

Solution 2

HUDM 4125

2.1-3

$$\begin{aligned} \text{(a)} \quad \sum_{x=1}^{\infty} r^x &= \frac{r}{1-r}; \\ \sum_{x=1}^{\infty} \frac{1}{4}^x &= \frac{\frac{1}{4}}{1-\frac{1}{4}} = \frac{1}{3}; \\ c &= 3. \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad \sum_{x=0}^{\infty} f(x) &= c \sum_{x=0}^{\infty} \left(\frac{1}{x+1} - \frac{1}{x+2} \right); \\ \text{Notice this is a telescoping series, where } &\left(1 - \frac{1}{2} \right) + \left(\frac{1}{2} - \frac{1}{3} \right) + \dots +, \\ \text{The sum of the series is 1, } c &= 1. \end{aligned}$$

2.1-7

$$\begin{aligned} \text{(a)} \quad P(X=1) &= \frac{11}{36}; \\ P(X=2) &= \frac{9}{36} = \frac{1}{4}; \\ P(X=3) &= \frac{7}{36}; \\ P(X=4) &= \frac{5}{36}; \\ P(X=5) &= \frac{3}{36} = \frac{1}{12}; \\ P(X=6) &= \frac{1}{36}. \\ f(x) &= \frac{13-2x}{36}, x = 1, 2, \dots, 6. \end{aligned}$$

(c)

$$P(Y=0) = \frac{6}{36} = \frac{1}{6};$$

$$P(Y=1) = \frac{10}{36} = \frac{5}{18};$$

$$P(Y=2) = \frac{8}{36} = \frac{2}{9};$$

$$P(Y=3) = \frac{6}{36} = \frac{1}{6};$$

$$P(Y=4) = \frac{4}{36} = \frac{1}{9};$$

$$P(Y=5) = \frac{2}{36} = \frac{1}{18};$$

$$P(Y=y) = \frac{2(6-y)}{36}, \quad \text{for } y = 1, 2, 3, 4, 5,$$

$$P(Y = 0) = \frac{6}{36} = \frac{1}{6}.$$

2.1-10

$$\begin{aligned} \text{(a)} \quad & P(W = 0) = \frac{1}{8}; \\ & P(W = 1) = \frac{1}{8}; \\ & P(W = 2) = \frac{1}{8}; \\ & P(W = 3) = \frac{1}{8}; \\ & P(W = 4) = \frac{1}{8}; \\ & P(W = 5) = \frac{1}{8}; \\ & P(W = 6) = \frac{1}{8}; \\ & P(W = 7) = \frac{1}{8}; \end{aligned}$$

$$f_W(w) = \begin{cases} \frac{1}{8}, & \text{if } w = 0, 1, 2, 3, 4, 5, 6, 7 \\ 0, & \text{otherwise} \end{cases}$$

2.2-2

$$\begin{aligned} & f(-1) = \frac{4}{9}, f(0) = \frac{1}{9}, f(1) = \frac{4}{9}; \\ & E(X) = 0, \\ & E(X^2) = \frac{8}{9}, \\ & E(3X^2 - 2X + 4) = \frac{20}{3}. \end{aligned}$$

2.2-5

$$f(1) = 0.4, f(2) = 0.3, f(3) = 0.2, f(4) = 0.1,$$

$$P(X) = \begin{cases} 200, & \text{if } X = 1 \\ 400, & \text{if } X = 2 \\ 500, & \text{if } X = 3 \\ 600, & \text{if } X = 4 \end{cases}$$

$$E(P(X)) = 200 * 0.4 + 400 * 0.3 + 500 * 0.2 + 600 * 0.1 = 360$$

2.2-12

(a) The average class size is:

$$\frac{(16)(25) + (3)(100) + (1)(300)}{20} = 50$$

(b)

$$f(x) = \begin{cases} 0.4, & \text{if } x = 25 \\ 0.3, & \text{if } x = 100 \\ 0.3, & \text{if } x = 300 \end{cases}$$

(c)

$$E(X) = 25(0.4) + 100(0.3) + 300(0.3) = 130$$

2.3-1

$$\begin{aligned}(d) \quad E(X) &= \frac{7}{3}, \\ E(X^2) &= 6, \\ \text{Var}(X) &= \frac{5}{9}, \\ \gamma_1 &= \frac{E[(X-\mu)^3]}{\sigma^3} = \frac{-7\sqrt{5}}{25}.\end{aligned}$$

$$\begin{aligned}(f) \quad E(X) &= 0, \\ E(X^2) &= \frac{1}{2}, \\ \text{Var}(X) &= \frac{1}{2}, \\ \gamma_1 &= 0.\end{aligned}$$

2.3-8

$$\begin{aligned}E(X) &= \frac{50}{16} = 3.125, \\ E(X^2) &= \frac{170}{16} = 10.625, \\ \text{Var}(X) &= 0.859375, \\ \sigma &= \frac{\sqrt{55}}{8} = 0.926.\end{aligned}$$

2.3-11

$$f(x) = \begin{cases} \frac{2}{5}, & \text{if } x = 1 \\ \frac{1}{5}, & \text{if } x = 2 \\ \frac{2}{5}, & \text{if } x = 3 \\ 0, & \text{otherwise} \end{cases}$$

$$M'(t) = \frac{2}{5}e^t + \frac{1}{5} \cdot 2e^{2t} + \frac{2}{5} \cdot 3e^{3t}$$

$$M''(t) = \frac{2}{5}e^t + \frac{4}{5}e^{2t} + \frac{18}{5}e^{3t}$$

$$\begin{aligned}M'(0) &= E[X] = 2, \\ M''(0) &= \text{Var}(X) + E[X]^2 = \frac{24}{5} \\ \text{Var}(X) &= 4/5 = 0.8.\end{aligned}$$

2.3-17

(a) The pmf is:

$$f(x) = \frac{(x-1)}{2^x}, \quad x = 2, 3, \dots$$

(c) The mean μ and variance σ^2 are:

$$\mu = 4, \quad \sigma^2 = 4$$

(d) The probabilities are:

- (i) $P(X \leq 3) = \frac{1}{2}$
- (ii) $P(X \geq 5) = \frac{5}{16}$
- (iii) $P(X = 3) = \frac{1}{4}$

2.4-1

$$f(x) = \frac{7}{18} \left(\frac{x}{18}\right)^{11} \left(\frac{11}{18}\right)^{1-x}, x = 0, 1;$$

$$\mu = \frac{7}{18};$$

$$\sigma^2 = \frac{77}{324}.$$

2.4-3

$$(a) \quad \frac{1}{5} \left(\frac{2}{5}\right)^2 \left(\frac{4}{5}\right)^4 = 0.0164;$$

$$(b) \quad \frac{6!}{2!4!} \frac{1}{5} \left(\frac{2}{5}\right)^2 \left(\frac{4}{5}\right)^4 = 0.2458.$$

2.4-9

$$(a) \quad b(20, 0.80);$$

$$(b) \quad \mu = 16;$$

$$\sigma^2 = 3.2;$$

$$\sigma = 1.789.$$

$$(c) \quad (i) \quad P(X = 15) = 0.1746,$$

$$(ii) \quad P(X > 15) = 0.6296,$$

$$(iii) \quad P(X \leq 15) = 0.3704.$$

2.4-19

$$(a) \quad b(1, 2/3), \mu = 2/3, \sigma^2 = 2/9, \sigma = \sqrt{2}/3;$$

$$(b) \quad b(12, 0.75), \mu = 9, \sigma^2 = 2.25, \sigma = 1.5.$$