Solution 1

HUDM 4125

1.1-5

$(a)P(A) = 1/6.$

$(b)P(B) = 1 - P(A) = 5/6.$

$(c)P(A \cup B) = P(A) + P(B) - P(A \cap B) = 1/6 + 5/6 + 0 = 1.$
$P(A \cap B) = 0$ because event A and event B cannot occur at the same time. They are mutually exclusive.

1.1-7

$P(A) = P(A \cup B) - (1 - P(A \cap B)) = .76 - .13 = .63$
It is easy to see this with a Venndiagram.

1.1-11

$(a)S = \{00, 0, 1, 2, 3....., 34, 35, 36\}.$

$(b)P(A) = \frac{2}{38} = \frac{1}{19}.$

$(c)P(B) = \frac{4}{38} = \frac{2}{19}.$

$(d)P(D) = \frac{18}{38} = \frac{9}{19}.$
There are 18 odd numbers and the possible outcomes are 38.

1.2-3

$(a)26 * 26 * 10 * 10 * 10 * 10 = 6,760,000.$

$(b)26 * 26 * 26 * 10 * 10 * 10 = 17,576,000.$

1.2-5

$(a)4 * 3 * 2 * 1 = 24.$

$(b)4 * 4 * 4 * 4 = 256.$

1.2-12

$(a)0 = (1-1)^n = \sum_r^n \binom{n}{r} 1^{n-r}(-1)^r = \sum_{r=0}^n (-1)^r \binom{n}{r}.$

$(b)2^n = (1+1)^n = \sum_r^n \binom{n}{r} 1^{n-r} 1^r = \sum_{r=0}^n 1^n \binom{n}{r}.$

1.3-1

$(a)P(B_1) = \frac{5000}{1000000} = 0.005.$

$(b)P(A_1) = \frac{78515}{1000000} = 0.078515.$

$(c)P(A_1 \mid B_2) = \frac{P(A_1 \cap B_2)}{P(B_2)} = \frac{0.073630}{0.995} = 0.074.$

$(d)P(B_1 \mid A_1) = \frac{P(B_1 \cap A_1)}{P(A_1)} = \frac{0.004885}{0.078515} = 0.062.$

(e) Part c: the probability that people do not have HIV but tests positive is 0.074 among 1000000 randomly selected individuals;

Part d: that the probability that people, who test positive, have HIV is 0.062 among 1000000 randomly selected individuals.

1.3-9

$(a) P(match on 1st draw) = 1/4 * 3/3 * 2/2 * 1/1$;
$P(match on 2nd draw) = 3/4 * 1/3 * 2/2 * 1/1$; and so on.

$(b) P(A_i \cap A_j) = P(A_i)P(A_j \mid A_i) = \frac{3!}{4!}\frac{2!}{3!} = \frac{2!}{4!}$.

$(c) P(A_i \cap A_j \cap A_k) = P(A_i)P(A_j \mid A_i)P(A_k \mid A_i \cap A_j) = \frac{3!}{4!}\frac{2!}{3!}\frac{1!}{2!} = \frac{1!}{4!}$.

$(d) P(A \cup B \cup C \cup D) = P(A) + P(B) + P(C) + P(D) - P(A \cap B) - P(A \cap C) - P(A \cap D) - P(B \cap C) - P(B \cap D) - P(C \cap D) + P(A \cap B \cap C) + P(A \cap B \cap D) + P(A \cap C \cap D) + P(B \cap C \cap D) - P(A \cap B \cap C \cap D) = 4P(A_i) - 6P(A_i \cap A_j) + 4P(A_i \cap A_j \cap A_k) - P(A_1 \cap A_2 \cap A_3 \cap A_4) = \frac{4*3!}{4!} - \frac{6*2!}{4!} + \frac{4*1!}{4!} - \frac{1}{4!} = 1 - \frac{1}{2} + \frac{1}{3!} - \frac{1}{4!}$.

$(e) In this case, since the probabilities of all events are equal, and probabilities of all intersections are equal, $P(A_1 \cup A_2 \cup \cdots \cup A_n) = \binom{n}{1}P(A_1) + (-1)^1\binom{n}{2}P(A_1 \cap A_2) + (-1)^2\binom{n}{3}P(A_1 \cap A_2 \cap A_3) + \cdots + (-1)^{n-1}\binom{n}{n}P(A_1 \cap A_2 \cap \cdots \cap A_n)$.

(f) The series in parenthesis is the power series expansion of e^x, where X = -1.

1.3-12

$(a) P(B_1) = 1/18, P(B_5) = \frac{17}{18}\frac{16}{17}\frac{15}{16}\frac{14}{15}\frac{1}{14} = \frac{1}{18}$, same for $P(B_{18})$.

$(b)(a) P(B_1) = 2/18, P(B_5) = \frac{16}{18}\frac{15}{17}\frac{14}{16}\frac{13}{15}\frac{2}{14} + \binom{4}{1}\frac{16}{18}\frac{15}{17}\frac{14}{16}\frac{2}{15}\frac{1}{14} = \frac{2}{18}$, same for $P(B_{18})$.

1.4-3

(a) A and B are independent, $P(A \cap B) = P(A)P(B) = \frac{1}{6}$.

(b) A and B' are independent, $P(A \cap B') = P(A)P(B') = \frac{1}{4}(1 - \frac{2}{3}) = \frac{1}{12}$.

(c) A' and B' are independent, $P(A' \cap B') = P(A')P(B') = (1 - \frac{1}{4})(1 - \frac{2}{3}) = 1/4$.

(d) $P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{1}{4} + \frac{2}{3} - \frac{1}{6} = \frac{3}{4}$;
$P[(A \cup B)'] = 1 - P(A \cup B) = 1 - \frac{3}{4} = \frac{1}{4}$.

1.4-4(b)

$P(A' \cap B) = P(B)P(A' \mid B) = P(B)[1 - P(A \mid B)] = P(B)[1 - P(A)] = P(B)P(A')$

1.4-5

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$;
$P(A)P(B) = 0.8 \times 0.5 = 0.4 = P(A \cap B)$.
So, A and B are independent.

1.4-11

(a) A and B are mutually exclusive, $P(A \cap B) = 0$.
$P(A \cap B) = P(A)P(B) = 0.$;
At least one of P(A) or P(B) is zero if they want to be independent events.

(b) $A \subset B, A \cap B = A, P(A \cap B) = P(A)$.
A and B are independent only if P(A) = 0 or P(B) = 1.

1.5-2

(a) P(G) = 0.4 × 0.85 + 0.6 × 0.75 = 0.79.

$(b) P(A \mid G) = \frac{P(A)P(G|A)}{P(G)} = \frac{0.4 \times 0.85}{0.79} = 0.43.$

1.5-5

$P(C \mid D) = \frac{P(C)P(D|C)}{P(D)} = \frac{0.06}{0.095} = 0.632.$

1.5-9

$(a) P(D \mid P) = \frac{P(P)P(P|D)}{P(P)} = \frac{0.0005*0.99}{0.0005*0.99+0.03*0.9995} = 0.016.$

(b) 1 - 0.016 = 0.984