# FIT1045 Algorithms and programming in Python, S1-2019 Assignment 1 (value 12%)

Due: Friday 12th Apr 2019, 11:55 pm.

# **Objectives**

The objectives of this assignment are:

- To demonstrate the ability to implement algorithms using basic data structures and operations on them.
- To gain experience in designing an algorithm for a given problem description and implementing that algorithm in Python.

## **Submission Procedure**

- 1. Put you name and student ID on each page of your solution.
- 2. Save your files into a zip file called yourFirstName\_yourLastName.zip
- 3. Submit your zip file containing your solution to Moodle.
- 4. Your assignment will not be accepted unless it is a readable zip file.

Important Note: Please ensure that you have read and understood the university's policies on plagiarism and collusion available at http://www.monash.edu.au/students/policies/academic-integrity.html. You will be required to agree to these policies when you submit your assignment.

Marks: This assignment will be marked both by the correctness of your code and by an interview with your lab demonstrator, to assess your understanding. This means that although the quality of your code (commenting, decomposition, good variable names etc.) will not be marked directly, it will help to write clean code so that it is easier for you to understand and explain.

This assignment has a total of 30 marks and contributes to 12% of your final mark. Late submission will have 10% off the total assignment marks per day (including weekends) deducted from your assignment mark. (In the case of Assignment 1, this means that a late assignment will lose 3 marks for each day (including weekends)). Assignments submitted 7 days after the due date will normally not be accepted.

Detailed marking guides can be found at the end of each task. Marks are subtracted when you are unable to explain your code via a code walk-through in the assessment interview. Readable code is the basis of a convincing code walk-through.

# Task 1: Race to Pi (12 Marks)

### Background

The ratio of a circle's circumference to its diameter, usually denoted by the Greek letter  $\pi$ , is an extremely important constant in mathematical computations. Unfortunately, the exact value of  $\pi$  cannot be stored explicitly in a computer, because it is *irrational*, i.e., it does not correspond to any ratio between two integer values. To help us out, there are many different ways to *approximate*  $\pi$ , most of which based on computing a finite prefix of an infinite sum or product. For instance:

- 1. via a solution to the **basel** problem:  $\frac{\pi^2}{6} = \frac{1}{1} + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \dots$
- 2. derived from the **taylor** expansion of 1/(1+x):  $\frac{\pi}{4} = \frac{1}{1} \frac{1}{3} + \frac{1}{5} \frac{1}{7} + \dots$
- 3. via the wallis algorithm:  $\frac{\pi}{2} = \left(\frac{2\times2}{1\times3}\right) \times \left(\frac{4\times4}{3\times5}\right) \times \left(\frac{6\times6}{5\times7}\right) \times \left(\frac{8\times8}{7\times9}\right) \times \dots$
- 4. via the **spigot** algorithm:  $\frac{\pi}{2} = \left(\frac{1}{1}\right) + \left(\frac{1}{1} \times \frac{1}{3}\right) + \left(\frac{1}{1} \times \frac{1}{3} \times \frac{2}{5}\right) + \left(\frac{1}{1} \times \frac{1}{3} \times \frac{2}{5} \times \frac{3}{7}\right) + \dots$

Each of these sums (or the product in case of 3.) converges to the respective value on the left-hand-side of the equals sign. That is for every desired precision  $\epsilon > 0$ , the absolute difference between the left-hand-side and the sum/product on the right-hand-side is guaranteed to be less than  $\epsilon$  if a sufficient number of terms is computed. Thus, they are all valid ways to compute  $\pi$ . However, the speed of convergence, i.e., the number of terms that have to be computed to reach a desired precision is different for each method. (Note that in case of wallis and spigot every expression in parenthesis is considered one term; e.g., the first term of the wallis product equals 4/3, the third term of the spigot sum equals 2/15.) In this task, we will determine which of the above methods is preferable from this perspective.

#### Instructions

Create a Python module called pi.py. Within that module:

a) Write four different functions for approximating  $\pi$  corresponding to the four different approaches listed above (the functions must be called basel, taylor, wallis, and spigot). Each of these functions must obey the following input/output specification:

**Input**: a float parameter **precision** which specifies the precision to which  $\pi$  should be calculated, i.e., the absolute difference of the approximation to the Python constant **pi**.

