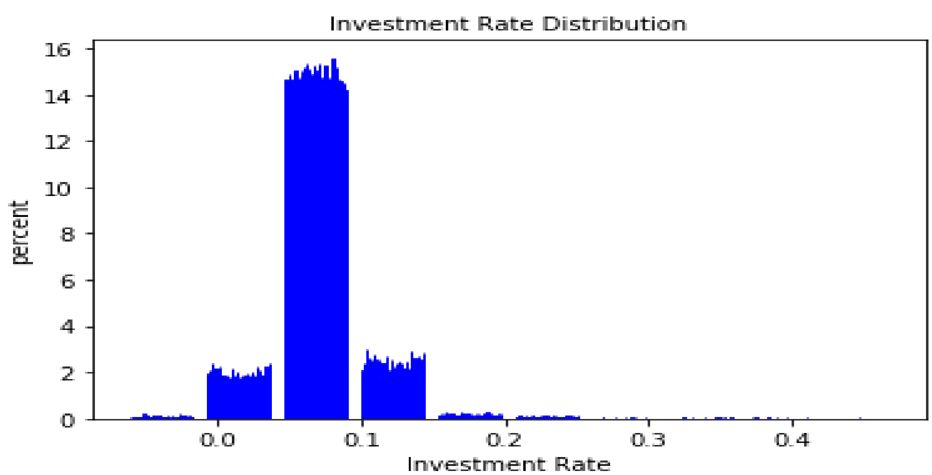

Figure 2: Simulated Model Investment Rate Distribution

4.3.2 Non-Convex Adjustment Cost

Since the neoclassical convex (quadratic) adjustment cost matches the data poorly, we do need to consider a different specification of adjustment costs. In this formulation, we consider the lump-sum fixed cost, which is proportional to the level of capital stock. We estimate two different specification $\Theta = (F)$ and $\Theta = (\gamma, F)$, using the same procedure described above. Table 4 summarizes the estimation outcomes including the first case examined in the previous section.

Table 4: Parameter Estimation

Spec.	W	Structural Parm. Est.		moments				
		γ	F	$corr(i, i_{-1})$	$corr(i, a)$	$spike^{+}$	$spike^{-}$	$\mathcal{L}(\hat{\Theta})$
data				0.058	0.143	0.186	0.018	
γ only	I	1.456	0	-0.016	0.616	0.0048	0.00	14.504
γ only	W^{2step}	1.735	0	0.011	0.623	0.0045	0.00	0.943
F only	I	0	0.218	-0.308	0.603	0.091	0.094	17.827
γ and F	I	0.035	0.046	-0.093	0.267	0.077	0.038	11.110

Table 4 shows that the extreme case of non-convex capital adjustment cost (F only) performs even worse judged from the values of the objective function. Nonetheless, we should note that the model with only non-convex adjustment cost does match the spikes better, indicating the investment bursts are more attributed to fixed costs rather than convex ones. It underestimates the positive and negative spike rates by smaller amounts and overshoots the correlation between the the profit shock and the investment rate. One reason why this specification records an even greater objective function value can be attributed to the estimate of the autocorrelation of investment rate, which is -0.308. This value is significantly far from our targeted value, 0.058. This result enables us to conclude that the convex component accounts for the larger part of the serial correlation of investment rates than the non-convex one, while non-convex cost better accounts for the spike rate.

Lastly, we estimate a hybrid model including both convex and non-convex adjustment costs. The outcome of this estimation supports our conjecture. The autocorrelation of investment rates is still negative, but is much smaller in absolute value. Even though the mixed model also overestimates the correlation between a and i, the range of the misspecification becomes smaller. This model does a great job in estimating $corr(i, a)$ and $spike^{-}$ and provides the minimum value of $\mathcal{L}(\hat{\Theta})$. This result implies that the coexistence of quadratic and fixed costs lower the correlation between profit shocks and investment rates. In general, the parameters estimated from the hybrid model are much smaller than those of the extreme models. This result is fairly close to that of Cooper and Haltiwanger (2005), despite the fact that we did not consider the irreversibility of investment, which is specified as the ratio between the selling price and the buying price. Incorporating the price difference in investment would create investment bursts and asymmetry of the distribution, because the loss from capital sales is more relevant when profitability shocks are below their mean.

On the other hand, our estimation result deviates from that of Cooper and Haltiwanger (2005) in a sense that it gives the negative autocorrelation of investment rates in most of our models. Incorporating the irreversibility and price difference between buying and borrowing capital is expected to provide a result similar to the conclusion of Cooper and Haltiwanger (2005). Changes in the domain of discretized K, the value of $\bar{a}$, and the method of constructing AR(1) process of A would also affect our estimation results, leaving room for our future work.

Nevertheless, our estimation results would provide helpful insights to the effects of investment tax subsidies. Those subsidies enter our model easily through variations in the cost of capital, changing the parameters of opportunity cost of holding capital or directly affecting the price of capital. One of the merits of this structural estimation is that the estimated parameters could be used for policy analysis. Our estimated model could offer predictions of the aggregate effects of an investment tax credit.

5 Conclusion

In this paper, we investigate the nature of adjustment process and structurally estimate key parameters that determine the shape those costs. Using SMM, we attempt to match our models with the pattern of investment rate data reflecting capital adjustment costs in the decision-making of firms. More precisely, the model is specified with convex capital adjustment cost (quadratic form), non-convex capital adjustment cost (fixed cost), and the price of investment. Investment rate data demonstrates a concentration around zero, the inactive stage, and two spikes at the end of the distribution. The convex and non-convex capital adjustment cost is estimated with data simulated from firms optimal investment decision. Our estimation results show that the model incorporating the convex and non-convex adjustment costs fits the data best, leading us to conclude that both of these components are necessary to shape the pattern of investment decisions of firms.

Since only quadratic adjustment costs and capital fixed costs are estimated, our ability to match the data is limited. Several shortcomings of our results are notable, yet absolutely possible to improve them if time permits. We can incorporate the opportunity cost parameter λ without much alteration of the model. In fact, we could try to estimate it together with the fixed cost. The irresponsibility of investment, specified as the ratio between selling and buying price, however, is hard to estimate because firms now need to solve three different bellman equations and to choose the one that gives th highest firm value. In addition, our analysis is also limited by data availability. We can apply the same method to other data, estimate some parameters that are calibrated in our work, and carry out this project with new data moments.

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