**Output**: two values (x,n) such that x is the first approximation to  $\pi$  with an absolute difference from pi of at most precision, and n is the number of terms of the sum/product that were computed in the process.

For example, if the input was 0.1, then the function should continue to compute terms of the series until the approximation x to  $\pi$  is less than 0.1 away from the Python constant pi (see more detailed examples below).

b) Write another function called race(precision, algorithms) that can be used to compare the convergence speed of different approximation algorithms via the following input/output specification:

Input: a float parameter precision and a list of Python functions algorithms.

Output: a list of integer pairs [(i1, n1), (i2, n2),..., (ik,nk)] such that

- for each pair (i,n) in the list, n is the number of steps that the i-th algorithm takes to approximate  $\pi$  within the given precision
- the list is in *ascending order* with respect to the number of steps (the second number of the pair) with ties resolved arbitrarily.

Note that the input to race are functions and not strings. That is, in principle race can be used for evaluating arbitrary functions for approximating  $\pi$ , not only the four covered here.

c) Write a function print\_results which takes as input the output of the function race and prints the results in a human readable format. Each line of the output should be "Algorithm i finished in n steps" where i is first element of the integer pair, and n is the second.

The only two import statements allowed in your module are from math import pi and from math import sqrt. Note that the subtasks can be solved independently and the given order (a, b, c) is only a recommendation.

### Examples

a) Calling wallis (0.2) returns (2.972154195011337, 4) as the result of approximating  $\pi$  with

$$2.972154195011337 = 2\underbrace{\left(\frac{2\times2}{1\times3}\times\frac{4\times4}{3\times5}\times\frac{6\times6}{5\times7}\times\frac{8\times8}{7\times9}\right)}_{\text{4 terms}}$$

using 4 terms because pi - 2.972154195011337 < 0.2 and non of the three prior wallis approximations satisfies this condition.

b) Calling basel (0.1) returns (3.04936163598207, 10) as the result of approximating  $\pi$  with

$$3.04936163598207 = \sqrt{6\left(\frac{1}{1} + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \dots + \frac{1}{100}\right)}$$

using 10 terms because pi - 3.04936163598207 < 0.1 and non of the nine prior basel approximations satisfies this condition.

c) Calling taylor (0.2) returns (3.3396825396825403, 5) as the result of approximating  $\pi$  with

$$3.3396825396825403 = 4\underbrace{\left(\frac{1}{1} - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9}\right)}_{5 \text{ terms}}$$

using 5 terms because pi - 3.3396825396825403 < 0.2 and non of the four prior taylor approximations satisfies this condition.

d) Calling spigot (0.1) returns (3.0476190476190474, 4) as the result of approximating  $\pi$  with

$$3.0476190476190474 = 2\underbrace{\left(\frac{1}{1} + \left(\frac{1}{1} \times \frac{1}{3}\right) + \left(\frac{1}{1} \times \frac{1}{3} \times \frac{2}{5}\right) + \left(\frac{1}{1} \times \frac{1}{3} \times \frac{2}{5} \times \frac{3}{7}\right)\right)}_{A \text{ towns}}$$

using 4 terms because pi - 3.0476190476190474 < 0.1 and non of the prior three spigot approximations satisfies this condition.

- e) Calling race(0.01, [taylor, wallis, basel]) returns [(2,78),(3,96),(1,100)] because functions taylor, basel, and wallis need 100, 96, and 78 terms, respectively, to approximate  $\pi$  within the required precision of 0.01.
- f) Calling print\_results([(2,78),(3,96),(1,100)]) prints

Algorithm 2 finished in 78 steps

Algorithm 3 finished in 96 steps

Algorithm 1 finished in 100 steps

#### Marking Guide (total 12 marks)

Marks are given for the correct behaviour of the different functions:

- (a) 2 marks each for basel, taylor, wallis, and spigot
- (b) 3 marks for race
- (c) 1 mark for print\_results

All functions are assessed independently. For instance, even if function race does not always produce the correct output, function print\_results can still be marked as correct.

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# Task 2: Reversi (18 Marks)

#### Background

Reversi is a game for two players (Black and White), which is played on a squared board of 8 rows/columns with stones that are white on one side and black on the other. In the beginning of the game, stones of Player 1 (B) and Player 2 (W) are placed in a cross arrangement at the center of the board as shown in Table ??. Then the players take turns placing stones on empty fields on the board with their colour facing up according to the following rules:

- 1. All opponent's stones that are enclosed in a straight line (horizontal, vertical, or diagonal) between the newly placed stone and another stone of the same color are flipped (see Table ??).
- 2. A move is only valid if it turns at least one opponent's stone.
- 3. If a player has no valid moves, their turn is skipped (see example in Table ??).
- 4. The game ends when both players have no legal moves.

When the game ends, each player scores points equal to the number of stones which have their colour face up, and the player with a higher score wins (draw if scores equal). In this task you will create a Python program that allows one or two players to play a game of Reversi.

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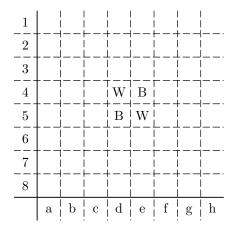


Table 1: Initial board state at the beginning of game. Player 1 (B) has the first move.

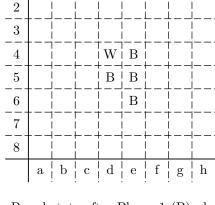


Table 2: Board state after Player 1 (B) played "e6" as her first move.

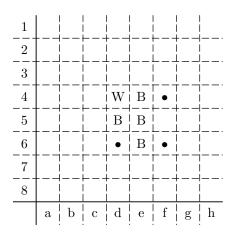


Table 3: Valid moves (indicated by •) for Player 2 (W) following "e6" from Player 1.

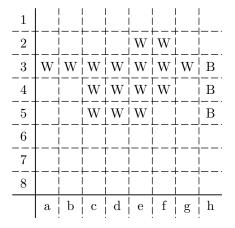


Table 4: Board state with no valid move for Player 2 (W), who, in case it is her turn, has to pass back to Player 1 (B), whose only valid move is "e1".

## Instructions

Create a python module reversi.py. The only import statement allowed in your module is import copy. Your module must at least contain the nine functions described in the subtasks below, but you are allowed, and in fact, encouraged, to implement additional functions as you deem appropriate.

- a) Create three functions new\_board(), print\_board(board), and score(board) that represent the basic elements of Reversi as follows. The function new\_board() takes no parameters and returns a fresh board configuration representing the Reversi starting position. The other two functions each take as input a board configuration. The function score(board) returns as output a pair of integers (s1, s2) where s1 represents the number of stones of Player 1 in the given board configuration and s2 represents the number of stones of Player 2. The function print\_board(board) has no return value but prints the given board configuration in human-readable form to the console (in particular, all rows of the board must be printed in separate lines of the console and labelled by their names 1, 2, ... and all columns must be properly aligned and labelled by their names a, b, ...— similar to the illustrations above). For these and all other functions of this task, a board configuration must be represented as a list of lists in the following way:
  - A board configuration is a list of length 8, the elements of which represent the rows of the board from top to bottom: the first element represents the topmost row of the board, row 1, the second represents row 2, etc..
  - Each row, i.e., each element of a list representing a board configuration, is also a list of length 8. These inner lists represent the rows of the board from left to right: the first element represents the field in the leftmost column, Column A, the second element represents the field in second column, Column B, etc.
  - Each field, i.e., each element of the inner lists representing the rows, is an integer from the set  $\{0,1,2\}$  representing the state of that field: 0 represents an empty square, 1 represents a square with a Black stone, and 2 represents a square with a White stone.

See example ?? for an illustration of this representation of a board configuration.

- b) Create three functions enclosing(board, player, pos, direct), valid\_moves(board, player), and next\_state(board, player, pos) that represent the rules of Reversi according to the following specifications.
  - A board **position** is represented by a pair (r, c) where r and c are each integers from the range range(8) representing a row index and a column index, respectively. For instance, the position "b3" is represented by (2, 1). The row and column order of the "b3" style and the specification of **position** is reversed. This is because it is natural to constuct lists of lists in such a way that the outer lists correspond to rows, and the inner list positions correspond to column positions, so we reference them with row first, then column. This row, column convention is quite common across maths and computer science.
  - A direction is represented by a pair (dr, dc) where dr and dc are each integers from the set {-1, 0, 1}. For instance, "horizontal to the left" is described by (0, -1), "diagonal to the right and down" is described by (1, 1), and so on).
  - The function enclosing(board, player, pos, dir) represents whether putting a player's stone on a given position would enclose a straight line of opponent's stones in a given direction:
    - Input: A board configuration board, an integer player from the set {1, 2}, a board position pos and a direction dir.
    - Output: True if board contains a stone of player in direction dir from position pos and all positions on the straight line between that stone and pos contain stones of the other player, and there is at least one stone belonging to the other player on the straight line; False otherwise.
  - The function valid\_moves(board, player) has:
    - Input: A board configuration board as well as an integer player from the set {1, 2}.
    - Output: A list of board positions (represented as pairs of integers as described above) [(r1, c1), (r2, c2), ..., (rk, ck)] such that the elements of that list correspond to all valid positions that Player player is allowed to drop a stone on in the given input configuration board (in particular, the list is empty if the player has not valid moves in the given board).
  - The function next\_state(board, player, pos) has:
    - Input: A board configuration board as well as an integer player from the set {1, 2} and a board position representing the move of Player player to place one of their stones on the board configuration board on the field described by pos.
    - Output: A pair (next\_board, next\_player) such that next\_board is the board configuration resulting from Player placing a stone on position pos on the input board configuration board, and next\_player is an integer from the set {0, 1, 2} corresponding to the player who gets to move next or 0 in case the game ends after the input move is carried out.

- c) Create functions position(string), run\_two\_players() and run\_single\_player() that put all of the above together and that allow one or two players to play a game of Reversi through the console as follows.
  - The function position(string) accepts as input any string string (e.g., "b5" or "Genghis Khan") and has as output the board position (r, c) described by the string or None in case the string does not correspond to a valid board position.
  - Running run\_two\_players() should repeatedly do the following until the user selects to stop:
    - 1: Print the current board state
    - 2: Ask the player whose turn it is to input a position to drop a stone (in the format "d4"). The player can also choose to quit by entering "q"
    - 3: Your program should then respond in one of the following ways:
      - If the move is not valid, print "invalid move"
      - If the move is valid, place the move on the board.
    - 4: If the game ended (no valid move remaining for either player), print the game result and exit
    - 5: If the game did not end, the current player's turn is over. It is now the other player's turn
  - Running run\_single\_player() should behave identical to run\_two\_players() except that all moves of Player 2 are carried out automatically such that the resulting board configuration has the *maximal score* for Player 2 (among all their valid moves).

#### Examples

- a) Calling score(new\_board()) returns (2, 2) because the score in the starting configuration is 2 for Player 1 (B) and 2 for player 2 (W).
- b) Calling enclosing(new\_board(), 1, (4, 5), (0, -1)) returns True because Player 1 can enclose one opponent's stone to the left when placing a stone on field "f5". In contrast, there is no straight line of opponent's stones to enclose on the diagonal line to the bottom right. Therefore, enclosing(new\_board(), 1, (4, 5), (1, 1)) returns False.
- c) Calling next\_state(new\_board(), 1, (4, 5)) will return (next\_board, 2) where next\_board is the board configuration

```
[[0,0,0,0,0,0,0,0],
[0,0,0,0,0,0,0,0],
[0,0,0,0,0,0,0],
[0,0,0,2,1,0,0,0],
[0,0,0,1,1,1,0,0],
[0,0,0,0,0,0,0,0],
[0,0,0,0,0,0,0,0],
```

because that is the configuration resulting from Player 1 putting a stone on field "f5" (encoded by integer pair (4, 5)) in the starting configuration, and Player 2 has the next move afterwards.

- d) Calling valid\_moves(next\_state(new\_board(), 1, (4, 5))[0], 2) returns the list [(3, 5), (5, 3), (5, 5)] or any of its permutations (list with same elements but different order) because the valid moves of Player 2 after Player 1 played "f5" in Turn 1 are "d6", "f4", and "f6".
- e) Calling position("e3") returns the pair (2, 4) because that is the internal representation of board position "e3". Calling position("l1"), position("a0"), or position("Genghis Khan") all return None because all these input do not correspond to valid board positions.

## Marking Guide (total 18 marks)

Marks are given for the correct behaviour of the different functions:

- a) 1 mark each for new\_board, print\_board, and score
- b) 4 marks for enclosing, 3 marks for valid\_moves, and 2 marks next\_state
- c) 1 mark for position, 2 marks for run\_two\_players and 3 marks for run\_single\_player

All functions are assessed independently to the degree possible. For instance, even if function valid\_moves does not always produce the correct output, function run\_two\_players can still be marked as correct.